

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 2



2011 Trial Examination

SOLUTIONS

SECTION 1: Multiple-choice questions (1 mark each)

Question 1

Answer : A

Explanation :

$$\text{centre} = (-1, 1) \quad , \quad a = (-1) - (-3) = 2$$

$$\frac{b}{a} = 2 \quad \therefore \quad b = 4 \quad \therefore \quad \frac{(x+1)^2}{4} - \frac{(y-1)^2}{16} = 1$$

Question 2

Answer : B

Explanation :

$$y = \sin(2 \cos^{-1}(2x))$$

$$-\frac{\pi}{2} \leq 2 \cos^{-1}(2x) \leq \frac{\pi}{2} \quad \therefore \quad -\frac{\pi}{4} \leq \cos^{-1}(2x) \leq \frac{\pi}{4}$$

$$\text{But } 0 \leq \cos^{-1}(2x) \leq \pi \quad \therefore \quad 0 \leq \cos^{-1}(2x) \leq \frac{\pi}{4} \quad \therefore \quad \frac{\sqrt{2}}{2} \leq 2x \leq 1$$

$$\therefore \quad \frac{\sqrt{2}}{4} \leq x \leq \frac{1}{2} \quad \therefore \quad \text{dom} = \left[\frac{\sqrt{2}}{4}, \frac{1}{2} \right]$$

Question 3*Answer : B**Explanation :*

$$y = \sec(a(b-x)) = \sec(a(x-b))$$

dilation of factor 2 from the y axis, hence $a = \frac{1}{2}$

translation $\frac{\pi}{2}$ to the right, hence $b = \frac{\pi}{2}$

Question 4*Answer : B**Explanation :*

The rays $\text{Arg}(z-i) = \frac{\pi}{4}$ and $\text{Arg}(z-2) = \frac{\pi}{2}$ intersect at $z = 2 + 3i$

$$\text{Arg}(z+1) = \text{Arg}(3+3i) = \frac{\pi}{4}$$

Question 5*Answer : D**Explanation :*

$$1+i, \quad 1-i \quad \text{and} \quad 2$$

$$Q(z) = P(i\bar{z})$$

$$i\bar{z} = 1+i, \quad 1-i \quad \text{and} \quad 2$$

$$\bar{z} = \frac{1+i}{i}, \quad \frac{1-i}{i} \quad \text{and} \quad \frac{2}{i}$$

$$\bar{z} = 1-i, \quad -1-i \quad \text{and} \quad -2i$$

$$z = 1+i, \quad -1+i \quad \text{and} \quad 2i$$

Question 6*Explanation :*

$$w = \frac{2\bar{z}^2(1+i)}{z^2}, \quad \text{Arg}(\bar{z}) = \frac{\pi}{16} \quad \therefore \quad \text{Arg}(z) = -\frac{\pi}{16}$$

$$\text{Arg}(w) = 2 \times \frac{\pi}{16} + \frac{\pi}{4} - 2 \times \left(-\frac{\pi}{16}\right) = \frac{\pi}{8} + \frac{\pi}{4} + \frac{\pi}{8} = \frac{\pi}{2}$$

Question 7*Answer : D**Explanation :*

$$T = \{z : |z+1| \geq |z-i|, z \in C\} \cap \{z : |z-2| \leq |z-i|, z \in C\} \cap \{z : |z| \leq 1, z \in C\}$$

$|z+1| \geq |z-i|$ is the half plane above the left line

$|z-2| \leq |z-i|$ is the half plane below the right line

$|z| \leq 1$ is the interior of the circle

Question 8*Answer : A**Explanation :*

When $\underline{a}$, $\underline{b}$ and $\underline{c}$ are linearly dependent, one of them, for example $\underline{c}$, is a linear combination of the other two.

Therefore $\underline{c}$ is in the plane determined by $\underline{a}$ and $\underline{b}$.

