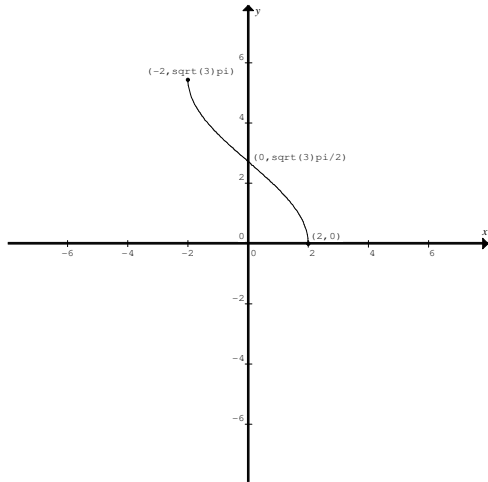
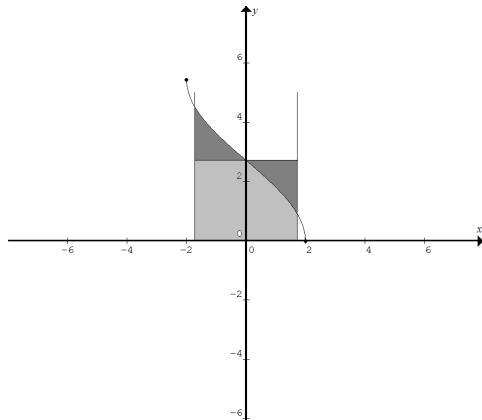


Q1a $y = \sqrt{3} \cos^{-1}\left(\frac{x}{2}\right)$



Q1b



From the graph, $\int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{3} \cos^{-1}\left(\frac{x}{2}\right) dx = \text{area under graph}$
 $= \text{area of rectangle} = 2\sqrt{3} \times \frac{\pi\sqrt{3}}{2} = 3\pi$

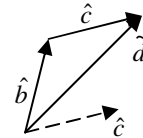
Q2a $A(-2,1,0)$, $B(-1,2,-2)$ and $C(0,-3,4)$

$\vec{OA} = -2\tilde{i} + \tilde{j}$, $\vec{OB} = -\tilde{i} + 2\tilde{j} - 2\tilde{k}$, $\vec{OC} = -3\tilde{j} + 4\tilde{k}$
 $\vec{AB} = \vec{OB} - \vec{OA} = \tilde{i} + \tilde{j} - 2\tilde{k}$, $\vec{BC} = \vec{OC} - \vec{OB} = \tilde{i} - 5\tilde{j} + 6\tilde{k}$
 $\vec{BC} \neq m\vec{AB}$, $\therefore A, B$ and C are not collinear.

Q2b $\vec{OA} = -2\tilde{i} + \tilde{j}$, $\vec{OB} = -\tilde{i} + 2\tilde{j} - 2\tilde{k}$, $\vec{OC} = -3\tilde{j} + 4\tilde{k}$,
 $\vec{OA} - 2\vec{OB} - \vec{OC} = 0$. $\therefore \vec{OA}$, $\vec{OB}$ and $\vec{OC}$ are linearly dependent. Hence the three position vectors are coplanar.

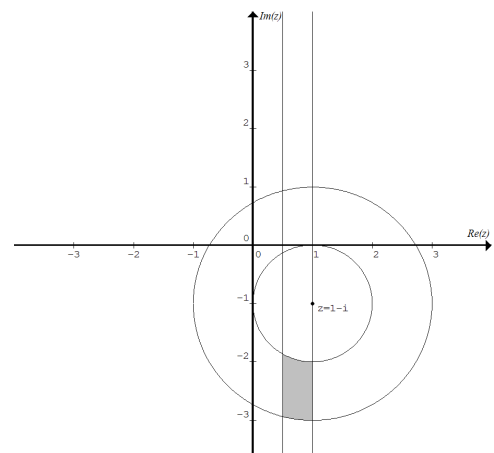
Q2c Let $\hat{b}$ be a unit vector in the direction of $\vec{OB}$ and $\hat{c}$ a unit vector in the direction of $\vec{OC}$.

