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Specialist Mathematics

2011

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working or explanation **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

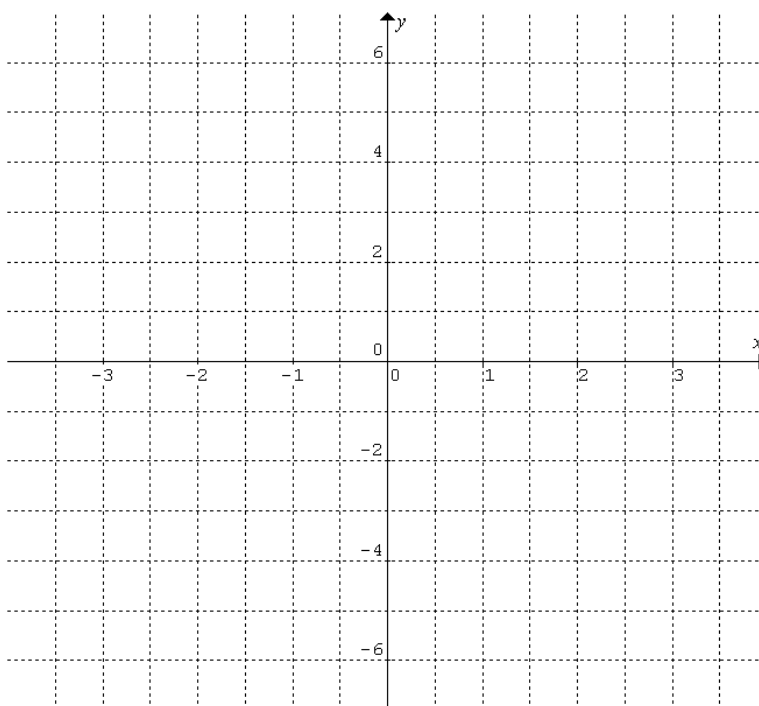
Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1

- a. Sketch the graph of $y = \sqrt{3} \cos^{-1}\left(\frac{x}{2}\right)$. Clearly label the end-points and axis-intercepts with exact coordinates.

(Use $\pi \approx 3.14$ and $\sqrt{3} \approx 1.73$ to help you to sketch)

2 marks



- b. Find the exact value of $\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3} \cos^{-1}\left(\frac{x}{2}\right) dx$.

2 marks

Question 2

Let $A(-2,1,0)$, $B(-1,2,-2)$ and $C(0,-3,4)$ be three points in 3-dimensional space.

a. Determine whether A , B and C are *collinear* or not.

2 marks

b. Determine whether the position vectors of A , B and C are *coplanar* or not.

2 marks

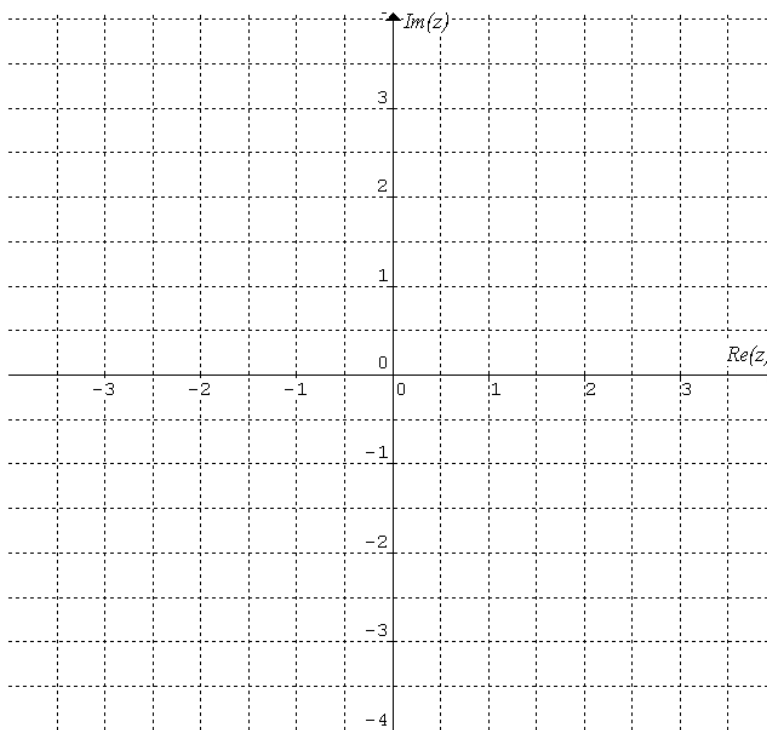
c. Find a vector that bisects the angle between the position vectors of B and C .

