

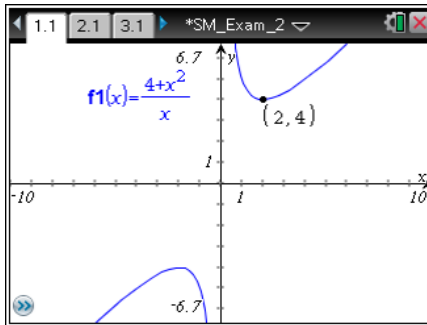
The Mathematical Association of Victoria
Trial Examination 2011
Specialist Maths Examination 2 - SOLUTIONS

SECTION 1**Answers**

1. E 2. B 3. A 4. D 5. E 6. A
 7. C 8. A 9. B 10. D 11. B 12. D
 13. C 14. E 15. D 16. B 17. C 18. B
 19. B 20. C 21. E 22. B

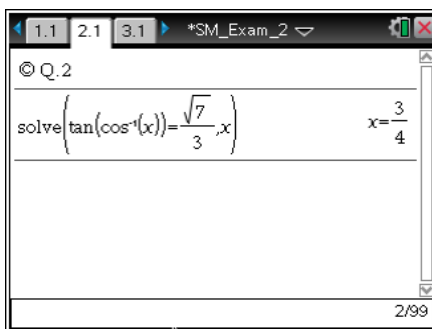
Question 1**Answer E**

Substituting $x = 2$ for each option gives the result $f(2) = 4$ for option E only.

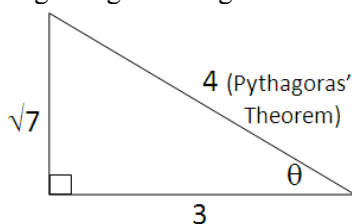
**Question 2****Answer B**

Using CAS, or knowledge of trigonometry,

$$\tan\left(\cos^{-1}\left(\frac{3}{4}\right)\right) = \frac{\sqrt{7}}{3}$$

**Alternatively,**

The situation can be represented geometrically on a right angled triangle.

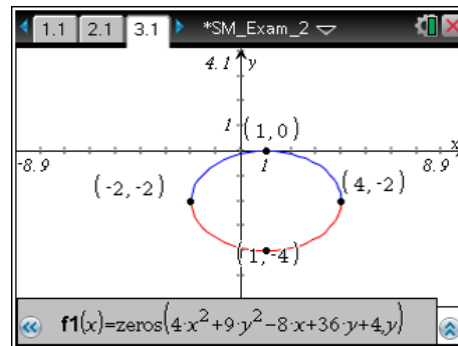


$$\frac{p}{q} = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3}{4}$$

Question 3**Answer A**

By graphing, using a conics application on CAS or by completing the square:

Centre is $(1, -2)$, horizontal semi-axis is 3 and vertical semi-axis is 2.

**Alternatively,**

$$4x^2 + 9y^2 - 8x + 36y + 4 = 0$$

$$4(x^2 - 2x + 1) + 9(y^2 + 4y + 4) - 36 = 0$$

$$4(x-1)^2 + 9(y+2)^2 = 36$$

$$\frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = 1$$

Question 4**Answer D**

$$x = t^2 \text{ and } y = t(t^2 + 1), \quad t \geq 0$$

$$y = x^{\frac{1}{2}}(x+1)$$

$$y = x^{\frac{3}{2}} + x^{\frac{1}{2}}$$

Question 5**Answer E**

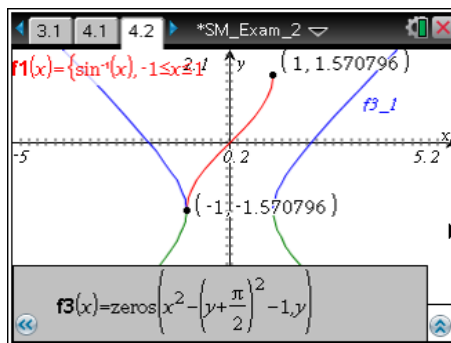
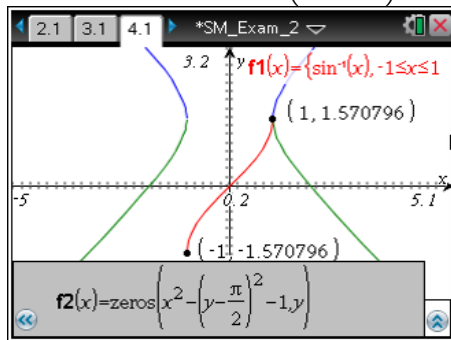
The graph of g has endpoints at

$\left(1, \frac{\pi}{2}\right), \left(-1, -\frac{\pi}{2}\right)$. The hyperbola $x^2 - y^2 = 1$

has vertices at $(1, 0)$ and $(-1, 0)$ and needs to be

translated $\frac{\pi}{2}$ units up or down to intersect with the

graph of g . Hence $x^2 - \left(y \pm \frac{\pi}{2}\right)^2 = 1$.

**Question 6****Answer A**

The point of intersection of $\operatorname{Re}(z) = -2$

and $\operatorname{Im}(z) = i$ is $z = -2 + i$.

Question 7**Answer C**

Using the conjugate root theorem, the roots

include $z = 1$, $z = 2\sqrt{2}$ and $z = 1 \pm i\sqrt{3}$.

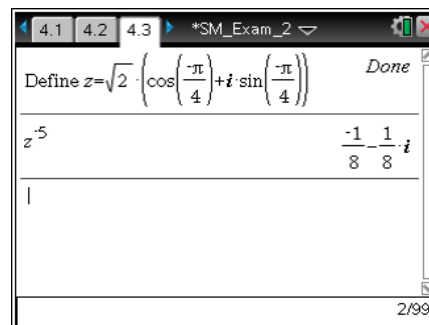
Therefore the degree of the polynomial must be at least 4.

Question 8**Answer A**

$$z = \sqrt{2} \operatorname{cis}\left(-\frac{\pi}{4}\right)$$

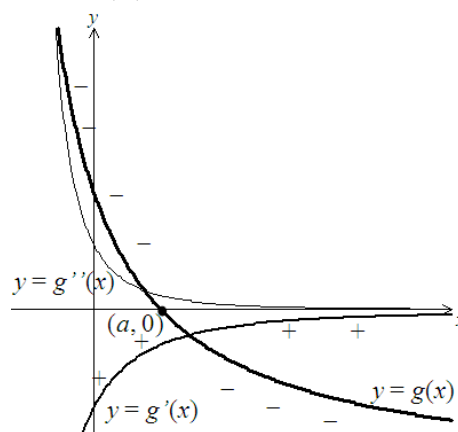
$$z^{-5} = (\sqrt{2})^{-5} \operatorname{cis}\left(\frac{5\pi}{4}\right)$$

$$z^{-5} = -\frac{1}{8}(1+i)$$

**Question 9****Answer B**

Consider the sketch graphs of $y = g'(x)$ and

$y = g''(x)$.