Question 9*Answer : C**Explanation :*

$$\underline{b} \cdot \underline{c} = |\underline{b}|^2 + \underline{a} \cdot \underline{b} \quad \therefore \quad \underline{b} \cdot \underline{c} - \underline{a} \cdot \underline{b} = |\underline{b}|^2 \quad \therefore \quad \underline{b} \cdot (\underline{c} - \underline{a}) = |\underline{b}|^2 \quad \text{true}$$

$$\frac{\underline{a} \cdot \underline{c}}{|\underline{c}|^2} \underline{c} + \frac{\underline{b} \cdot \underline{c}}{|\underline{c}|^2} \underline{c} = \underline{c} \quad \therefore \quad \frac{\underline{a} \cdot \underline{c} + \underline{b} \cdot \underline{c}}{|\underline{c}|^2} \underline{c} = \underline{c} \quad \text{true}$$

$$\underline{a} - \frac{\underline{a} \cdot \underline{c}}{|\underline{c}|^2} \underline{c} = \left(\underline{b} - \frac{\underline{b} \cdot \underline{c}}{|\underline{c}|^2} \underline{c} \right) \quad \text{false}$$

the vector resolutes of $\underline{a}$ and $\underline{b}$ in the direction $\perp$ to $\underline{c}$ are opposite not equal.

$$\frac{\underline{c} \cdot \underline{a}}{|\underline{c}|^2} \underline{c} + \frac{\underline{c} \cdot \underline{b}}{|\underline{c}|^2} \underline{c} = \underline{c} \quad \therefore \quad \frac{\underline{c} \cdot (\underline{a} + \underline{b})}{|\underline{c}|^2} \underline{c} = \underline{c} \quad \therefore \quad \frac{\underline{c} \cdot \underline{c}}{|\underline{c}|^2} \underline{c} = \underline{c} \quad \text{true}$$

$$\underline{c} \cdot \underline{a} = |\underline{c}|^2 - \underline{b} \cdot \underline{c} \quad \therefore \quad \underline{c} \cdot (\underline{a} + \underline{b}) = |\underline{c}|^2 \quad \text{true}$$

Question 10*Answer : B**Explanation :*

$$\underline{a} = \underline{i} + \sqrt{3}\underline{j} \quad , \quad \underline{b} = 3\underline{k}$$

$$\hat{\underline{a}} = \frac{1}{2}(\underline{i} + \sqrt{3}\underline{j}) \quad , \quad \hat{\underline{b}} = \underline{k}$$

$$\hat{\underline{a}} + \hat{\underline{b}} = \frac{1}{2}(\underline{i} + \sqrt{3}\underline{j}) + \underline{k}$$

$$|\hat{\underline{a}} + \hat{\underline{b}}| = \sqrt{2}$$

Hence the required vector is $\frac{1}{2\sqrt{2}}(\underline{i} + \sqrt{3}\underline{j}) + \frac{1}{\sqrt{2}}\underline{k}$

Question 11*Answer : B**Explanation :*

$$\frac{1 - \sin(2\theta)}{\cos(2\theta)} = \tan\left(\frac{\pi}{4} - \theta\right) \quad \text{Change } \theta \text{ to } \theta - \frac{\pi}{4}$$

$$\frac{1 - \sin\left(2\theta - \frac{\pi}{2}\right)}{\cos\left(2\theta - \frac{\pi}{2}\right)} = \tan\left(\frac{\pi}{4} - \theta + \frac{\pi}{4}\right) \quad \therefore \frac{1 + \cos(2\theta)}{\sin(2\theta)} = \cot(\theta)$$

Question 12*Answer : E**Explanation :*

$$\underline{r}(t) = \tan(t)\underline{i} + \sec(t)\underline{j}$$

$$x = \tan(t) \quad y = \sec(t)$$

$$\sec^2(t) - \tan^2(t) = 1 \quad \therefore y^2 - x^2 = 1 \quad \therefore \text{hyperbola}$$

Question 13*Answer : D**Explanation :*

$$v^2 = -4x$$

$$a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left(\frac{1}{2} \times (-4x)\right) = -2 \quad \therefore a = \text{const}$$

Initially $v = 8$. Since $a < 0$ v decreases

Question 14*Answer : D**Explanation :*

$$\int_0^1 x^3 e^{x^2} dx \quad u = x^2 \quad \therefore \quad du = 2x dx$$

$$\int_0^1 x^3 e^{x^2} dx = \int_0^1 x^2 x e^{x^2} dx = \int_0^1 u e^u \frac{du}{2} = \frac{1}{2} \int_0^1 u e^u du$$

Question 15*Answer : A**Explanation :*

$$\underline{r}(t) = -\sin(2\pi t)\underline{i} + \cos(\pi t)\underline{j} \quad \therefore \quad \underline{v}(t) = -2\pi \cos(2\pi t)\underline{i} - \pi \sin(\pi t)\underline{j}$$

$$\underline{v}\left(\frac{3}{2}\right) = -2\pi \cos\left(2\pi \times \frac{3}{2}\right)\underline{i} - \pi \sin\left(\pi \times \frac{3}{2}\right)\underline{j}$$

$$\underline{v}\left(\frac{3}{2}\right) = 2\pi \underline{i} + \pi \underline{j} \quad \therefore \quad \left| \underline{v}\left(\frac{3}{2}\right) \right| = \pi\sqrt{5}$$