2 marks

Question 3

Shade clearly the region in the complex plane for $\{z : 1 \leq z + \bar{z} \leq 2\} \cap \{z : 1 \leq |z - 1 + i| \leq 2\}$.

3 marks



Question 4

Let $z = \cos \theta + i \sin \theta$.

a. Find $|iz - z|$. 1 mark

b. Find $\arg(iz - z)$. 2 marks

c. Hence, or otherwise, show that $\cos \theta - \sin \theta = \sqrt{2} \sin\left(\frac{3\pi}{4} + \theta\right)$. 1 mark

Question 5

The position of a particle is given by $\tilde{r} = \frac{t}{2} \tilde{i} + (49t - 4.9t^2) \tilde{j}$, where time $t \geq 0$ is measured in seconds and distance is in metres. $\tilde{i}$ is a horizontal unit vector to the right and $\tilde{j}$ a vertical vector pointing upward.

a. Find the initial speed (correct to the nearest whole number) of the particle. 1 mark

b. Show that the acceleration of the particle is constant. 1 mark

c. Find the horizontal displacement when the particle returns to the same level as its initial position. 1 mark

d. Find the cartesian equation of the path of the particle's motion. 1 mark

Question 6

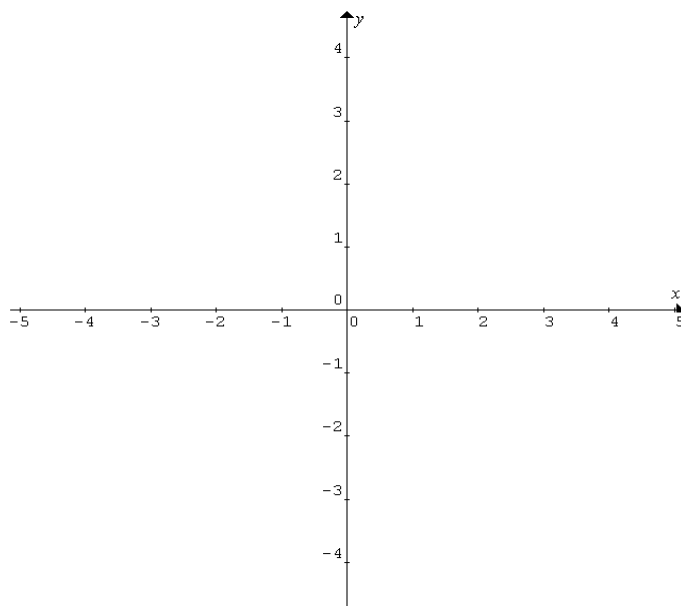
y is a differentiable function of x and satisfies $8x^3 - y^3 = 1 - 6xy^2 + 12x^2y$.

a. Find $\frac{dy}{dx}$.

2 marks

b. Sketch the graph of $8x^3 - y^3 = 1 - 6xy^2 + 12x^2y$.

2 marks

**Question 7**

Let $f(x) = \frac{\log_e(x^2)}{|x|}$ where $x \in \mathbb{R} \setminus \{0\}$.

a. Find the exact value of $f'(-e)$.

