

$$\begin{aligned} \text{Q1 } \int \frac{1+x}{9-x^2} dx &= \int \left(\frac{\frac{2}{3}}{3-x} - \frac{\frac{1}{3}}{3+x} \right) dx \quad (\text{partial fractions}) \\ &= -\frac{2}{3} \log_e |3-x| - \frac{1}{3} \log_e |3+x| = -\frac{1}{3} (2 \log_e |3-x| + \log_e |3+x|) \\ &= -\frac{1}{3} \log_e (3-x)^2 |3+x| \end{aligned}$$

Alternatively, $-\frac{1}{3} \log_e |3-x| |9-x^2|$

$$\text{Q2 } y = kxe^{2x}, \frac{dy}{dx} = ke^{2x} + 2kxe^{2x}, \frac{d^2y}{dx^2} = 2ke^{2x} + 2\frac{dy}{dx},$$

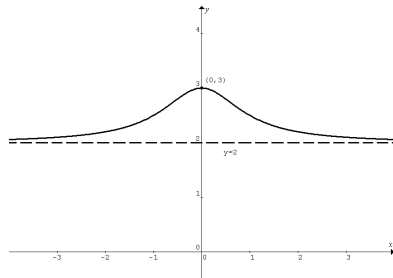
$$\therefore \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 2ke^{2x} + 5kxe^{2x}$$

$$\therefore e^{2x}(15x+6) = 2ke^{2x} + 5kxe^{2x}$$

$$e^{2x}(15x+6) = e^{2x}(2k+5kx), \therefore k = 3$$

$$\text{Q3a } f(x) = \frac{2x^2+3}{x^2+1} = \frac{2(x^2+1)+1}{x^2+1} = 2 + \frac{1}{x^2+1}$$

Q3b



Q3c Area

$$= 2 \times \int_0^1 \left(2 + \frac{1}{1+x^2} \right) dx = 2 \left[2x + \tan^{-1} x \right]_0^1 = 2 \left(2 + \frac{\pi}{4} \right) = 4 + \frac{\pi}{2}$$

$$\begin{aligned} \text{Q4 } z &= \frac{1-\sqrt{3}i}{-1+i} = \frac{2cis(-\frac{\pi}{3})}{\sqrt{2}cis(\frac{3\pi}{4})} = \sqrt{2}cis\left(-\frac{\pi}{3} - \frac{3\pi}{4}\right) \\ &= \sqrt{2}cis\left(-\frac{13\pi}{12}\right) = \sqrt{2}cis\left(\frac{11\pi}{12}\right), \therefore \text{Arg}(z) = \frac{11\pi}{12} \end{aligned}$$

$$\text{Q5 } x = 4 \sin t - 1, \frac{dx}{dt} = 4 \cos t, \sin t = \frac{x+1}{4}$$

$$y = 2 \cos t + 3, \frac{dy}{dt} = -2 \sin t, \cos t = \frac{y-3}{2}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2 \sin t}{4 \cos t} = \frac{-\frac{x+1}{4}}{y-3}$$

$$\text{At } (1, \sqrt{3}+3), \frac{dy}{dx} = \frac{-\frac{1}{2}}{\sqrt{3}} = -\frac{1}{2\sqrt{3}} = -\frac{\sqrt{3}}{6}$$

$$\begin{aligned} \text{Q6 } \int_0^1 e^x \cos(e^x) dx &= \int_1^e \cos(u) \frac{du}{dx} dx = \int_1^e \cos(u) du \\ &= [\sin(u)]_1^e = \sin(e) - \sin(1) \end{aligned}$$

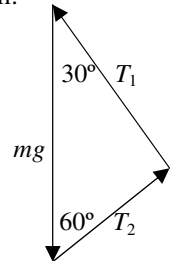
Q7a Add the force vectors head to tail.

$$\frac{T_1}{T_2} = \tan 60^\circ = \sqrt{3}, \therefore T_2 = \frac{T_1}{\sqrt{3}}$$

$$\text{Q7b } \cos 60^\circ = \frac{T_2}{mg}, \therefore \frac{1}{2} = \frac{T_2}{9.8m}$$

$$\text{At breaking point, } \frac{1}{2} = \frac{98}{9.8m}$$

$$\therefore m = 20 \text{ kg}$$



$$\text{Q8 Let } \cos^2\left(\frac{\pi x}{6}\right) = \frac{4}{3}, \sin^2\left(\frac{\pi x}{6}\right) = \frac{3}{4}, 1 - 2\sin^2\left(\frac{\pi x}{6}\right) = -\frac{1}{2}$$

$$\therefore \cos\left(\frac{\pi x}{3}\right) = -\frac{1}{2} \text{ where } 0 < x < 12, \text{ i.e. } 0 < \frac{\pi x}{3} < 4\pi$$

$$\therefore \frac{\pi x}{3} = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3},$$

$$\therefore x = 2, 4, 8, 10 \text{ and } y = \frac{4}{3}$$

The intersecting points are $\left(2, \frac{4}{3}\right), \left(4, \frac{4}{3}\right), \left(8, \frac{4}{3}\right)$ and $\left(10, \frac{4}{3}\right)$.

$$\text{Q9a } \tilde{a} = \tilde{i} - \tilde{j} + 2\tilde{k}, \tilde{b} = \tilde{i} + 2\tilde{j} + m\tilde{k}, \tilde{c} = \tilde{i} + \tilde{j} - \tilde{k}$$

$$|\tilde{b}| = 2\sqrt{3}, |\tilde{b}|^2 = 12, \therefore 1 + 4 + m^2 = 12, m = \pm\sqrt{7}$$

$$\text{Q9b } \tilde{a} \cdot \tilde{b} = 0, \therefore 1 - 2 + 2m = 0, m = \frac{1}{2}$$

$$\text{Q9ci } 3\tilde{c} - \tilde{a} = 3(\tilde{i} + \tilde{j} - \tilde{k}) - (\tilde{i} - \tilde{j} + 2\tilde{k}) = 2\tilde{i} + 4\tilde{j} - 5\tilde{k}$$

$$\text{Q9cii } \tilde{a}, \tilde{b} \text{ and } \tilde{c} \text{ are linearly dependent when } 3\tilde{c} - \tilde{a} = n\tilde{b}, \text{ where } n \text{ is a real constant. } \therefore 2\tilde{i} + 4\tilde{j} - 5\tilde{k} = n(\tilde{i} + 2\tilde{j} + m\tilde{k})$$

$$\therefore n = 2 \text{ and } nm = -5, \therefore m = -\frac{5}{2}$$

$$\text{Q10 } y \log_e x = e^{2y} + 3x - 4$$

$$\text{By implicit differentiation, } \frac{dy}{dx} \log_e x + \frac{y}{x} = 2e^{2y} \frac{dy}{dx} + 3$$

$$\frac{dy}{dx} (\log_e x - 2e^{2y}) = 3 - \frac{y}{x}. \text{ At } (1, 0), \frac{dy}{dx} (-2) = 3, \therefore \frac{dy}{dx} = -\frac{3}{2}.$$

$$\text{Q11 } V = \int_0^{\frac{\pi}{6}} \pi \sin^2 x dx = \frac{\pi}{2} \int_0^{\frac{\pi}{6}} (1 - \cos 2x) dx = \frac{\pi}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{\frac{\pi}{6}}$$

$$\frac{\pi}{2} \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4} \right) = \frac{\pi}{24} (2\pi - 3\sqrt{3})$$

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