Question 16*Answer : D**Explanation :*

For $y = \text{const}$ the gradient follows a sin or cos shape

when $x = y$, gradient is zero, hence sin shape

when $x = 1$ and $y = 0$, gradient is positive,

hence $\sin(x - y)$ applies

Question 17*Answer : B**Explanation :*

$$d = \frac{1}{2}at^2 \quad , \quad t=1 \quad \therefore \quad d = \frac{1}{2}a$$

$$d_n = \frac{1}{2}an^2 - \frac{1}{2}a(n-1)^2 = \frac{1}{2}a(n^2 - (n-1)^2) \quad \therefore \quad d_n = d(2n-1)$$

Question 18*Answer : C**Explanation :*

$$mg - mkv = ma \quad \therefore \quad a = g - kv$$

$$\frac{g}{2} = g - kv \quad \therefore \quad v = \frac{g}{2k}$$

$$\frac{dv}{dt} = g - kv \quad \therefore \quad t = \int_0^{\frac{g}{2k}} \frac{dv}{g - kv} = \frac{1}{k} \log_e 2$$

Question 19*Answer : E**Explanation :*

$$\left. \begin{array}{l} 2mg - T = 2ma \\ T - mg = ma \end{array} \right\}$$

$$\Rightarrow mg = 3ma \quad \therefore \quad a = \frac{g}{3}$$

$$T - mg = m\frac{g}{3} \quad \therefore \quad T = \frac{4}{3}mg$$

Question 20*Answer : C**Explanation :*

$$\left. \begin{array}{l} F - \mu(m+M)g - \mu mg = Ma \\ \mu mg = ma \end{array} \right\} \Rightarrow F - \mu(m+M)g - \mu mg = \mu mg \quad \therefore F = 2\mu(M+m)g$$

Question 21*Answer : A**Explanation :*

$$Q^2 = R^2 + P^2$$

$$\text{if } \theta = 45^\circ \text{ then } R = P = \frac{Q}{\sqrt{2}}$$

Question 22*Answer : A**Explanation :*

$$p = m\dot{y} \quad , \quad \dot{p} = \frac{dp}{dt} = \frac{d}{dt}(m\dot{y}) \quad \therefore \quad \dot{p} = m\frac{d\dot{y}}{dt} + \dot{y}\frac{dm}{dt}$$

$$\dot{a} = \frac{d\dot{y}}{dt} \quad \therefore \quad \dot{p} = m\dot{a} + \dot{y}\frac{dm}{dt}$$

SECTION 2

Question 1

a.

$$\left. \begin{aligned} z + \frac{1}{z} &= -2\sin\theta, \quad -\frac{\pi}{4} \leq \theta \leq 0 \quad \therefore \quad z^2 + 2z\sin\theta + 1 = 0 \\ z &= -\sin\theta \pm \sqrt{\sin^2\theta - 1} \quad \therefore \quad z = -\sin\theta \pm i\cos\theta \end{aligned} \right\} \dots\dots[M1]$$

$$\left. \begin{aligned} z &= -\sin\theta + i\cos\theta \quad \text{or} \quad z = -\sin\theta - i\cos\theta \\ \text{But } \cos\left(\frac{\pi}{2} + \theta\right) &= -\sin\theta \quad \text{and} \quad \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta \end{aligned} \right\} \dots\dots[M1]$$

$$\therefore u = \text{cis}\left(\frac{\pi}{2} + \theta\right) \quad \text{as} \quad \frac{\pi}{2} + \theta > 0 \quad \therefore w = \bar{u} \quad \dots\dots[M1]$$

$$\therefore u = \text{cis}\left(\frac{\pi}{2} + \theta\right) \quad \text{and} \quad w = \text{cis}\left(-\frac{\pi}{2} - \theta\right) \quad \text{as required} \quad \dots\dots[A1]$$