$\hat{b} = \frac{1}{3}(-\tilde{i} + 2\tilde{j} - 2\tilde{k})$, $\hat{c} = \frac{1}{5}(-3\tilde{j} + 4\tilde{k})$

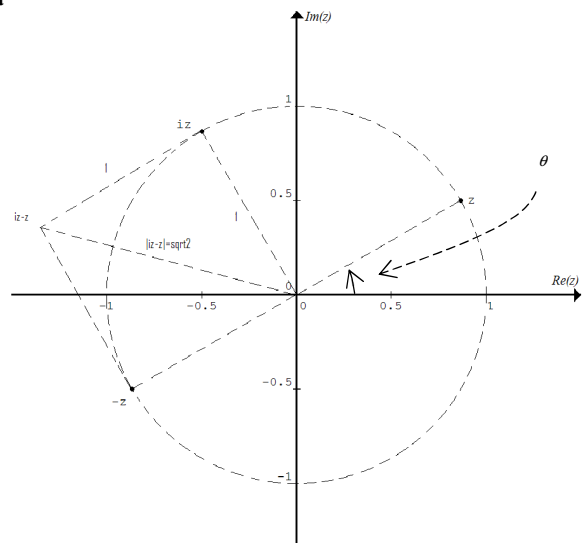


A vector that bisects the angle between $\vec{OB}$ and $\vec{OC}$ is
 $\tilde{d} = \hat{b} + \hat{c} = -\frac{1}{3}\tilde{i} + \frac{1}{15}\tilde{j} + \frac{2}{15}\tilde{k}$.

Q3 $\{z : 1 \leq z + \bar{z} \leq 2\} \cap \{z : 1 \leq |z - 1 + i| \leq 2\}$.



Q4a



Refer to the diagram above, $|iz - z| = \sqrt{2}$.

Q4b $\arg(iz - z) = \theta + \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4} + \theta$

Q4c $z = \cos \theta + i \sin \theta$, $iz = -\sin \theta + i \cos \theta$
 $\therefore iz - z = (-\sin \theta - \cos \theta) + i(\cos \theta - \sin \theta)$

From parts **a** and **b**,

$$iz - z = \sqrt{2} \operatorname{cis}\left(\frac{3\pi}{4} + \theta\right) = \sqrt{2} \cos\left(\frac{3\pi}{4} + \theta\right) + i\sqrt{2} \sin\left(\frac{3\pi}{4} + \theta\right)$$

$$\therefore \cos \theta - \sin \theta = \sqrt{2} \sin\left(\frac{3\pi}{4} + \theta\right)$$

Q5a $\tilde{r} = \frac{t}{2} \tilde{i} + (49t - 4.9t^2) \tilde{j}$, $\tilde{r} = \frac{1}{2} \tilde{i} + (49 - 9.8t) \tilde{j}$

At $t = 0$, $\tilde{r} = \frac{1}{2} \tilde{i} + 49 \tilde{j}$

$$\therefore \text{initial speed} = |\tilde{r}| = \sqrt{0.5^2 + 49^2} \approx 49 \text{ ms}^{-1}$$

Q5b $\tilde{a} = \tilde{r} = -9.8 \tilde{j}$ a constant vector

Q5c $\tilde{r} = \frac{t}{2} \tilde{i} + (49t - 4.9t^2) \tilde{j}$

At $t = 0$, $\tilde{r} = \tilde{0}$ and the $\tilde{j}$ -component of $\tilde{r}$ is 0.

Let $49t - 4.9t^2 = 0$ and $t > 0$, $\therefore t = 10$ and $\tilde{r} = 5 \tilde{i}$.

$$\therefore \text{displacement} = 5 \tilde{i} - \tilde{0} = 5 \tilde{i}$$

Q5d $x = \frac{t}{2}$, $\therefore t = 2x$

$$y = 49t - 4.9t^2 = 49(2x) - 4.9(2x)^2$$

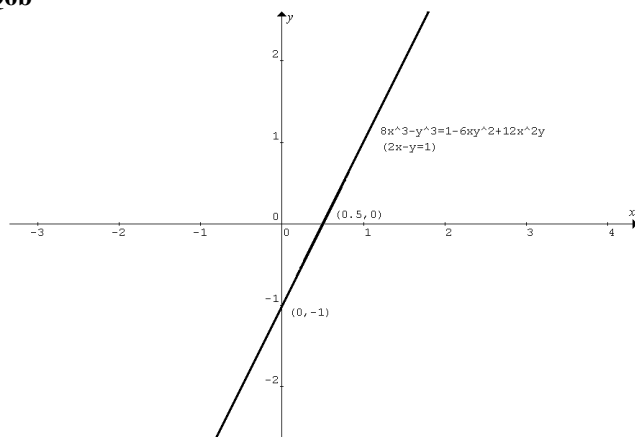
$$\therefore y = 98x - 19.6x^2, x \geq 0$$

Q6a $8x^3 - y^3 = 1 - 6xy^2 + 12x^2y$

$$8x^3 - 12x^2y + 6xy^2 - y^3 = 1, (2x - y)^3 = 1$$

$$\therefore 2x - y = 1, \frac{dy}{dx} = 2$$

Q6b



Intercepts are $(0.5, 0)$ and $(0, -1)$.

