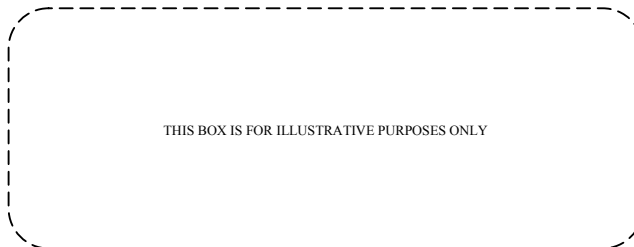




2011 Trial Examination



THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER

Figures

Words

Letter

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SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 12 pages.
- Working space is provided throughout the book.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

On an Argand diagram, O is the origin and point A represents a complex number $a + ib$ in the fourth quadrant. Point B represents the complex number $-\sqrt{3} - 3i$. If the triangle OAB is equilateral, find the values of a and b .

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

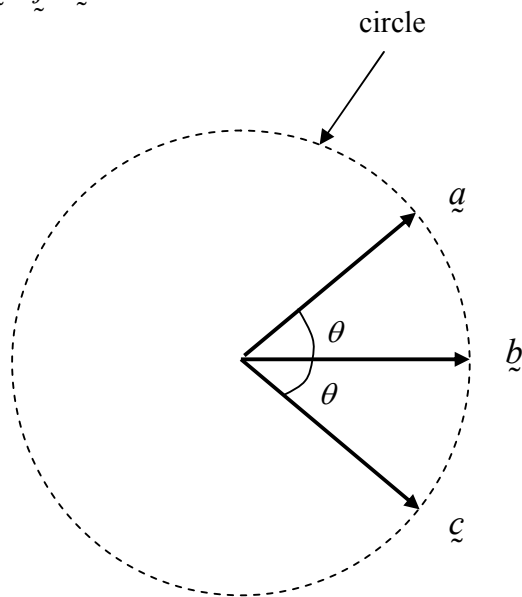
3 marks

TURN OVER

Question 2

The diagram below shows three coplanar vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ with the same magnitude.

$$\underline{a} = \underline{i} - \underline{j} + \underline{k} \quad \text{and} \quad \underline{b} = \underline{i} + \underline{j} + \underline{k}.$$



The angles separating $\underline{a}$ and $\underline{b}$ and $\underline{b}$ and $\underline{c}$ are equal. Find vector $\underline{c}$.

[illegible]

4 marks

Question 3

Consider the identity: $\sin^{-1} x + \frac{\pi}{6} = \sin^{-1} \left(\frac{\sqrt{3}}{2} x + \frac{1}{2} \sqrt{1-x^2} \right)$

a. Check that this identity is true for $x = \frac{\sqrt{3}}{2}$

2 marks

b. Prove that the identity is true for all values of x , $0 \leq x \leq 1$

3 marks

TURN OVER

Question 4

Find the equation of the normal to the curve with equation $3x^2 - 2xy + 2y^3 = 7$ at the point where $y=1$ and $x < 0$.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

4 marks

A graph of a function $y=f(x)$ on a coordinate plane. The x-axis is labeled x and the y-axis is labeled y . The function is represented by a thick black curve. Two points on the curve are highlighted: one at $x=f^{-1}(a)$ and $y=a$, and another at $x=f^{-1}(b)$ and $y=b$. Dashed lines connect these points to the axes. An arrow points from the label $y=f(x)$ to the curve.

$$\int_{f^{-1}(a)}^{f^{-1}(b)} f(x)dx + \int_a^b f^{-1}(y)dy = b f^{-1}(b) - a f^{-1}(a)$$

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Question 6

Use a geometrical interpretation, or otherwise, to evaluate the integral $\int_{-3}^{-1} \log_e |x| dx$

[illegible]

3 marks

Question 7

Two particles A and B are moving with constant velocities $\underline{v}_A = -2\underline{i} + 3\underline{j}$ and $\underline{v}_B$ respectively. Particle A starts at the point $(4, -3)$ and particle B starts at the point $(-1, 1)$. The particles collide after 1 second.

- a. Find the velocity $\underline{v}_B$ of particle B.

3 marks

- b. Find the distance covered by particle B until collision.

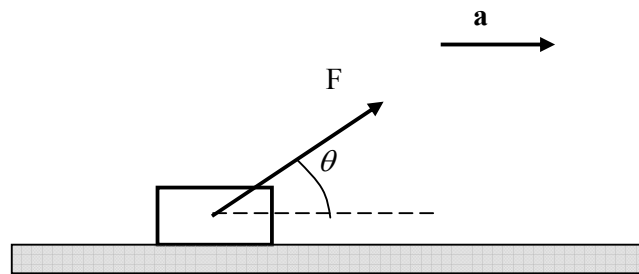
1 mark

TURN OVER

Question 8

A block of mass m lies on a rough horizontal table, with coefficient of friction μ .

A force of magnitude F , acting on the object at an angle θ to the horizontal, causes the block to accelerate on the table, as seen in the diagram below.



- a. Draw in all the forces acting on the block.

1 mark

- b. Show that the acceleration of the block is $a = \frac{F}{m}(\cos \theta + \mu \sin \theta) - \mu g$

4 marks

Question 9

The path of a particle is described by the cartesian equation: $\frac{x^2}{4} + \frac{y^2}{9} = 1$. Both x and y coordinates are circular functions of time t , with the same period. The particle starts moving anticlockwise from the point $(-2, 0)$, with a speed of 6 m/s.

a. Find the parametric equations for x and y .

3 marks

b. Find the speed of the particle after a quarter of a period.

2 marks

TURN OVER

Question 10

A particle moving in a straight line slows down under the influence of an acceleration $a \text{ ms}^{-2}$ such that $a = -kv$, where $v \text{ ms}^{-1}$ is the velocity of the particle at any instant. The initial velocity is u . After 2 seconds the speed of the particle has decreased by a factor of e , where e is Euler's number.

- a. Find the constant k and the expression for the velocity v at any time t .

2 marks

- b. Find the distance covered by the particle in the first 2 seconds in terms of u , the initial speed.

2 marks

END OF QUESTION AND ANSWER BOOK