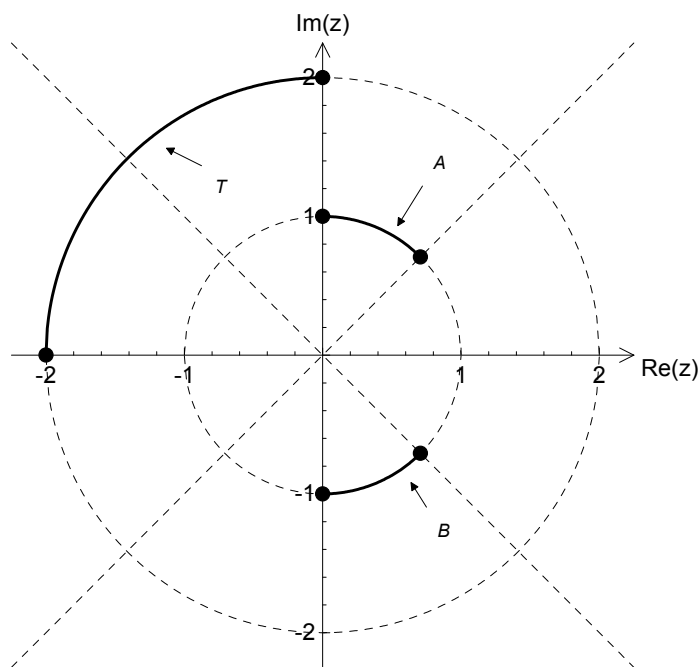
b.

See the diagram to the left

region A[A1]

region B[A1]

region T[A1]



c.

$$|z_1 - z_2|_{\min} = |2i - i| = 1 \quad \dots\dots[A1]$$

$$|z_1 - z_2|_{\max} = \left| \cos\left(\frac{\pi}{4}\right) + 2 \right| = \sqrt{\left(\cos\left(\frac{\pi}{4}\right) + 2\right)^2 + \sin^2\left(\frac{\pi}{4}\right)} = \sqrt{2\sqrt{2} + 5} \quad \dots\dots[A1]$$

d.

i.

$$u^n - w^n = u^n - (\bar{u})^n = 2i \operatorname{Im}(u^n) \quad \dots\dots[M1]$$

$$u^n - w^n = 2i \sin\left(n\left(\frac{\pi}{2} + \theta\right)\right) = 2i \sin\left(\frac{n\pi}{2} + n\theta\right) \text{ as required } \quad \dots\dots[A1]$$

ii.

$$u^4 - w^4 = 2i \sin\left(\frac{4\pi}{2} + 4 \times \frac{\pi}{4}\right) = 2i \sin(3\pi) = 0 \dots\dots[A1]$$

Question 2

a.

$$\left. \begin{array}{l} \text{To show that } MNPQ \text{ is a parallelogram, it is enough to show that } \overrightarrow{MN} = \overrightarrow{QP} \\ \overrightarrow{MN} = \frac{1}{2} \overrightarrow{AB} + \frac{1}{2} \overrightarrow{BC} \quad \text{But } \overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AD} + \overrightarrow{DC} = -\underline{a} + \underline{b} + \underline{c} \\ \overrightarrow{MN} = \frac{1}{2} \underline{a} + \frac{1}{2} (-\underline{a} + \underline{b} + \underline{c}) \quad \therefore \overrightarrow{MN} = \frac{1}{2} (\underline{b} + \underline{c}) \end{array} \right\} \dots\dots[M1]$$

$$\overrightarrow{QP} = \frac{1}{2} \overrightarrow{AD} + \frac{1}{2} \overrightarrow{DC} = \frac{1}{2} (\underline{b} + \underline{c}) \quad \therefore \overrightarrow{MN} = \overrightarrow{QP} \quad \dots\dots[A1]$$

b.

i.

$$\left. \begin{array}{l} \overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} \quad \therefore \overrightarrow{AC} = 5\underline{i} + 3\underline{j} \\ \overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB} \quad \therefore \overrightarrow{DB} = -(2\underline{i} + 3\underline{j}) + 8\underline{i} \quad \therefore \overrightarrow{DB} = 6\underline{i} - 3\underline{j} \end{array} \right\} \dots\dots[A1]$$

ii.

$$\left. \begin{aligned} \overrightarrow{AC} &= 5\hat{i} + 3\hat{j} & \overrightarrow{DB} &= 6\hat{i} - 3\hat{j} \\ \overrightarrow{AC} \cdot \overrightarrow{DB} &= |\overrightarrow{AC}| \times |\overrightarrow{DB}| \times \cos \theta \\ 5 \times 6 - 3 \times 3 &= \sqrt{5^2 + 3^2} \times \sqrt{6^2 + 3^2} \times \cos \theta \end{aligned} \right\} \dots\dots [M1]$$

$$\cos \theta = \frac{21}{\sqrt{34} \times \sqrt{45}} \quad \therefore \quad \theta = 57.53^\circ \approx 58^\circ \quad \dots\dots [A1]$$

c.