Q7a $f(x) = \frac{\log_e(x^2)}{|x|}$,

$$f'(x) = \frac{|x|(\frac{2}{x}) - \log_e(x^2) \times \frac{d|x|}{dx}}{x^2} = \begin{cases} \frac{-2 + \log_e(x^2)}{x^2}, & x < 0 \\ \frac{2 - \log_e(x^2)}{x^2}, & x > 0 \end{cases}$$

$$\therefore f'(-e) = \frac{-2 + \log_e(-e)^2}{(-e)^2} = 0$$

Q7b For $x < 0$, $f(x) = \frac{\log_e(x^2)}{|x|} = \frac{2\log_e|x|}{|x|} = -\frac{2\log_e|x|}{x}$

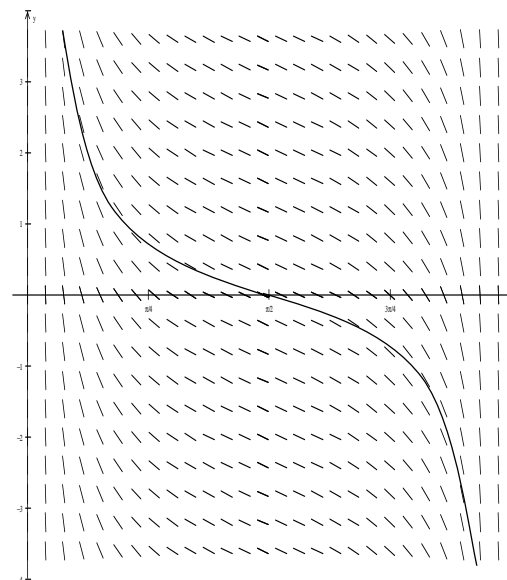
$$\therefore \int_{-e}^{-1} f(x) dx = \int_{-e}^{-1} -\frac{2\log_e|x|}{x} dx$$

$$= \int_{-e}^{-1} -2u \frac{du}{dx} dx = \int_1^0 -2udu$$

$$= \int_0^1 2udu = [u^2]_0^1 = 1$$

Let $u = \log_e|x|$, $\frac{du}{dx} = \frac{1}{x}$

Q8a



Q8b $y = \cot x = \frac{\cos x}{\sin x}$

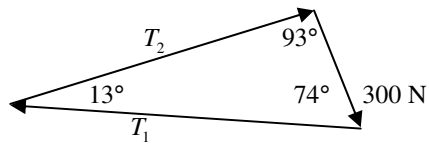
$$\frac{dy}{dx} = \frac{(\sin x)(-\sin x) - (\cos x)(\cos x)}{\sin^2 x}$$

$$= \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = -\frac{1}{\sin^2 x}$$

$$= -\operatorname{cosec}^2 x$$

$$\therefore f(x) = -\operatorname{cosec}^2 x$$

Q9a



Q9b The sine rule: $\frac{T_1}{\sin 93^\circ} = \frac{300}{\sin 13^\circ}$

$$T_1 = \frac{300 \sin 93^\circ}{\sin 13^\circ} \approx 1332 \text{ N}$$

Q10a $v = \pm \sqrt{10 - 8x - 2x^2}$, $\therefore \frac{1}{2}v^2 = 5 - 4x - x^2$

$$a = \frac{d}{dx} \left(\frac{1}{2}v^2 \right) = -4 - 2x$$

At $x=0$, $a = -4$

Q10b Resultant force $F = ma = 0.2 \times -4 = -0.8 \text{ N}$

Q10c Maximum speed occurs when $a = 0$, i.e. $-4 - 2x = 0$
 $x = -2$

$$\therefore \text{maximum speed} = \sqrt{10 + 16 - 8} = 3\sqrt{2} \text{ ms}^{-1}.$$

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors