



Victorian Certificate of Education 2010

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Letter

Figures

Words

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SPECIALIST MATHEMATICS

Written examination 2

Monday 1 November 2010

Reading time: 3.00 pm to 3.15 pm (15 minutes)

Writing time: 3.15 pm to 5.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 23 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1**Instructions for Section 1**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

An ellipse has a horizontal semi-axis length of 1 and a vertical semi-axis length of 2. The centre of the ellipse is at the point with coordinates $(3, -5)$.

A possible equation for the ellipse is

- A. $(x + 3)^2 + 4(y - 5)^2 = 4$
- B. $(x - 3)^2 + 4(y + 5)^2 = 4$
- C. $4(x + 3)^2 + (y - 5)^2 = 4$
- D. $4(x - 3)^2 + (y + 5)^2 = 4$
- E. $4(x - 3)^2 + (y + 5)^2 = 1$

Question 2

Each of the following equations represents a hyperbola.

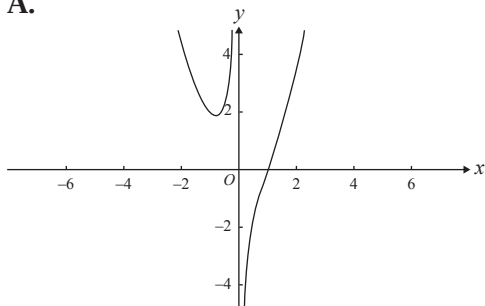
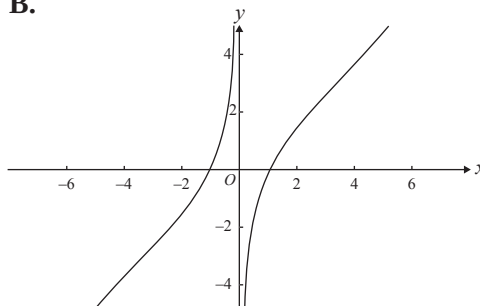
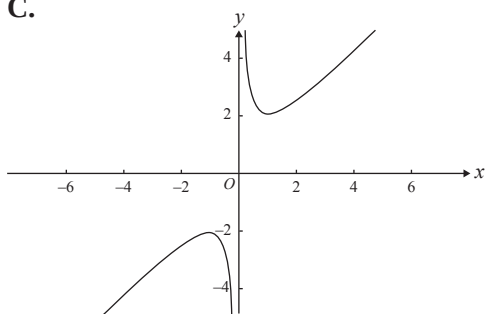
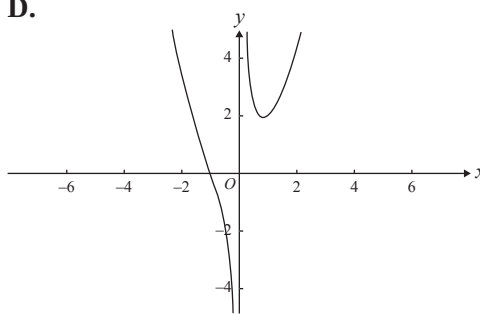
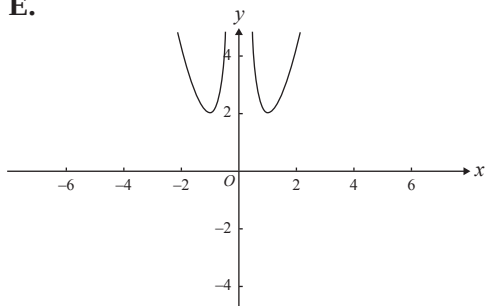
Which hyperbola **does not** have perpendicular asymptotes?

- A. $(x - 1)^2 - (y + 2)^2 = 1$
- B. $x^2 - 2x - y^2 + 4y = 4$
- C. $(x - 1)^2 - (y + 2)^2 = 9$
- D. $(y - 1)^2 - (x + 2)^2 = 1$
- E. $2x^2 - 4x - y^2 - 4y = 4$

Question 3

Let $f(x) = \frac{x^k + a}{x}$, where k and a are real constants.