i.

$$\left. \begin{aligned} \overrightarrow{AC} &= 5\hat{i} + 3\hat{j} & \overrightarrow{DB} &= 6\hat{i} - 3\hat{j} & , & \overrightarrow{AE} = m\overrightarrow{AC} & \text{ and } & \overrightarrow{DE} = n\overrightarrow{DB} \\ \overrightarrow{AE} + \overrightarrow{EB} &= \overrightarrow{AB} & \therefore & m\overrightarrow{AC} + (1-n)\overrightarrow{DB} &= 8\hat{i} \end{aligned} \right\} \dots\dots [M1]$$

$$m(5\hat{i} + 3\hat{j}) + (1-n)(6\hat{i} - 3\hat{j}) = 8\hat{i} \quad \therefore \quad (5m - 6n + 6)\hat{i} + (3m + 3n - 3)\hat{j} = 8\hat{i} \quad \dots\dots [M1]$$

$$\left. \begin{aligned} 5m - 6n + 6 &= 8 \\ 3m + 3n - 3 &= 0 \end{aligned} \right\} \Rightarrow \left. \begin{aligned} 5m - 6n + 6 &= 8 \\ 6m + 6n - 6 &= 0 \end{aligned} \right\} \quad \therefore \quad 11m = 8 \quad \therefore \quad m = \frac{8}{11} \quad \dots\dots [A1]$$

$$n = 1 - m \quad \therefore \quad n = \frac{3}{11} \quad \dots\dots [A1]$$

ii.

$$\left. \begin{aligned} \overrightarrow{AC} &= 5\hat{i} + 3\hat{j} \\ \overrightarrow{AE} &= m\overrightarrow{AC} \quad \therefore \quad \overrightarrow{AE} = \frac{8}{11}(5\hat{i} + 3\hat{j}) = \frac{40}{11}\hat{i} + \frac{24}{11}\hat{j} \end{aligned} \right\} \dots\dots [M1]$$

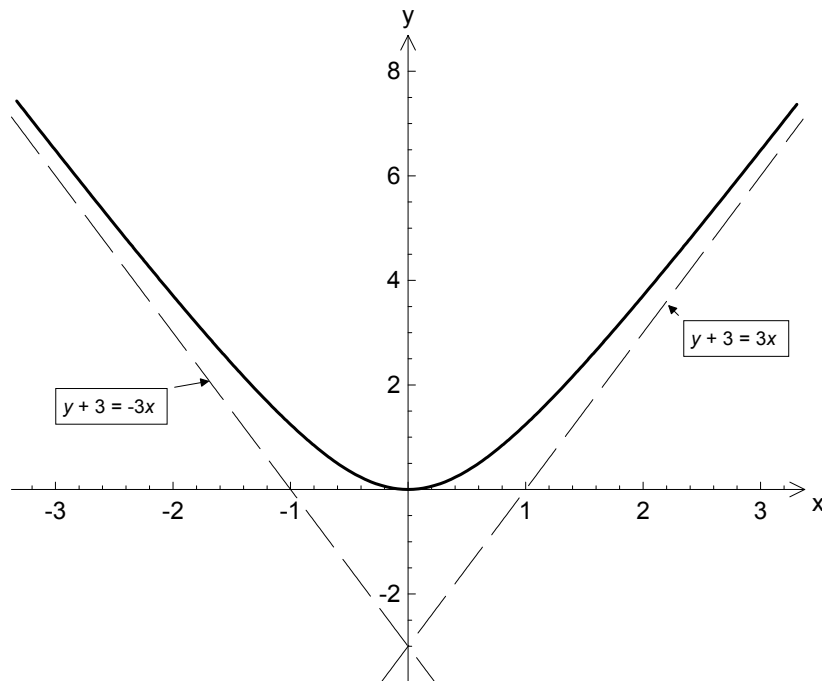
$$A_{\triangle ABE} = \frac{1}{2} \times 8 \times \frac{24}{11} = \frac{96}{11} \text{ sq. units} \quad \dots\dots [A1]$$

Question 3**a.**

See the diagram below

shape (one arm only)[A1]

$$\frac{(y+3)^2}{9} - x^2 = 1 \quad a=1 \text{ and } b=3 \quad \dots\dots[A1]$$

asymptotes $y+3 = \pm 3x$ [A2]**b.****i.**

$$y+3 = \pm 3\sqrt{x^2+1} \quad \therefore y = -3 + 3\sqrt{x^2+1} \text{ as } y \geq 0 \quad \dots\dots[M1]$$

$$f(x) = \frac{dy}{dx} = \frac{d}{dx}(-3 + 3\sqrt{x^2+1}) = \frac{3x}{\sqrt{x^2+1}} \text{ as required } \dots\dots[A1]$$

ii.