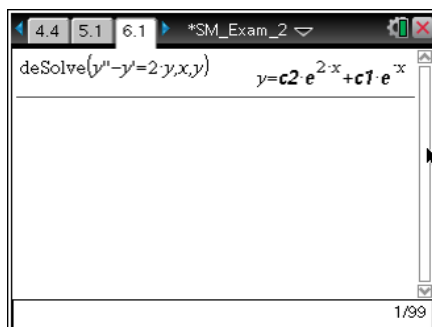


$$g(a) = 0, \quad g'(a) < 0, \quad g''(a) > 0$$

Therefore, $g''(a) > g(a) > g'(a)$

Question 10**Answer D**

The solution to the differential equation is of the form $y = Ae^{-x} + Be^{2x}$. Therefore, $a = 1$, $b = 2$.

**Question 11****Answer B**

$$\begin{aligned} \text{Area} &= \int_0^{\frac{\pi}{2}} (\sin(x) \cos^3(x)) dx \\ &= \int_0^{\frac{\pi}{2}} (\sin(x)(1 - \sin^2(x)) \cos(x)) dx \end{aligned}$$

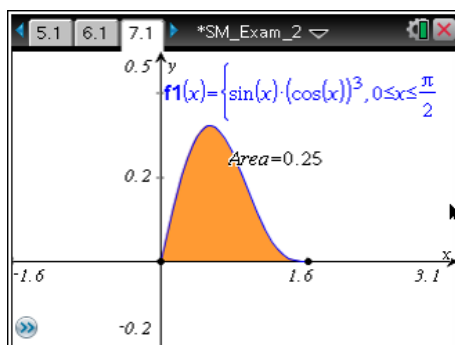
Let $u = \sin(x)$ therefore $\frac{du}{dx} = \cos(x)$

Terminals: $x = 0$, $u = \sin(0) = 0$ and

$$x = \frac{\pi}{2}, u = \sin\left(\frac{\pi}{2}\right) = 1$$

Substituting,

$$\text{Area} = \int_0^1 u(1 - u^2) \frac{du}{dx} dx$$

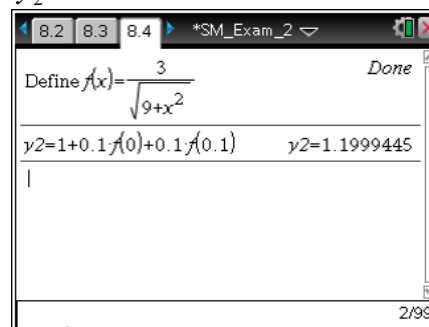
**Question 12****Answer D**

$h = 0.1$, therefore $n = 2$ for $x = 0.2$

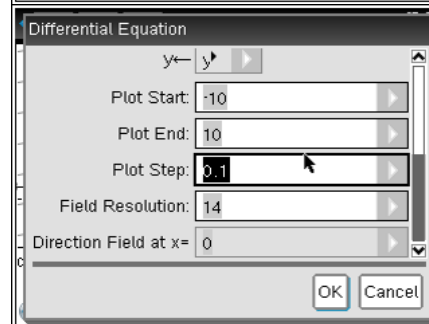
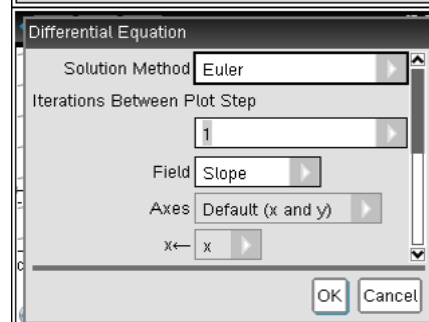
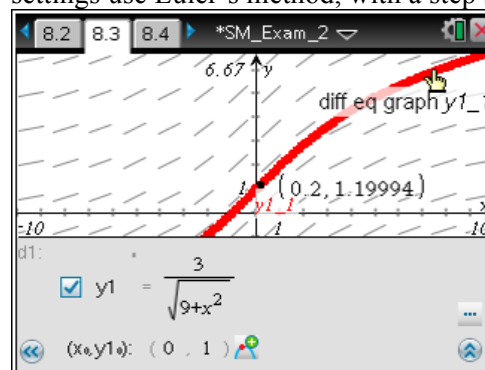
$$y_1 = 1 + 0.1f(0) \text{ and}$$

$$y_2 = (1 + 0.1f(0)) + 0.1f(0.1)$$

$$y_2 \approx 1.1999$$



Alternatively, use the *Differential Equation Graph* functionality of your CAS, ensuring that the settings use Euler's method, with a step size of 0.1.



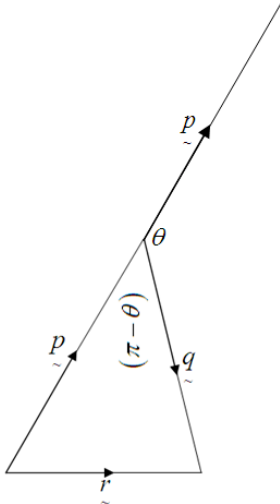
Question 13**Answer C**

The direction field could be a family of parabolas of the form $y = ax^2 + c$, where $a > 0$, $c \in R$.

From the options available, the only differential equation that could have a solution of this form is

$$\frac{dy}{dx} = x$$

$$y = \int x dx = \frac{1}{2}x^2 + c.$$

Question 14**Answer E**

$$\vec{p} \cdot \vec{q} = |\vec{p}| \times |\vec{q}| \cos(\theta) \quad \dots \text{equation (1)}$$

Using the cosine rule

$$|\vec{r}|^2 = |\vec{p}|^2 + |\vec{q}|^2 - 2|\vec{p}| \times |\vec{q}| \cos(\pi - \theta)$$

$$\cos(\pi - \theta) = \frac{|\vec{p}|^2 + |\vec{q}|^2 - |\vec{r}|^2}{2|\vec{p}| \times |\vec{q}|}$$

$$\text{However, } \cos(\pi - \theta) = -\cos(\theta)$$

$$\cos(\theta) = -\left(\frac{|\vec{p}|^2 + |\vec{q}|^2 - |\vec{r}|^2}{2|\vec{p}| \times |\vec{q}|} \right) \quad \dots \text{equation (2)}$$

Substituting equation (2) in equation (1)

$$\vec{p} \cdot \vec{q} = |\vec{p}| \times |\vec{q}| \times -\left(\frac{|\vec{p}|^2 + |\vec{q}|^2 - |\vec{r}|^2}{2|\vec{p}| \times |\vec{q}|} \right)$$

$$\vec{p} \cdot \vec{q} = -\frac{1}{2} \left(|\vec{p}|^2 + |\vec{q}|^2 - |\vec{r}|^2 \right)$$

