



Victorian Certificate of Education 2011

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures

Words

Letter

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SPECIALIST MATHEMATICS

Written examination 1

Thursday 10 November 2011

Reading time: 9.00 am to 9.15 am (15 minutes)

Writing time: 9.15 am to 10.15 am (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
11	11	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 9 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

Find an antiderivative of $\frac{1+x}{9-x^2}$, $x \in \mathbb{R} \setminus \{-3, 3\}$.

3 marks

Question 2

Find the value of the real constant k given that kxe^{2x} is a solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2x}(15x + 6).$$

3 marks

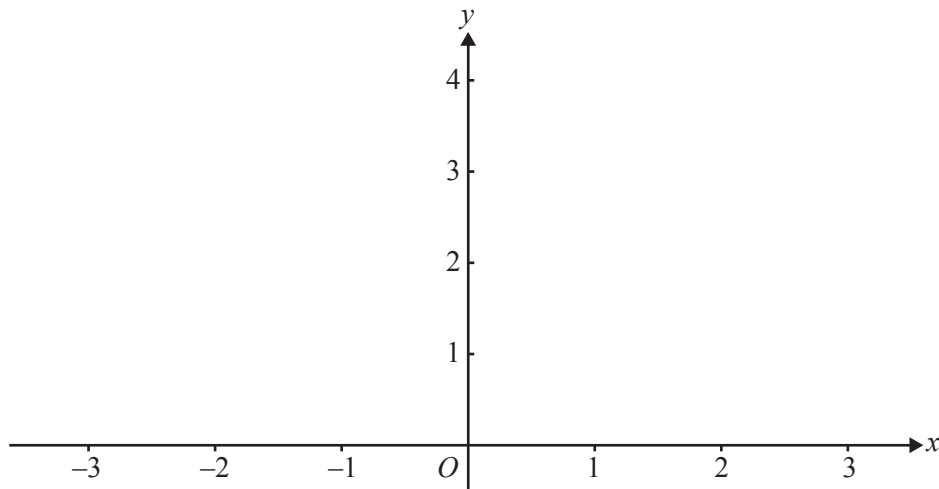
TURN OVER

Question 3

- a. Show that $f(x) = \frac{2x^2 + 3}{x^2 + 1}$ can be written in the form $f(x) = 2 + \frac{1}{x^2 + 1}$.

1 mark

- b. Sketch the graph of the relation $y = \frac{2x^2 + 3}{x^2 + 1}$ on the axes below.
Label any asymptotes with their equations and label any intercepts with the axes, writing them as coordinates.



3 marks

- c. Find the area enclosed by the graph of the relation $y = \frac{2x^2 + 3}{x^2 + 1}$, the x -axis, and the lines $x = -1$ and $x = 1$.

3 marks

Question 4

Consider $z = \frac{1 - \sqrt{3}i}{-1 + i}$, $z \in \mathbb{C}$.

Find the principal argument of z in the form $k\pi$, $k \in \mathbb{R}$.

3 marks

Question 5

For the curve with parametric equations

$$\begin{aligned} x &= 4 \sin(t) - 1 \\ y &= 2 \cos(t) + 3 \end{aligned}$$

find $\frac{dy}{dx}$ at the point $(1, \sqrt{3} + 3)$.

3 marks

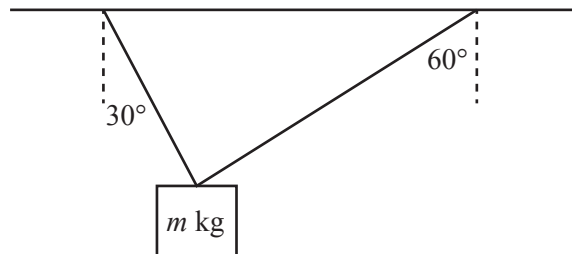
Question 6

Evaluate $\int_0^1 e^x \cos(e^x) dx$.

2 marks

Question 7

A flowerpot of mass m kg is held in equilibrium by two light ropes, both of which are connected to a ceiling. The first rope makes an angle of 30° to the vertical and has tension T_1 newtons. The second makes an angle of 60° to the vertical and has tension T_2 newtons.



- a. Show that $T_2 = \frac{T_1}{\sqrt{3}}$.

1 mark

- b. The first rope is strong, but the second rope will break if the tension in it exceeds 98 newtons. Find the maximum value of m for which the flowerpot will remain in equilibrium.

3 marks

Question 8

Find the **coordinates** of the points of intersection of the graph of the relation

$$y = \operatorname{cosec}^2\left(\frac{\pi x}{6}\right) \text{ with the line } y = \frac{4}{3}, \text{ for } 0 < x < 12.$$

3 marks

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Question 9

Consider the three vectors

$$\underline{\underline{a}} = \underline{\underline{i}} - \underline{\underline{j}} + 2\underline{\underline{k}}, \quad \underline{\underline{b}} = \underline{\underline{i}} + 2\underline{\underline{j}} + m\underline{\underline{k}} \quad \text{and} \quad \underline{\underline{c}} = \underline{\underline{i}} + \underline{\underline{j}} - \underline{\underline{k}}, \quad \text{where } m \in \mathbb{R}.$$

- a. Find the value(s) of m for which $|\underline{\underline{b}}| = 2\sqrt{3}$.

2 marks

- b. Find the value of m such that $\underline{\underline{a}}$ is perpendicular to $\underline{\underline{b}}$.

1 mark

- c. i. Calculate $3\underline{\underline{c}} - \underline{\underline{a}}$.

- ii. **Hence** find a value of m such that $\underline{\underline{a}}$, $\underline{\underline{b}}$ and $\underline{\underline{c}}$ are **linearly dependent**.

1 + 1 = 2 marks

Question 10

Consider the relation $y \log_e(x) = e^{2y} + 3x - 4$.

Evaluate $\frac{dy}{dx}$ at the point $(1, 0)$.

4 marks

Question 11

The region in the first quadrant enclosed by the curve $y = \sin(x)$, the line $y = 0$ and the line $x = \frac{\pi}{6}$ is rotated about the x -axis.

Find the volume of the resulting solid of revolution.

3 marks

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc \sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$

$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$

$\sin(2x) = 2 \sin(x) \cos(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x-y) = \sin(x) \cos(y) - \cos(x) \sin(y)$

$\cos(x-y) = \cos(x) \cos(y) + \sin(x) \sin(y)$

$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$

$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \quad (\text{de Moivre's theorem})$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), \quad x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration:} \quad v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

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Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$