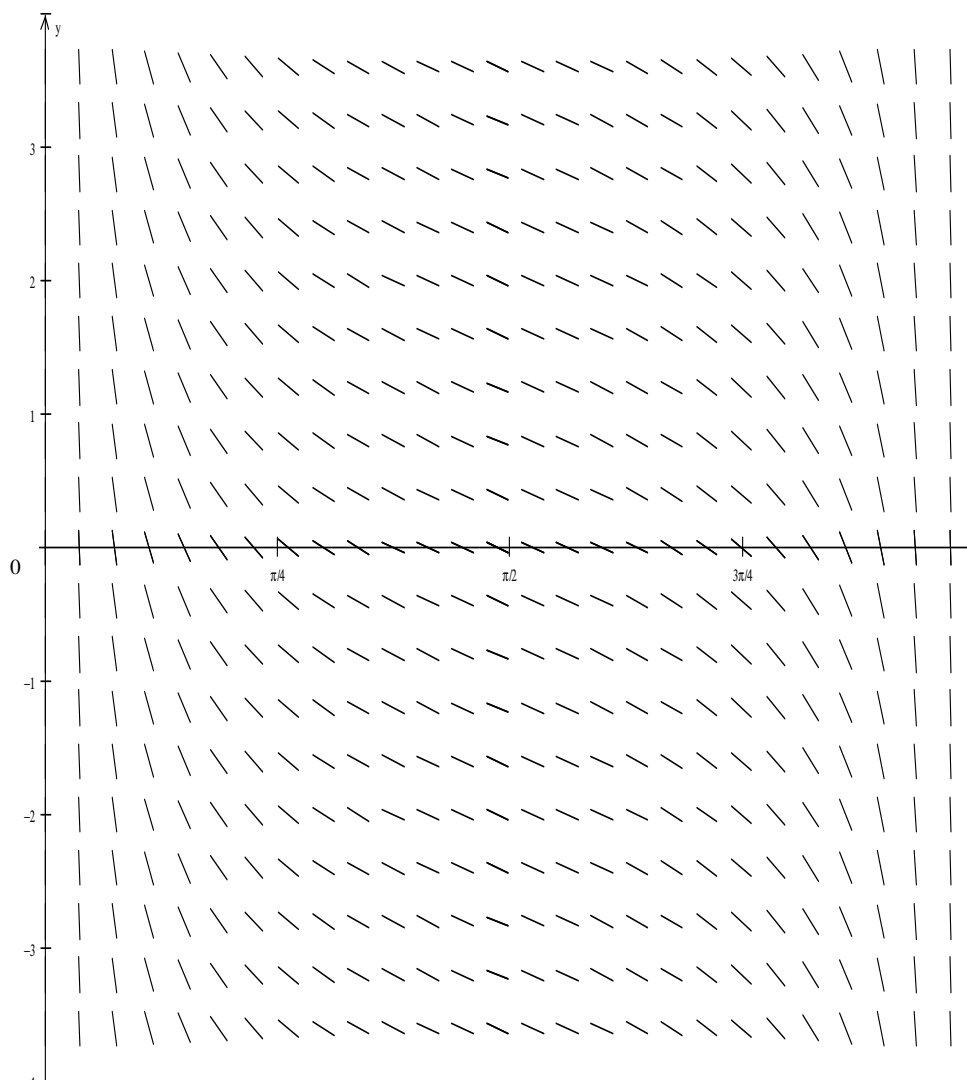
2 marks

b. Find the exact value of $\int_{-e}^{-1} f(x) dx$.

2 marks

Question 8

The slope field for an unknown differential equation $\frac{dy}{dx} = f(x)$ is shown below, where $x = 0$ and $x = \pi$ are asymptotes.



a. Sketch a solution curve to the differential equation $\frac{dy}{dx} = f(x)$.

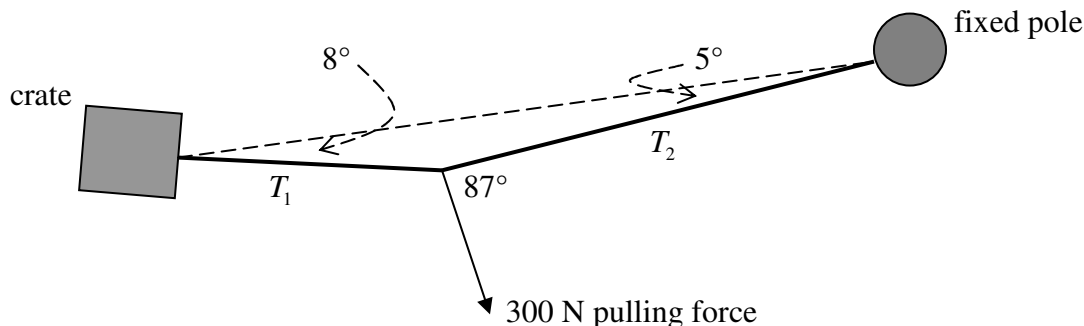
1 mark

b. Hence determine $f(x)$ in simplest form.

2 marks

Question 9

A strong rope connects a large crate to a fixed pole. The crate is about to move when a person pulls the rope with a force of 300 N. The following diagram shows the aerial view of the situation. Ignore the weight of the rope in your calculations.



- a. Draw a clear triangle of forces to represent the pulling force and the tension forces T_1 and T_2 . Show the measure of each of the interior angles of the triangle of forces.

2 marks

- b. Use the triangle of forces to find tension T_1 to the nearest newton.

2 marks

Question 10

A 0.2-kg particle is moving along the x -axis. Its velocity at position x is given by $v = \pm\sqrt{10 - 8x - 2x^2}$. Time is measured in seconds and distance in metres.

- a. Find the acceleration of the particle at the origin ($x = 0$).

1 mark

- b. Find the resultant force on the particle at the origin.

1 mark

- c. Find the maximum speed of the particle.

2 marks

End of Exam 1