Question 15**Answer D**

Since the vectors are perpendicular,

$$(a\vec{i} + 6\vec{j} + 2\vec{k}) \cdot (2\vec{i} - 3\vec{j} + 6\vec{k}) = 0$$

$$2a - 18 + 12 = 0$$

$$a = 3$$

For the unit vector to $3\vec{i} + 6\vec{j} + 2\vec{k}$,

$$m = \frac{1}{\sqrt{3^2 + 6^2 + 2^2}} = \pm \frac{1}{7}$$

Question 16**Answer B**

$$\vec{a} = 2\vec{i} + 4\vec{k} \text{ and } \vec{b} = 3\vec{j} - 5\vec{k}.$$

Consider option A. If $\vec{c} = 2\vec{i} - 3\vec{j} - \vec{k}$ is linearly dependent with $\vec{a}$ and $\vec{b}$, then $\vec{c} = \alpha\vec{a} + \beta\vec{b}$. But

$$\vec{c} = \alpha\vec{a} + \beta\vec{b} = 2\alpha\vec{i} - 3\beta\vec{j} - \alpha\vec{k} + 3\beta\vec{k}$$

Consider option B. If $\vec{c} = -4\vec{i} - 9\vec{j} + 7\vec{k}$ is

linearly dependent with $\vec{a}$ and $\vec{b}$, then

$$\vec{c} = -2\alpha\vec{a} - 3\beta\vec{b}.$$

$$\vec{c} = -4\vec{i} - 9\vec{j} + (-8 + 15)\vec{k}$$

$$\vec{c} = -4\vec{i} - 9\vec{j} + 7\vec{k}, \text{ as required. Hence B.}$$

Alternatively,

$\vec{c}$ can be expressed as:

$$\vec{c} = \alpha\vec{a} + \beta\vec{b} \quad (1)$$

$$\therefore \vec{c} = 2\alpha\vec{i} + 3\beta\vec{j} + (4\alpha - 5\beta)\vec{k}$$

For option A, using the above equation (1)

$$2\alpha = 2 \rightarrow \alpha = 1; 3\beta = -3 \rightarrow \beta = -1$$

However, substituting these values into the $\vec{k}$ term above

gives the wrong answer

because $4\alpha - 5\beta = 4 + 5 = 9$.

For option B, using the above equation (1)

$$2\alpha = -4 \rightarrow \alpha = -2; 3\beta = -9 \rightarrow \beta = -3$$

For the $\vec{k}$ term, $4\alpha - 5\beta = -8 + 15 = 7$.

This is consistent with the $\vec{k}$ term given in option B.

Therefore option B is correct.

Question 17**Answer C**

$$\frac{dv}{dt} = -3v$$

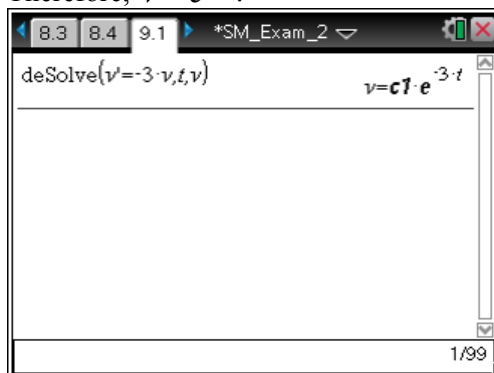
$$t = \int \frac{1}{-3v} dv$$

$$t = -\frac{1}{3} \log_e(|v|) + C$$

Given that $v(0) = 1$,

$$0 = -\frac{1}{3} \log_e(1) + C$$

$$C = 0$$

Therefore, $v = e^{-3t}$.**Question 18****Answer B**Let m be the mass of the body and g be acceleration due to gravity.

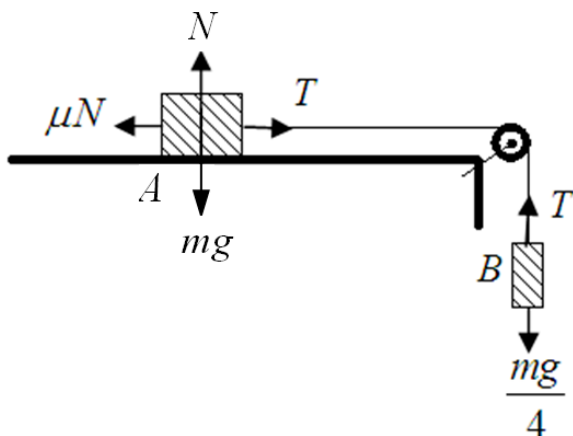
$$N = mg \cos(60^\circ)$$

When the body is on the point of sliding,

$$\mu N = mg \sin(60^\circ)$$

$$\mu(mg \cos(60^\circ)) = mg \sin(60^\circ)$$

$$\mu = \tan(60^\circ) = \sqrt{3}$$

Question 19**Answer B**

Consider the resultant force at B.

$$R = ma$$

$$\frac{mg}{4} - T = \frac{m}{4} \times \frac{g}{5}$$

$$T = \frac{mg}{4} - \frac{mg}{20}$$

$$T = \frac{mg}{5}$$

Question 20**Answer C**Using calculus or kinematics formulas, $v = \sqrt{492}$.

$$p = \frac{\sqrt{492}}{5} = 4.44 \text{ ms}^{-1}$$

Using kinematics formula

$$v^2 = u^2 + 2as$$

$$v^2 = 10^2 + 2 \times -9.8 \times -20$$

$$v = \sqrt{492}$$

$$p = mv$$

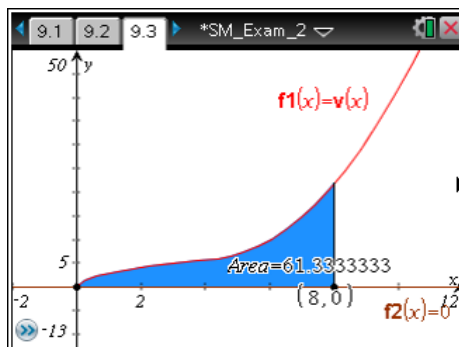
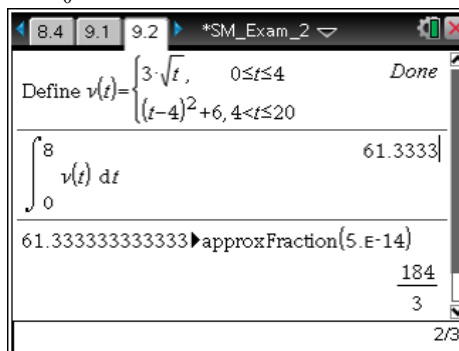
$$p = \frac{\sqrt{492}}{5}$$

$$p = 4.44 \text{ kgms}^{-1} \text{ (correct to 2 decimal places).}$$

Question 21**Answer E**

$$v(t) = \begin{cases} 3\sqrt{t} & 0 \leq t \leq 4 \\ (t-4)^2 + 6 & 4 < t \leq 20 \end{cases}$$

$$x = \int_0^8 v(t) dt = \frac{184}{3} \text{ m}$$



Question 22**Answer B**

Consider the resultant force on the vehicle:

$$R = ma$$

$$-4800 = 1200a$$

$$a = -4 \text{ ms}^{-2}$$

To find the braking time, use calculus or kinematics formulas.

$$v = u + at$$

$$0 = 25 - 4t$$

$$t = \frac{25}{4} = 6.25 \text{ s}$$

To find the braking distance, use calculus or kinematics formulas.