If k is an odd integer which is greater than 1 and $a < 0$, a possible graph of f could be

A.**B.****C.****D.****E.****Question 4**

The position vector of a particle at time $t \geq 0$ is given by $\underline{r} = \sin(t)\underline{i} + \cos(2t)\underline{j}$.

The path of the particle has cartesian equation

A. $y = 2x^2 - 1$

B. $y = 1 - 2x^2$

C. $y = \sqrt{1 - x^2}$

D. $y = \sqrt{x^2 - 1}$

E. $y = 2x\sqrt{1 - x^2}$

Question 5

For $-\frac{\pi}{2} < x < \frac{\pi}{2}$, the graphs of the two curves given by $y = 2 \sec^2(x)$ and $y = 5|\tan(x)|$ intersect

- A. only at the one point $(\arctan(2), 10)$
- B. only at the two points $(\pm \arctan(2), 10)$
- C. only at the one point $\left(\arctan\left(\frac{1}{2}\right), \frac{5}{2}\right)$
- D. only at the two points $\left(\pm \arctan\left(\frac{1}{2}\right), \frac{5}{2}\right)$
- E. at the two points $\left(\pm \arctan\left(\frac{1}{2}\right), \frac{5}{2}\right)$, as well as at the two points $(\pm \arctan(2), 10)$

Question 6

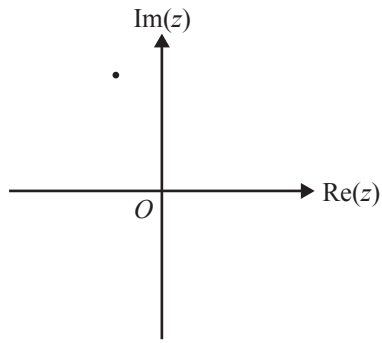
Let $z = \operatorname{cis}\left(\frac{5\pi}{6}\right)$.

The imaginary part of $z - i$ is

- A. $-\frac{i}{2}$
- B. $-\frac{1}{2}$
- C. $-\frac{\sqrt{3}}{2}$
- D. $-\frac{3}{2}$
- E. $-\frac{3i}{2}$

Question 7

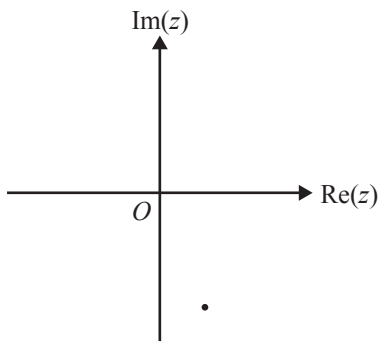
A particular complex number z is represented by the point on the following argand diagram.



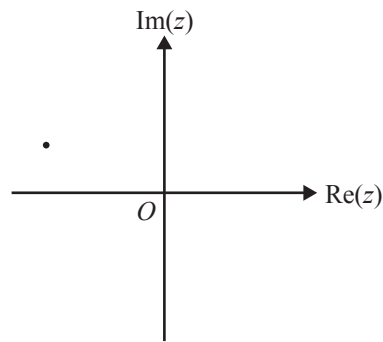
All axes below have the **same scale** as those in the diagram above.

The complex number $i\bar{z}$ is best represented by

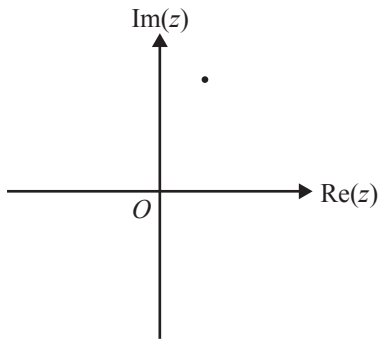
A.



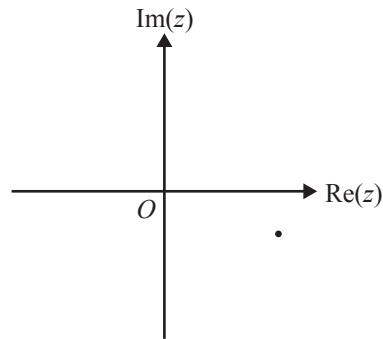
B.