It can be seen from the graph that the gradient to the hyperbola changes from -3 to 3 , hence $a = -3$ and $b = 3$ [A1]

c.

i.

$$V = \pi \int_0^h x^2 dy \quad , \quad (y+3)^2 - 9x^2 = 9 \quad \therefore \quad x^2 = \frac{(y+3)^2}{9} - 1 \quad \dots\dots[\text{M1}]$$

$$V = \pi \int_0^h \left(\frac{(y+3)^2}{9} - 1 \right) dy \quad \dots\dots[\text{A1}]$$

ii

$$V = \pi \int_0^h \left(\frac{(y+3)^2}{9} - 1 \right) dy = \pi \left[\frac{(y+3)^3}{27} - y \right]_0^h = \pi \left(\frac{(h+3)^3}{27} - h - 1 \right)$$

$$V(h) = \pi \left(\frac{h^3}{27} + \frac{h^2}{3} \right) \text{ as required} \quad \dots\dots[\text{A1}]$$

d.

Consider dV , a very small change in volume, that produces dh , a very small change in depth.

$$dV = A \times dh \quad \therefore \quad A = \frac{dV}{dh} \quad \dots\dots[\text{A1}]$$

Question 4

a.

$$\left. \begin{aligned} m\vec{a} &= \vec{W} + \vec{F} \\ m(a_x \hat{i} + a_y \hat{j}) &= mg \hat{j} - mk(v_x \hat{i} + v_y \hat{j}) \end{aligned} \right\} \quad \dots\dots[\text{M1}]$$

$$\left. \begin{aligned} a_x \hat{i} + a_y \hat{j} &= -k v_x \hat{i} + (g - k v_y) \hat{j} \\ a_x &= -k v_x \quad \text{and} \quad a_y = g - k v_y \\ \frac{dv_x}{dt} &= -k v_x \quad \text{and} \quad \frac{dv_y}{dt} = g - k v_y \quad \text{as required} \end{aligned} \right\} \quad \dots\dots[\text{A1}]$$

b.

$$\left. \begin{aligned} \frac{dv_x}{dt} &= -k v_x \quad \therefore \quad -kt = \int \frac{dv_x}{v_x} \quad \therefore \quad -kt = \log_e v_x + c \\ \text{At } t=0, \quad v_x &= u \quad \therefore \quad 0 = \log_e u + c \quad \therefore \quad c = -\log_e u \end{aligned} \right\} \dots\dots[M1]$$

$$-kt = \log_e v_x - \log_e u \quad \therefore \quad \log_e \left(\frac{v_x}{u} \right) = -kt \quad \therefore \quad v_x(t) = ue^{-kt} \quad \dots\dots[A1]$$

$$\left. \begin{aligned} \frac{dv_y}{dt} &= g - k v_y \quad \therefore \quad t = \int \frac{dv_y}{g - k v_y} \quad \therefore \quad t = -\frac{1}{k} \log_e (g - k v_y) + c \\ \text{At } t=0, \quad v_y &= 0 \quad \therefore \quad 0 = -\frac{1}{k} \log_e (g) + c \end{aligned} \right\} \dots\dots[M1]$$

$$\left. \begin{aligned} t &= -\frac{1}{k} \log_e (g - k v_y) + \frac{1}{k} \log_e (g) \quad \therefore \quad t = \frac{1}{k} \log_e \left(\frac{g}{g - k v_y} \right) \\ \frac{g}{g - k v_y} &= e^{kt} \quad \therefore \quad g - k v_y = g e^{-kt} \quad \therefore \quad v_y(t) = \frac{g}{k} (1 - e^{-kt}) \end{aligned} \right\} \dots\dots[A1]$$

c.