$$x = ut + \frac{1}{2}at^2$$

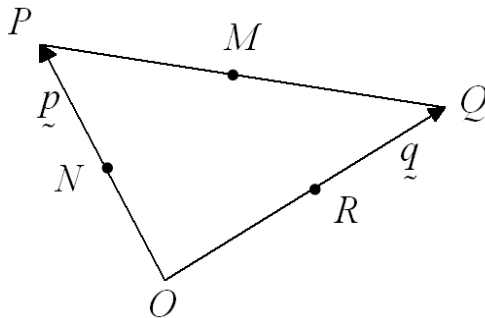
$$x = 25 \times 6.25 - \frac{1}{2} \times 4 \times (6.25)^2$$

$$x = \frac{625}{8} = 78.125 \text{ m}$$

END OF SECTION 1 SOLUTIONS

SECTION 2 SOLUTIONS

Question 1



a.

$$|\underline{p}| = |\underline{q}| = \sqrt{9 + 4 + 36}$$

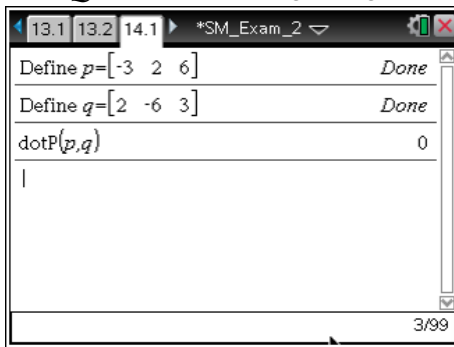
$$|\underline{p}| = |\underline{q}| = 7, \text{ therefore } \triangle POQ \text{ is isosceles.}$$

1A

$$\underline{p} \cdot \underline{q} = (-3 \times 2) + (2 \times -6) + (6 \times 3) = 0, \text{ therefore } \angle POQ = 90^\circ$$

1A

$\triangle POQ$ is an isosceles right-angles triangle.



b. i.

$$\vec{OM} = \vec{OP} + \frac{1}{2} \vec{PQ}$$

$$\vec{OM} = \underline{p} + \frac{1}{2}(\underline{q} - \underline{p})$$

$$\vec{OM} = \frac{1}{2}(\underline{p} + \underline{q}) \quad (\text{This result effectively proves that } \vec{NM} = \vec{OR} = \vec{RQ})$$

$$\vec{OM} = -\frac{1}{2}\underline{i} - 2\underline{j} + \frac{9}{2}\underline{k}$$

1A

b. ii.

From the previous result,

$$\vec{NM} = \vec{OR} = \frac{1}{2}\underline{q}, \text{ therefore}$$

$$\vec{MN} = -\vec{OR} = -\frac{1}{2}\underline{q}$$

$$\vec{MN} = -\underline{i} + 3\underline{j} - \frac{3}{2}\underline{k}$$

1A

Alternatively,

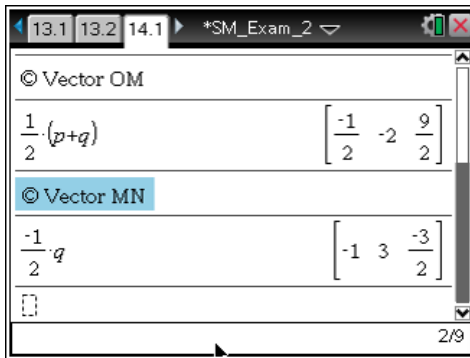
$$\vec{MN} = -\vec{OM} + \vec{ON}$$

From part b.i.

$$\vec{MN} = -\frac{1}{2}(\vec{p} + \vec{q}) + \frac{1}{2}\vec{p}$$

$$\vec{MN} = -\frac{1}{2}\vec{q}$$

$$\vec{MN} = -\vec{i} + 3\vec{j} - \frac{3}{2}\vec{k}$$

**c.**Required to show that $\vec{OM} \perp \vec{PQ}$

$$\vec{OM} = \vec{OP} + \frac{1}{2}\vec{PQ}$$

$$= \vec{p} + \frac{1}{2}(\vec{q} - \vec{p})$$

$$= \frac{1}{2}(\vec{q} + \vec{p})$$

Therefore,

1M

$$\vec{OM} \cdot \vec{PQ} = \frac{1}{2}(\vec{q} + \vec{p}) \cdot (\vec{q} - \vec{p})$$

$$= \frac{1}{2}(\vec{q} \cdot \vec{q} - \vec{p} \cdot \vec{p})$$

$$= \frac{1}{2}(|\vec{q}|^2 - |\vec{p}|^2)$$

1A

$$= 0 \quad (\text{because } |\vec{q}| = |\vec{p}|: \text{isosceles triangle})$$

Therefore the line OM is perpendicular to the hypotenuse PQ , as required.**d.** $\vec{s} \cdot \vec{p} = 0$ because $\vec{s}$ is perpendicular to $\vec{p}$. Therefore

$$-3a + 2b + 6c = 0 \quad \dots \text{equation 1}$$

1M $\vec{s} \cdot \vec{q} = 0$ because $\vec{s}$ is perpendicular to $\vec{q}$. Therefore

$$2a - 6b + 3c = 0 \quad \dots \text{equation 2}$$

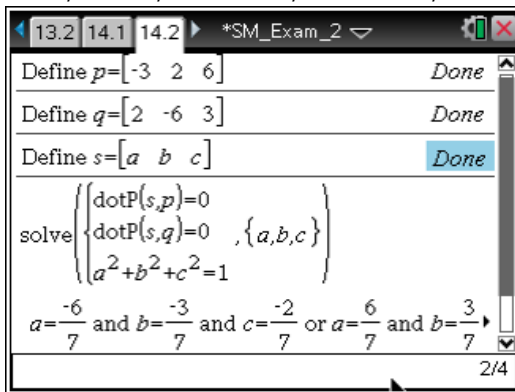
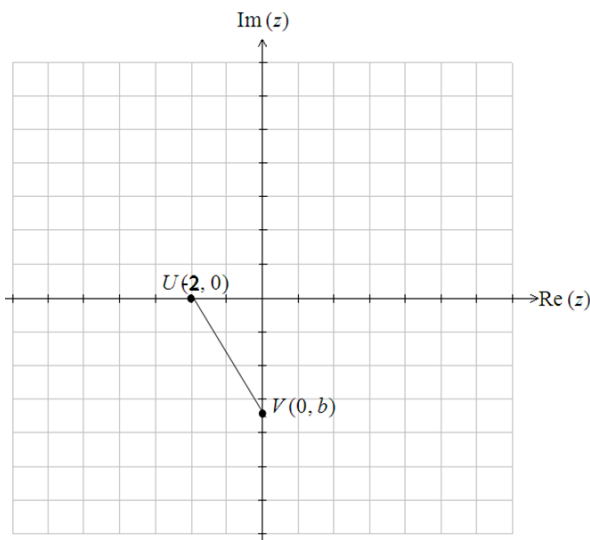
1M $\vec{s}$ is a unit vector, therefore

$$a^2 + b^2 + c^2 = 1 \quad \dots \text{equation 3}$$

1A

Solve equations 1, 2 and 3 simultaneously

$$a = \frac{6}{7}, b = \frac{3}{7} \text{ and } c = \frac{2}{7} \text{ or } a = -\frac{6}{7}, b = -\frac{3}{7} \text{ and } c = -\frac{2}{7}$$