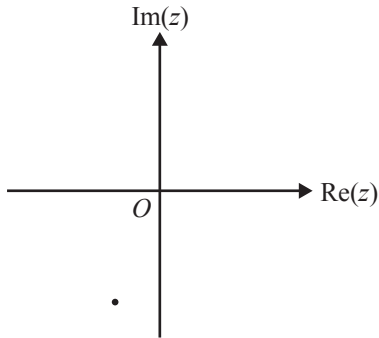
C.



D.



E.



Question 8

The polynomial equation $P(z) = 0$ has one complex coefficient. Three of the roots of this equation are $z = 3 + i$, $z = 2 - i$ and $z = 0$.

The **minimum degree** of $P(z)$ is

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

Question 9

Given that $z = 4\text{cis}\left(\frac{2\pi}{3}\right)$, it follows that $\text{Arg}(z^5)$ is

- A. $\frac{10\pi}{3}$
- B. $\frac{4\pi}{3}$
- C. $\frac{7\pi}{3}$
- D. $-\frac{\pi}{3}$
- E. $-\frac{2\pi}{3}$

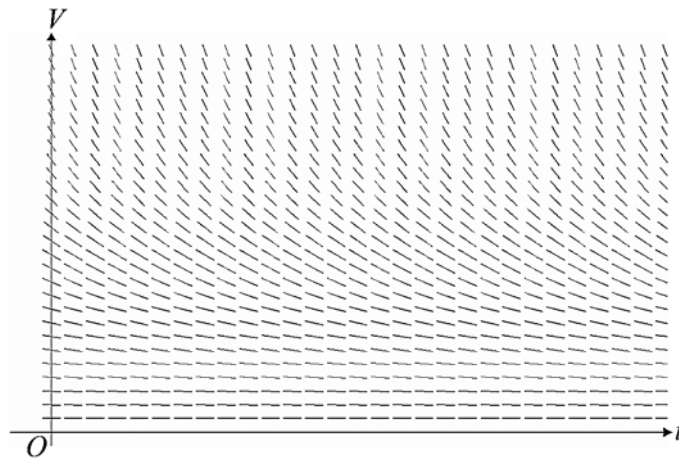
Question 10

On an argand diagram, a set of points which lies on a circle of radius 2 centred at the origin is

- A. $\{z \in C : z\bar{z} = 2\}$
- B. $\{z \in C : z^2 = 4\}$
- C. $\{z \in C : \text{Re}(z^2) + \text{Im}(z^2) = 4\}$
- D. $\{z \in C : (z + \bar{z})^2 - (z - \bar{z})^2 = 16\}$
- E. $\{z \in C : (\text{Re}(z))^2 + (\text{Im}(z))^2 = 16\}$

Question 11

A direction field for the volume of water, V megalitres, in a reservoir t years after 2010 is shown below.



According to this model, for $k > 0$, $\frac{dV}{dt}$ is equal to

- A. $-kt^2$
- B. $\frac{k}{V}$
- C. $-kV^2$
- D. kV^2
- E. $-\frac{k}{V}$

Question 12

Let $\frac{dy}{dx} = \frac{x+2}{x^2+2x+1}$ and $(x_0, y_0) = (0, 2)$.

Using Euler's method, with a step size of 0.1, the value of y_1 correct to two decimal places is

- A. 0.17
- B. 0.20
- C. 1.70
- D. 2.17
- E. 2.20

Question 13

The amount of a drug, x mg, remaining in a patient's bloodstream t hours after taking the drug is given by the differential equation

$$\frac{dx}{dt} = -0.15x.$$

The number of hours needed for the amount x to halve is

- A. $2\log_e\left(\frac{20}{3}\right)$
- B. $\frac{20}{3}\log_e(2)$
- C. $2\log_e(15)$
- D. $15\log_e\left(\frac{3}{2}\right)$
- E. $\frac{3}{2}\log_e(200)$

Question 14

Use a suitable substitution to show that the definite integral $\int_0^2 \frac{x}{\sqrt{x^2-1}} dx$ can be simplified to

- A. $\frac{1}{2} \int_{-1}^3 u^{-\frac{1}{2}} du$
- B. $2 \int_{-1}^3 u^{-\frac{1}{2}} du$
- C. $\frac{1}{2} \int_0^2 u^{-\frac{1}{2}} du$
- D. $2 \int_0^2 u^{-\frac{1}{2}} du$
- E. $\int_0^2 u^{-\frac{1}{2}} du$