$$v_y(t) = \frac{g}{k} (1 - e^{-kt}) \quad \therefore \quad y = \int \frac{g}{k} (1 - e^{-kt}) dt \quad \therefore \quad y = \frac{g}{k} \left(t + \frac{1}{k} e^{-kt} \right) + c \quad \dots\dots[M1]$$

$$\left. \begin{aligned} \text{At } t=0, \quad y &= 0 \quad \therefore \quad 0 = \frac{g}{k} \times \frac{1}{k} + c \quad \therefore \quad c = -\frac{g}{k} \times \frac{1}{k} \\ y &= \frac{g}{k} \left(t + \frac{1}{k} e^{-kt} \right) - \frac{g}{k^2} \quad \therefore \quad y(t) = \frac{g}{k} t + \frac{g}{k^2} (e^{-kt} - 1) \end{aligned} \right\} \dots\dots[A1]$$

d.**i.**

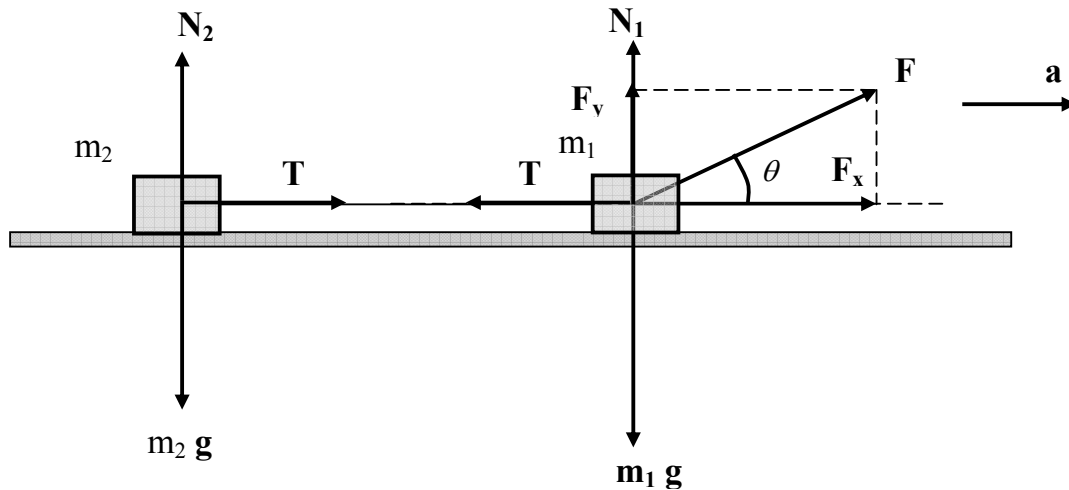
$$h = \frac{gt^2}{2} \quad \therefore \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 490}{9.8}} = 10s \quad \dots\dots[A1]$$

ii.

$$490 = 100t + \frac{10000}{9.8}(e^{-0.01 \times 9.8t} - 1) \quad \therefore t = 11.9s \quad \dots\dots[A1]$$

Question 5

a.



See the diagram above

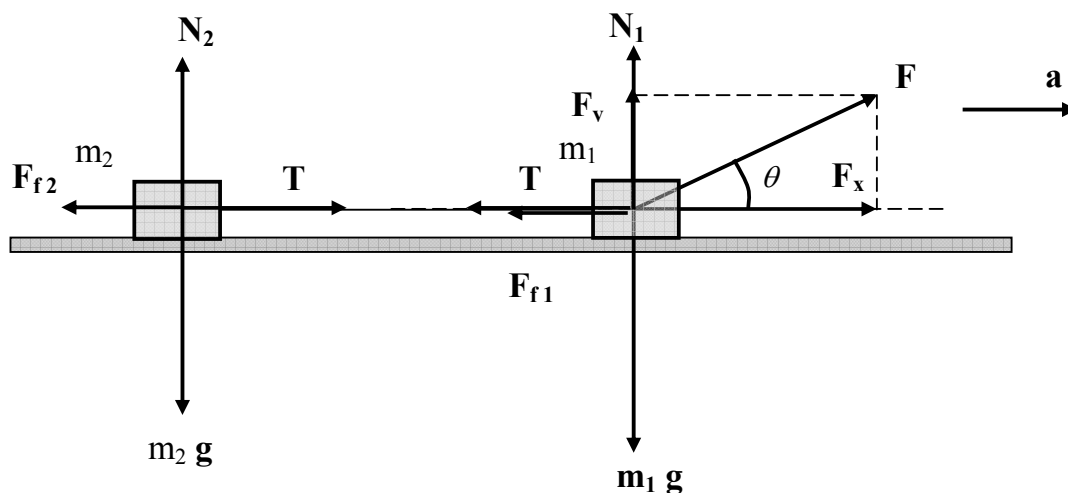
Apply Newton's laws on both axes for both objects

$$\left\{ \begin{array}{l} N_1 + F \sin \theta - m_1 g = 0 \\ F \cos \theta - T = m_1 a \\ N_2 - m_2 g = 0 \\ T = m_2 a \end{array} \right\} \quad \dots\dots[M1]$$

$$F \cos \theta = (m_1 + m_2)a \quad \therefore a = \frac{F \cos \theta}{m_1 + m_2} \quad \dots\dots[M1]$$

$$\therefore a = \frac{300 \cos 45^\circ}{60} \quad \therefore a = 3.5 \text{ms}^{-2} \quad \dots\dots[A1]$$

b.