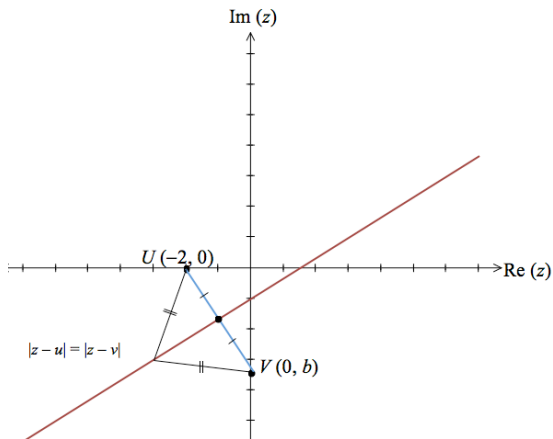
1A**Question 2****a. i.****1A****a. ii.**Let the equation of the line segment be of the form $y = mx + c$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b}{2}$$

$$c = b$$

$$\text{Equation is } y = \frac{b}{2}x + b, \text{ Domain is } [-2, 0]$$

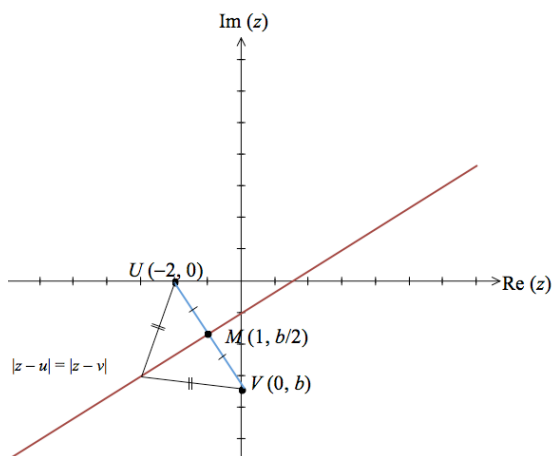
1A

b. i.**1A****b. ii.**

By definition, P is equidistant from U and V . Every point in P will therefore be perpendicular to UV , as illustrated in the diagram below.

Alternatively, from part **a.ii.** above, $m_{uv} = \frac{b}{2}$ and from part **b.iii.** below, $m_p = -\frac{2}{b}$.

$m_p \times m_{uv} = -1$, therefore P is perpendicular to UV .

1A**b. iii.**

$$|z - u| = |z - v|$$

$$|x + iy + 2| = |x + i(y - b)|$$

$$\sqrt{(x+2)^2 + y^2} = \sqrt{x^2 + (y-b)^2}$$

1M

$$x^2 + 4x + 4 + y^2 = x^2 + y^2 - 2by + b^2$$

$$2by + 4x + 4 - b^2 = 0, \text{ as required}$$

1A

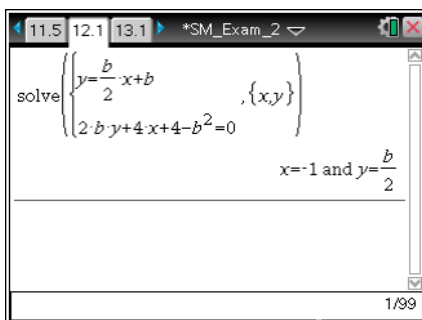
c. Solve simultaneously for x and y ,

$$y = \frac{b}{2}x + b \text{ and } 2by + 4x + 4 - b^2 = 0$$

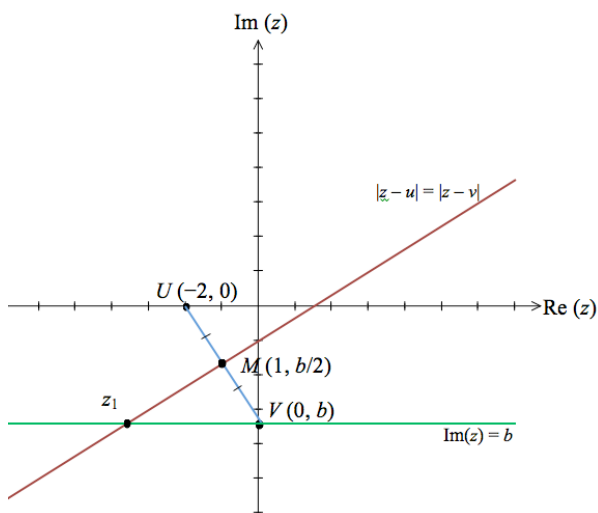
1M

$$M\left(-1, \frac{b}{2}\right)$$

1A



d. i.



1A

d. ii.

The cartesian equation of P is $2by + 4x + 4 - b^2 = 0$ (equation 1)

At the point of intersection with $y = b$ (equation 2), substitute equation 2 in equation 1 **1M**

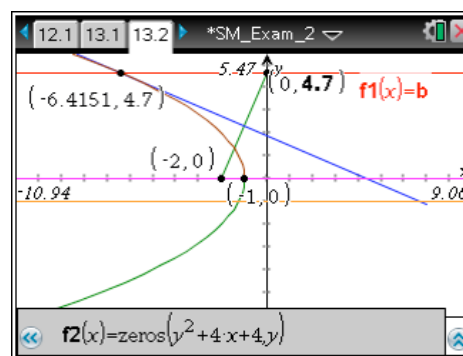
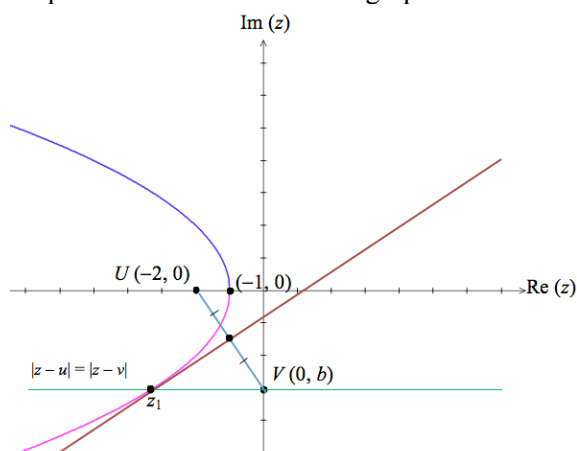
$$2y^2 + 4x + 4 - y^2 = 0$$

1A

$$y^2 + 4x + 4 = 0, \text{ as required.}$$

d. iii.

The equation of the curve can be graphed on a CAS, as shown below.



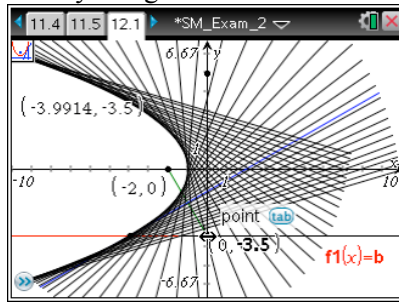
Correct shape

1A

Correct vertex and correctly labelled

1A

Alternatively, the path of can be explored using a dynamic geometry application on a CAS device, as illustrated in the screen dump below. This clearly shows that the vertex of the curve is at $(-1,0)$, and that P is always tangential to the curve.