Question 15

The scalar resolute of $\underline{a} = 3\underline{i} - \underline{k}$ in the direction of $\underline{b} = 2\underline{i} - \underline{j} - 2\underline{k}$ is

- A. $\frac{8}{\sqrt{10}}$
- B. $\frac{8}{9}(2\underline{i} - \underline{j} - 2\underline{k})$
- C. 8
- D. $\frac{4}{5}(3\underline{i} - \underline{k})$
- E. $\frac{8}{3}$

Question 16

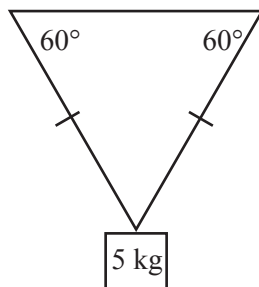
The square of the magnitude of the vector $\underline{d} = 5\underline{i} - \underline{j} + \sqrt{10}\underline{k}$ is

- A. 6
- B. 34
- C. 36
- D. 51.3
- E. $\sqrt{34}$

Question 17

The angle between the vectors $\underline{a} = \underline{i} + \underline{k}$ and $\underline{b} = \underline{i} + \underline{j}$ is exactly

- A. $\frac{\pi}{6}$
- B. $\frac{\pi}{4}$
- C. $\frac{\pi}{3}$
- D. $\frac{\pi}{2}$
- E. π

Question 18

A 5 kg mass is suspended from a horizontal ceiling by two strings of equal length. Each string makes an angle of 60° to the ceiling, as shown in the above diagram. Correct to one decimal place, the tension in each string is

- A. 24.5 newtons
- B. 28.3 newtons
- C. 34.6 newtons
- D. 49.0 newtons
- E. 84.9 newtons

Question 19

An object is moving in a northerly direction with a constant acceleration of 2 ms^{-2} . When the object is 100 m due north of its starting point, its velocity is 30 ms^{-1} in the northerly direction.

The exact initial velocity of the object could have been

- A. $10\sqrt{5} \text{ ms}^{-1}$
- B. $5\sqrt{10} \text{ ms}^{-1}$
- C. $10\sqrt{7} \text{ ms}^{-1}$
- D. $-10\sqrt{7} \text{ ms}^{-1}$
- E. $7\sqrt{10} \text{ ms}^{-1}$

Question 20

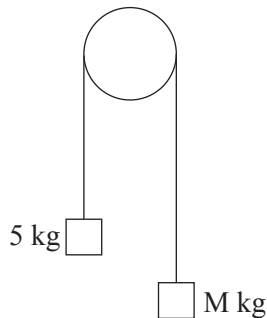
The acceleration, $a \text{ ms}^{-2}$, of a particle moving in a straight line is given by $a = \frac{v}{\log_e(v)}$, where v is the velocity of the particle in ms^{-1} at time t s. The initial velocity of the particle was 5 ms^{-1} .

The velocity of the particle, in terms of t , is given by

- A. $v = e^{2t}$
- B. $v = e^{2t} + 4$
- C. $v = e^{\sqrt{2t} + \log_e(5)}$
- D. $v = e^{\sqrt{2t + (\log_e 5)^2}}$
- E. $v = e^{-\sqrt{2t + (\log_e 5)^2}}$

Question 21

A light inextensible string passes over a smooth, light pulley. A mass of 5 kg is attached to one end of the string and a mass of M kg is attached to the other end, as shown below.



The M kg mass accelerates downwards at $\frac{7}{5} \text{ ms}^{-2}$.

