



Puffy Publications (P²) Trial Exam 2011

	STUDENT NUMBER								Letter
Figures									
Words									

Specialist Mathematics

Written Examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, blank sheets of paper, white out liquid/tape or a calculator of any type.

Materials supplied

- Question and answer book of 11 pages, with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Candidates are reminded that this Exam, produced by Puffy Publications, is NOT an official VCAA Paper for the 2011 Specialist Mathematics Examination 1. This paper has been produced with the sole purpose of assisting students in exam preparation and is not endorsed or supported by the VCAA.

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Instructions

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1

- a. Find an expression for $\frac{dy}{dx}$ in terms of x and y for $kxy^3 - 3y^2 = 8x$

2 marks

- b. Hence, find the exact value of $\frac{dy}{dx}$ at the point $(-\frac{1}{2}, 1)$

2 marks

TURN OVER

Question 2

Find the exact solutions to the equation $\cot^2\left(x + \frac{\pi}{3}\right) = 3$ for the domain $[0, 2\pi]$

2 marks

Question 3

Evaluate $\int_{\frac{3}{\pi}}^{\frac{6}{\pi}} \left(\frac{1}{x^2} \sin \left(\frac{1}{x} \right) \right) dx$

[illegible]

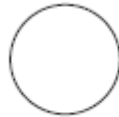
2 marks

TURN OVER

Question 4

Tarundeep is trying to impress Karishma by showing off his strength. He throws a 2 kilogram medicine ball vertically upwards with an initial velocity of 10m/s. The air resistance acting on the medicine ball is equal to $\frac{2v^2}{g}$ Newtons.

- a. On the diagram below, show all the forces acting on the medicine ball on its flight up



1 mark

- b. Show that, on its upwards path, the acceleration is, $a = -g - \frac{v^2}{g}$

1 mark

- c. Solve a suitable differential equation to express the velocity, v , in terms of the displacement, x .

3 marks

TURN OVER

Question 5

Let $\underline{u} = 2\underline{i} + \underline{j} + 3\underline{k}$ and $\underline{v} = \underline{i} - 2\underline{j} - \underline{k}$.

- a. If $\underline{x} = a\underline{i} + b\underline{j} + c\underline{k}$ is perpendicular to both $\underline{u}$ and $\underline{v}$, write two equations in terms of a , b and c .

2 marks

- b. Let $c = \gamma$, find a and b in terms of the parameter γ

2 marks

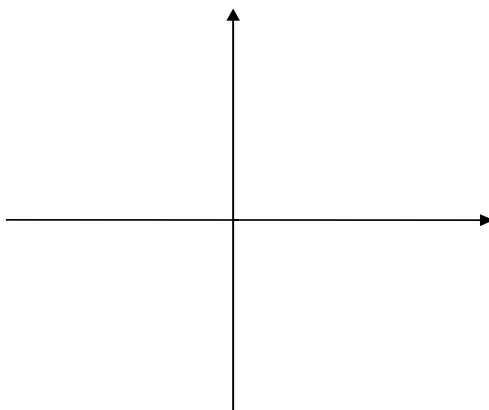
- c. Hence, express the vector $\underline{x}$ in terms of γ

1 mark

TURN OVER

Question 6

- a. On the axes below, sketch the graph of $y = 2 \sin^{-1}(x - 1)$, showing endpoints and intercepts with the co-ordinate axes.



1 mark

- b. Find the area bounded by the curve, the y-axis and the line $y = \pi$

1 mark

- c. The area described above is now rotated around the y-axis in order to produce Tarundeep's champagne glass. Find the volume of the produced solid.

3 marks

TURN OVER

Question 7

- a. A block of mass m kilograms is placed on a plane inclined at 30° to the horizontal. If the block experiences an acceleration of 4ms^{-2} down the plane, find the value of the co-efficient of friction, in terms of g .

2 marks

- b. Tarundeep now attempts to pull the block up the slope with a force of T Newtons. If the block has a mass of 1kg and is on the point of moving up the slope, find the value of T .

2 marks

TURN OVER

Question 8

- a.** Given that $z + \frac{1}{z} = k$, where k is a real constant, show that z either lies on the $\text{Re}(z)$ axis, or the Unit Circle centred at the Origin.

[illegible]

2 marks

- b.** If z lies on the $\text{Re}(z)$ axis, show that $|k| \geq 2$.

[illegible]

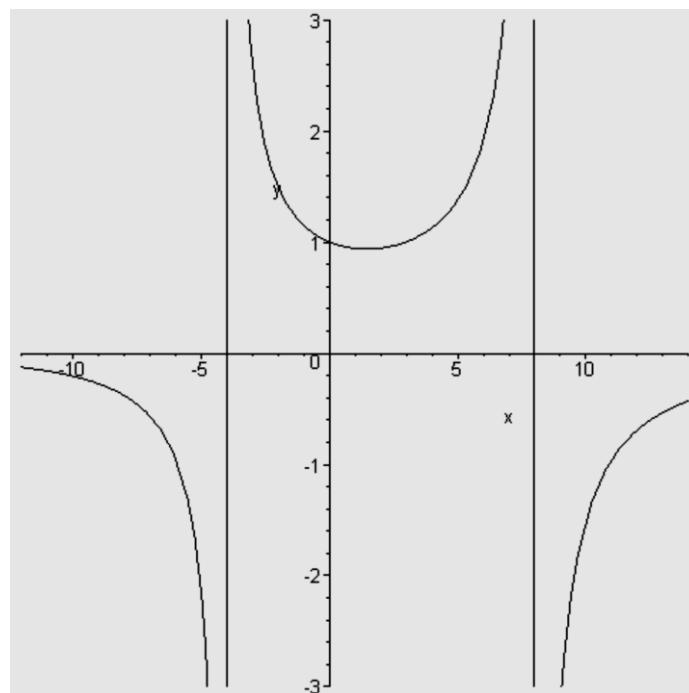
2 marks

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Question 9

- a. Find the asymptotes of the graph of $y = \frac{x+32}{-x^2+4x+32}$

1 mark



- b. The graph of the function is shown above. Decompose the expression for the function into partial fractions.

2 mark

TURN OVER

- c. The area bounded by the graph, the co-ordinate axes and the line $x=4$ can be expressed as $a \log_e(2)$. Find the value of a .

[illegible]

2 mark

TURN OVER

Question 10

- a. Solve the equation $z^2 - 4z + 6 = 0$ over C

1 mark

- b. These two solutions are also solutions to a cubic equation, $z^3 + az^2 + bz + 6 = 0$.
Find the values of a and b .

2 marks

- c. Find the other solution to the above cubic

1 mark

END OF QUESTION AND ANSWER BOOK