Question 3

a. i. Consider the velocity of the particle as a function of its displacement.

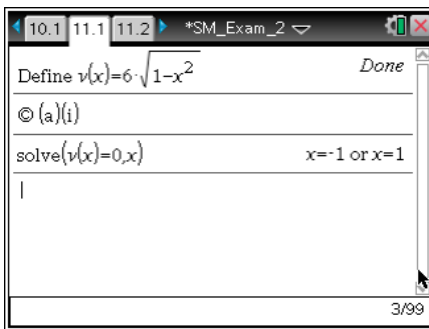
$$v(x) = \frac{dx}{dt}$$

$$0 = 6\sqrt{1-x^2}$$

$$1-x^2 = 0$$

$$x = 1 \text{ or } x = -1$$

1A



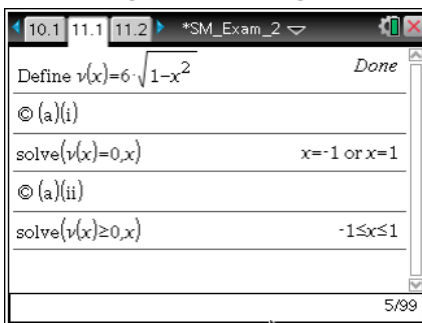
a. ii.

$$1-x^2 \geq 0$$

$$-1 \leq x \leq 1$$

$$\text{Domain is } \{x : x \in [-1, 1]\}$$

1A



b.i.

$$a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v \frac{dv}{dx}, \text{ as required}$$

1A

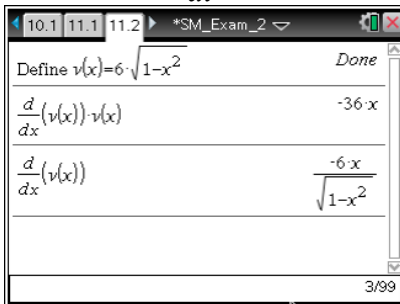
b.ii.

$$v = \frac{dx}{dt} = 6\sqrt{1-x^2}$$

$$a = v \frac{dv}{dx} = 6\sqrt{1-x^2} \times \frac{-6x}{\sqrt{1-x^2}} = -36x$$

1A

Therefore, $a = \frac{d^2x}{dt^2} + 36x = 0$, as required.

**c. i.**

$$\frac{dx}{dt} = 6\sqrt{1-x^2}$$

$$t = \frac{1}{6} \int \frac{dx}{\sqrt{1-x^2}}$$

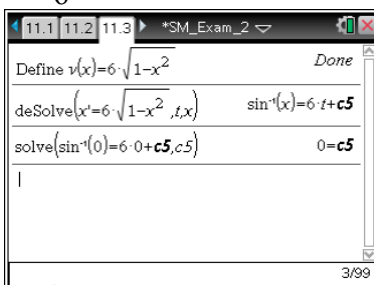
1M

$$t = \frac{1}{6} \sin^{-1}(x) + c$$

When $t = 0$, $x = 0$, therefore $c = 0$.

1A

$$t = \frac{1}{6} \sin^{-1}(x), \text{ as required.}$$

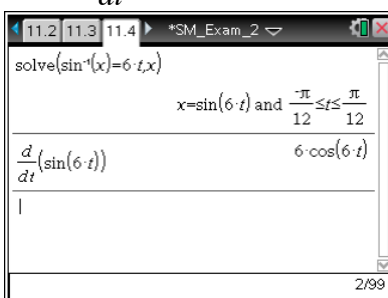
**c. ii.**

$$t = \frac{1}{6} \sin^{-1}(x)$$

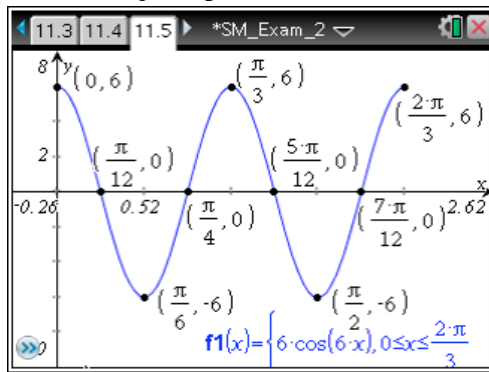
1M

$$x = \sin(6t)$$

$$v(t) = \frac{dx}{dt} = 6 \cos(6t)$$

1A**c. iii.**

$$\text{Period} = \frac{2\pi}{6} = \frac{\pi}{3}, \text{ amplitude} = 6.$$



Correct shape, amplitude and period

1A

Correct x-intercepts and coordinates of turning points

1A

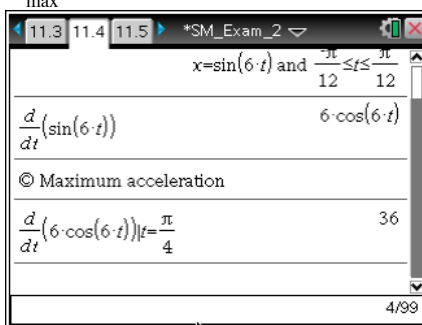
- d. The acceleration is a maximum where $\frac{dv}{dt}$ is greatest, that is, where the gradient of the velocity-time graph is steepest. By inspection, this occurs when $t = \frac{\pi}{4}$.

Maximum acceleration is given by $\frac{d(6\cos(6t))}{dt}$ at $t = \frac{\pi}{4}$.

1M

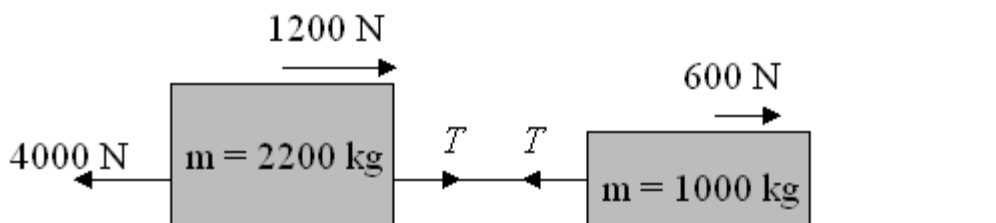
$$a_{\max} = 36$$

1A



Question 4

a.



1A

b.

Consider the resultant force on the vehicles

$$R = ma$$

1M

$$(4000 - 600 - 1200) = (2200 + 1000)a$$

$$a = \frac{2200}{3200}$$

$$a = \frac{11}{16} \text{ ms}^{-2} = 0.6875 \text{ ms}^{-2}$$

1A

c.