The value of M is

- A. $5\frac{2}{7}$
- B. $6\frac{2}{5}$
- C. $6\frac{2}{3}$
- D. $8\frac{2}{5}$
- E. $9\frac{4}{5}$

Question 22

A particle of mass m moves in a straight line under the action of a resultant force F where $F = F(x)$. Given that the velocity v is v_0 where the position x is x_0 , and that v is v_1 where x is x_1 , it follows that

A. $v_1 = \sqrt{\frac{2}{m} \int_{x_0}^{x_1} \sqrt{F(x)} dx} + v_0$

B. $v_1 = \sqrt{2 \int_{x_0}^{x_1} F(x) dx} + v_0^2$

C. $v_1 = \sqrt{2 \int_{\sqrt{x_0}}^{\sqrt{x_1}} F(x) dx} + v_0$

D. $v_1 = \sqrt{\frac{2}{m} \int_{x_0}^{x_1} F(x) dx} + v_0^2$

E. $v_1 = \sqrt{\frac{2}{m} \int_{x_0}^{x_1} (F(x) + v_0^2) dx}$

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

Unless otherwise specified an **exact** answer is required to a question.

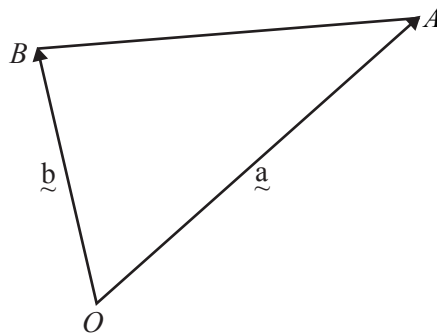
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

The diagram below shows a triangle with vertices O , A and B . Let O be the origin, with vectors $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$.



a. Find the following vectors in terms of $\mathbf{a}$ and $\mathbf{b}$.

i. $\vec{MA}$, where M is the midpoint of the line segment OA

ii. $\vec{BA}$

iii. $\vec{AQ}$, where Q is the midpoint of the line segment AB .

1 + 1 + 1 = 3 marks

- b. Let N be the midpoint of the line segment OB . Use a vector method to prove that the quadrilateral $MNQA$ is a parallelogram.

3 marks

Now consider the **particular** triangle OAB with $\vec{OA} = 3\mathbf{i} + 2\mathbf{j} + \sqrt{3}\mathbf{k}$ and $\vec{OB} = \alpha\mathbf{j}$ where α , which is greater than zero, is chosen so that triangle OAB is isosceles, with $|\vec{OB}| = |\vec{OA}|$.

- c. Show that $\alpha = 4$.

1 mark

- d. i. Find $\vec{OQ}$, where Q is the midpoint of the line segment AB .

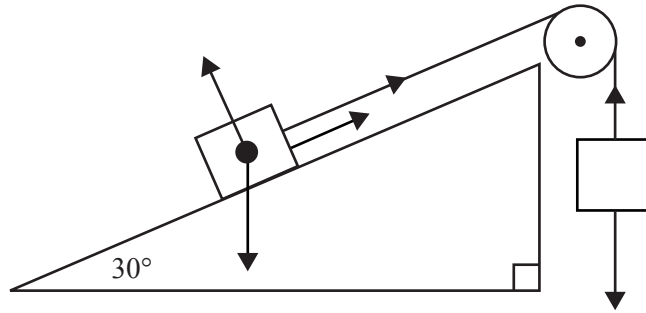
- ii. Use a vector method to show that $\vec{OQ}$ is perpendicular to $\vec{AB}$.

1 + 3 = 4 marks

Total 11 marks

Question 2

A block of m kg sits on a rough plane which is inclined at 30° to the horizontal. The block is connected to a mass of 10 kg by a light inextensible string which passes over a frictionless light pulley.



- a. If the block is on the point of moving **down the plane**, clearly label the forces which are shown in the diagram.

2 marks

- b. Write down, **but do not attempt to solve**, equations involving the forces acting on the block, and the forces acting on the 10 kg mass, for the situation in **part a**.

3 marks

- c. Given that the coefficient of friction is $\frac{1}{4}$, show that the mass of the block in kg is given by $m = \frac{80}{4 - \sqrt{3}}$.

2 marks

In order to pull the block up the plane, the mass hanging from the string is replaced by a 20 kg mass, and a lubricant is spread on the inclined plane to decrease the coefficient of friction.

- d. If the block is now on the point of moving **up the plane**, show that the new value of the coefficient of friction, correct to three decimal places, is $\mu_1 = 0.077$.

2 marks

Before the block actually starts to move up the lubricated plane where $\mu_1 = 0.077$, the string breaks and the block begins to slide down the plane.