Apply Newton's laws on both axes for both objects

$$\left. \begin{cases} N_1 + F \sin \theta - m_1 g = 0 \\ F \cos \theta - T - F_{f1} = m_1 a \\ F_{f1} = \mu N_1 \end{cases} \right\} \dots [M1]$$

$$\left. \begin{cases} N_2 - m_2 g = 0 \\ T - F_{f2} = m_2 a \\ F_{f2} = \mu N_2 \end{cases} \right\} \dots [M1]$$

$$\Rightarrow \left. \begin{cases} N_1 = m_1 g - F \sin \theta \\ F \cos \theta - T - \mu(m_1 g - F \sin \theta) = m_1 a \\ N_2 = m_2 g \\ T - \mu m_2 g = m_2 a \end{cases} \right\} \dots [M1]$$

$$\left. \begin{aligned} F \cos \theta - \mu(m_1 g - F \sin \theta) - \mu m_2 g &= (m_1 + m_2) a \\ a &= \frac{F \cos \theta - \mu(m_1 g - F \sin \theta) - \mu m_2 g}{m_1 + m_2} \\ a &= \frac{F(\cos \theta + \mu \sin \theta) - \mu m_1 g - \mu m_2 g}{m_1 + m_2} \\ a &= \frac{F(\cos \theta + \mu \sin \theta)}{m_1 + m_2} - \mu g \end{aligned} \right\} \dots [M1]$$

$$\left. \begin{aligned} \mu = \tan \varphi \quad \therefore a &= \frac{F(\cos \theta + \tan \varphi \sin \theta)}{m_1 + m_2} - g \tan \varphi \\ a &= \frac{F \left(\cos \theta + \frac{\sin \varphi}{\cos \varphi} \sin \theta \right)}{m_1 + m_2} - g \tan \varphi = \frac{F(\cos \theta \cos \varphi + \sin \varphi \sin \theta)}{(m_1 + m_2) \cos \varphi} - g \tan \varphi \\ a &= \frac{F \cos(\theta - \varphi)}{(m_1 + m_2) \cos \varphi} - g \tan \varphi \text{ as required} \dots [A1] \end{aligned} \right\} \dots [M1]$$

c.**i.**

$$a = \frac{F \cos(\theta - 10^\circ)}{(m_1 + m_2) \cos 10^\circ} - g \tan 10^\circ$$

$$a = \max \quad \text{when} \quad \cos(\theta - 10^\circ) = 1 \quad \therefore \theta = 10^\circ \quad \dots [A1]$$

ii.

$$a_{\max} = \frac{F}{(m_1 + m_2) \cos 10^\circ} - g \tan 10^\circ = \frac{300}{60 \cos 10^\circ} - 9.8 \tan 10^\circ \quad \therefore a_{\max} = 3.3 \text{ ms}^{-2} \quad \dots [A1]$$

iii.

$$\left. \begin{aligned} a_1 &= \frac{F \cos(\theta_1 - 10^\circ)}{(m_1 + m_2) \cos 10^\circ} - g \tan 10^\circ \quad \text{and} \quad a_2 = \frac{F \cos(\theta_2 - 10^\circ)}{(m_1 + m_2) \cos 10^\circ} - g \tan 10^\circ \\ a_1 &= a_2 \quad \therefore \cos(\theta_1 - 10^\circ) = \cos(\theta_2 - 10^\circ) \end{aligned} \right\} \dots [M1]$$

$$\therefore \theta_1 - 10^\circ = 10^\circ - \theta_2 \quad \therefore \theta_2 = 20^\circ - \theta_1 \quad \therefore \theta_2 = 5^\circ \quad \dots [A1]$$

d.

The object is on verge of losing contact with the surface
when the normal reaction N_1 becomes zero.

$$N_1 = m_1 g - F \sin \theta \quad , \quad N_1 = 0 \quad \therefore 0 = m_1 g - F \sin \theta \quad \dots [M1]$$

$$\therefore F = \frac{m_1 g}{\sin \theta} = \frac{40 \times 9.8}{\sin 45^\circ} = 554.4 \text{ N} \quad \dots [A1]$$