For the car,

$$T - 600 = 1000a, \text{ with } a = \frac{11}{16} \text{ ms}^{-2} \quad \mathbf{1M}$$

$$T = 1000 \times \frac{11}{16} + 600$$

$$T = 1287.5 \text{ N} \quad \mathbf{1A}$$

Alternatively, for the truck,

$$4000 - 1200 - T = 2200a, \text{ with } a = \frac{11}{16} \text{ ms}^{-2}$$

$$T = 4000 - 1200 - 2200 \times \frac{11}{16}$$

$$T = 1287.5 \text{ N}$$

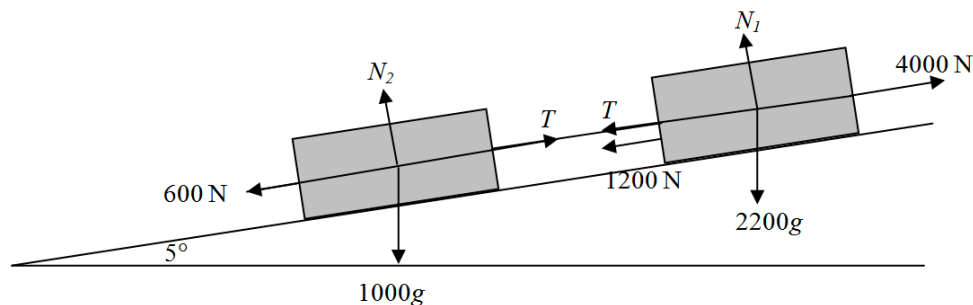
d.

The acceleration of the car and truck is constant, therefore

$$v = u + at$$

$$22 = 0 + \frac{11}{16}t$$

$$t = \frac{22 \times 16}{11} = 32 \text{ s} \quad \mathbf{1A}$$

e.The component of the weight of the vehicles parallel to the inclined road is $(2200 + 1000)g \sin(5^\circ)$

Resolving forces parallel to the plane,

$$\sum F = ma$$

$$4000 - 1200 - 600 - (2200 + 1000)g \sin(5^\circ) = (2200 + 1000)a \quad \mathbf{1M}$$

$$a = \frac{4000 - 1200 - 600 - (2200 + 1000)g \sin(5^\circ)}{(2200 + 1000)}$$

$$a \approx -0.167 \text{ ms}^{-2} \text{ (i.e. the speed of the vehicles is decreasing by } 0.17 \text{ ms}^{-1} \text{ every second)} \quad \mathbf{1A}$$

f.

$$a = \frac{F_T - kv}{m}, \text{ where } F_T = 4000 \text{ N, } m = 3200 \text{ kg and } k = 250.$$

$$\frac{dv}{dt} = \frac{4000 - 250v}{3200}$$

Solving the differential equation ,

$$v = Ae^{-\frac{5}{64}t} + 16$$

Starting from rest, $v = 0$ at $t = 0$

$$0 = Ae^{-\frac{5}{64} \times 0} + 16$$

$$A = -16$$

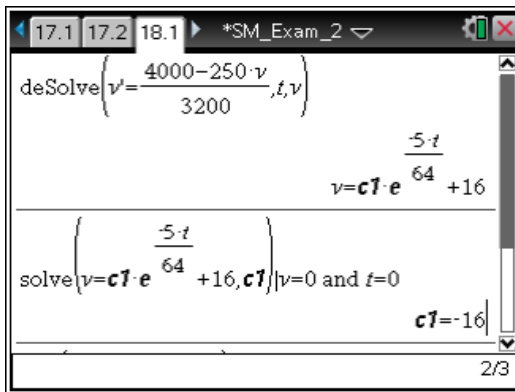
Therefore,

$$v = 16 \left(1 - e^{-\frac{5}{64}t} \right)$$

As $t \rightarrow \infty$, $e^{-\frac{5}{64}t} \rightarrow 0$ and $v \rightarrow 16$ The limiting (terminal) velocity is 16 ms^{-1}

1A

1A

**Question 5**

a.

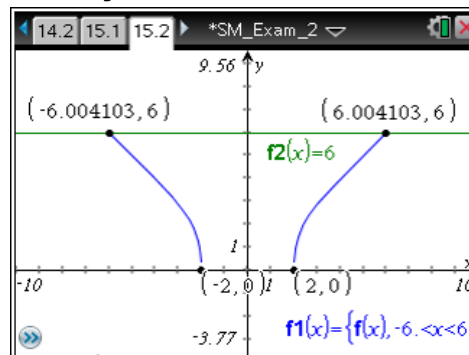
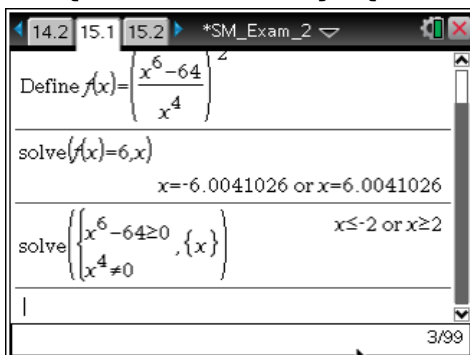
To find the maximal domain, $x^6 - 64 \geq 0$, but the range $0 \leq y < 6$. $(x^4 \neq 0)$, but this condition is made irrelevant by the maximal domain excluding the possibility of $x = 0$.)Solving the inequation $x^6 - 64 \geq 0$ for x , andsolving the equation $f(x) = 6$ for x , gives the results

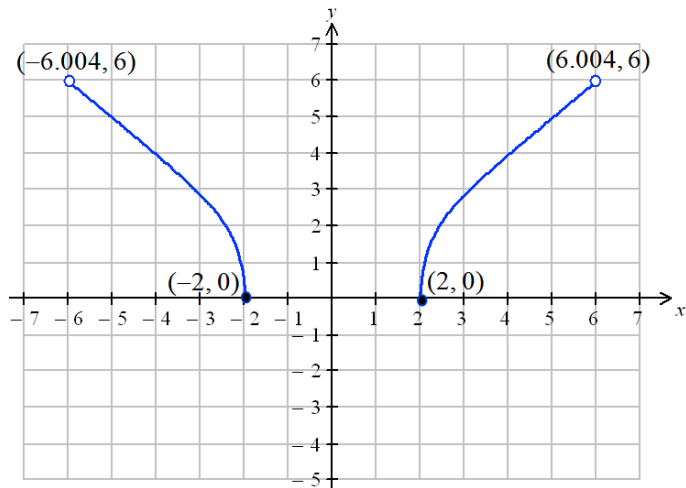
$$x \leq -2 \text{ or } x \geq 2 \text{ and } x \approx -6.004 \text{ or } x \approx 6.004$$

 $D = \{x : -6.004 < x \leq -2\} \cup \{x : 2 \leq x < 6.004\}$, as shown on the graph

1M

1A



b.

Correct shape and domain

1A

Endpoints correctly labelled and shown as open or closed

1A**c.i.**

$$\delta V \approx \pi y^2 \delta x$$

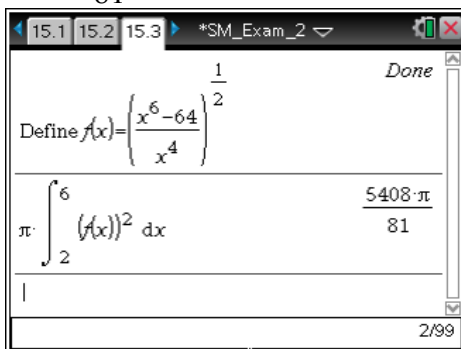
$$V = \pi \int_2^6 (f(x)^2) dx$$

$$V = \pi \int_2^6 \left(\frac{x^6 - 64}{x^4} \right) dx$$

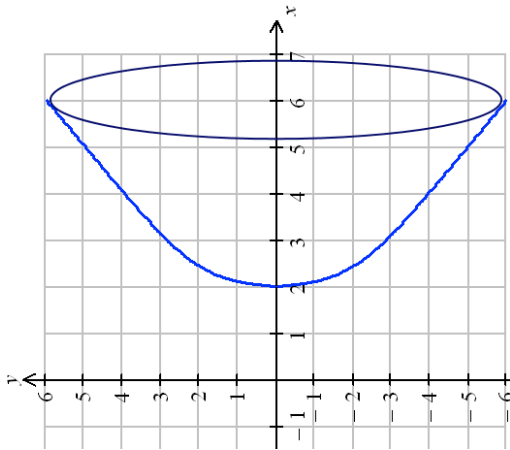
1A**c.ii.**

$$V = \pi \int_2^6 \left(\frac{x^6 - 64}{x^4} \right) dx$$

$$V = \frac{5408\pi}{81} \text{ L}$$

1A

d.



d.i.

$$\frac{dT}{dx} = k(80 - T), \quad k > 0$$

$$x = \frac{1}{k} \int \frac{dT}{80 - T}$$

$$x = -\frac{1}{k} \log_e(|80 - T|) + C$$

1M

When $x = 0$, $T = 15$, therefore $C = \frac{1}{k} \log_e(65)$

Substituting for C and simplifying the logarithms,

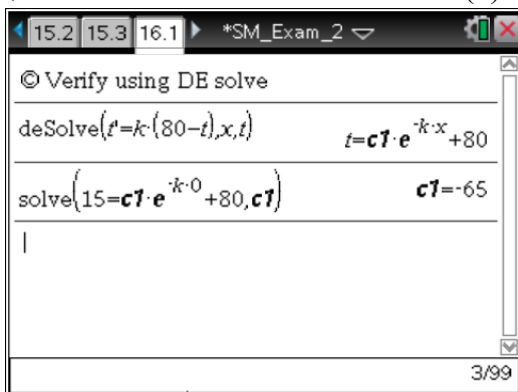
$$x = -\frac{1}{k} \log_e\left(\left|\frac{80 - T}{65}\right|\right)$$

$$\frac{80 - T}{65} = e^{-kx}$$

1A

$$T = 80 - 65e^{-kx}, \text{ as required}$$

(This meets the initial condition that $T(0) = 80 - 65e^{-k \times 0} = 15$)



d.ii.

$$T = 80 - 65e^{-kx}$$

When $T = 60$, $x = 10$

Solve for k , $60 = 80 - 65e^{-10k}$

$$k = \frac{1}{10} \log_e\left(\frac{13}{4}\right)$$

1A

deSolve($t=c1 \cdot e^{-k \cdot x} + 80$)

solve($15=c1 \cdot e^{-k \cdot 0} + 80, c1$) $c1 = -65$

© Find the value of k

solve($60=80-65 \cdot e^{-10 \cdot k}, k$) $k = \frac{\ln\left(\frac{13}{4}\right)}{10}$

5/99

e.i.

$$T = 55e^{-m(x-10)} + 5$$

When $T = 30$, $x = 10 + 15 = 25$

Solve for m , $30 = 55e^{-m(25-10)} + 5$

$$m = \frac{1}{15} \log_e\left(\frac{11}{5}\right) = \frac{\log_e(2.2)}{15}, \text{ as required}$$

1A

$t=c2 \cdot e^{-m \cdot x} + 5$ | $c2=55 \cdot e^{-10 \cdot m}$

$t=55 \cdot e^{-m \cdot x - 10 \cdot m} + 5$

© Find the value of m

solve($30=55 \cdot e^{-m \cdot 25 - 10 \cdot m} + 5, m$) $m = \frac{-\ln\left(\frac{11}{5}\right)}{15}$

|

6/99

e.ii.

Solve for x , $10 = 55e^{-\frac{\log_e(2.2)}{15}(x-10)} + 5$

$$x \approx 56$$

The time that the solution needs to be in the cooling room is $(56 - 10) = 46$ minutes

1A

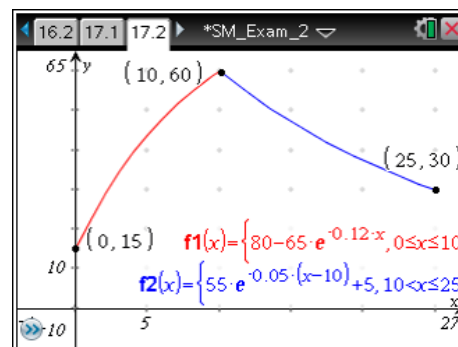
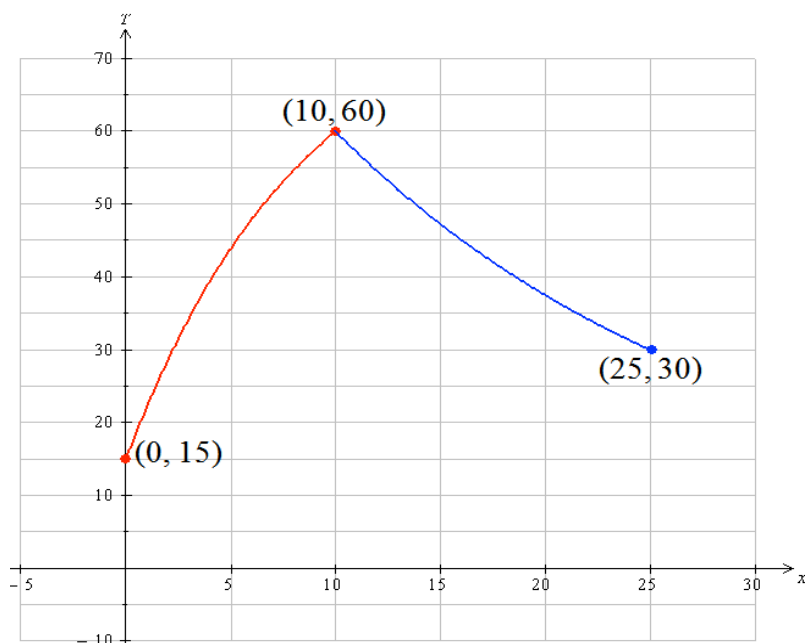
solve($30=55 \cdot e^{10 \cdot m - m \cdot 25} + 5, m$) $m = \frac{\ln\left(\frac{11}{5}\right)}{15}$

© Find the value of x when T=10

solve($10=55 \cdot e^{10 \cdot m - m \cdot x} + 5, x$) $x = 55.618737$

2/8

f.



Correct shape (growth followed by decay)

1A

Correct domains, with endpoints correctly labelled

1A

END OF SOLUTIONS