- e. Find the velocity of the block three seconds after the string breaks. Give your answer in ms^{-1} correct to one decimal place.

3 marks

Total 12 marks

Question 3

The population of a town is initially 20 000 people. This population would increase at a rate of one per cent per year, except that there is a steady flow of people arriving at and leaving from the town. The population P after t years may be modelled by the differential equation

$$\frac{dP}{dt} = \frac{P}{100} - k \quad \text{with the initial condition, } P = 20\,000 \text{ when } t = 0,$$

where k is the number of people leaving per year minus the number of people arriving per year.

- a.** Verify by substitution that for $k = 800$,

$$P = 20\,000(4 - 3e^{0.01t})$$

satisfies both the differential equation and the initial condition.

3 marks

- b.** For $k = 800$, find the time taken for the population to decrease to zero. Give your answer correct to the nearest whole year.

2 marks

$$\frac{dt}{dP} = \frac{100}{P - 100k} \text{ with } P = 20\,000 \text{ when } t = 0.$$

- $$P = (20\,000 - 100k)e^{0.01t} + 100k.$$

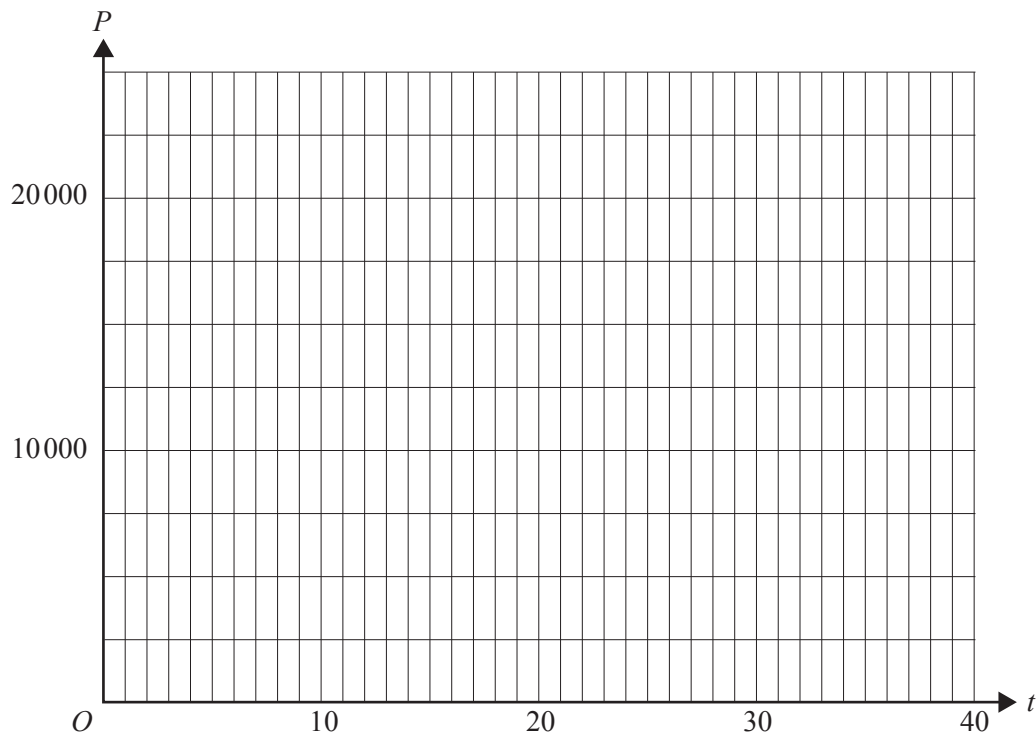
SECTION 2 – Question 3 – continued
TURN OVER

It can be shown that the solution given in **part c.** is valid for all real values of k .

d. For each of the values

- i.** $k = 800$
- ii.** $k = 200$
- iii.** $k = 100$

sketch the graph of P versus t on the set of axes below while the population exists, for $0 \leq t \leq 40$.



3 marks

e. i. Find the value of k if the population has increased to 22 550 after twelve years.

ii. Use the definition of k to interpret your answer to **part i.** in the context of the population model.

1 + 1 = 2 marks

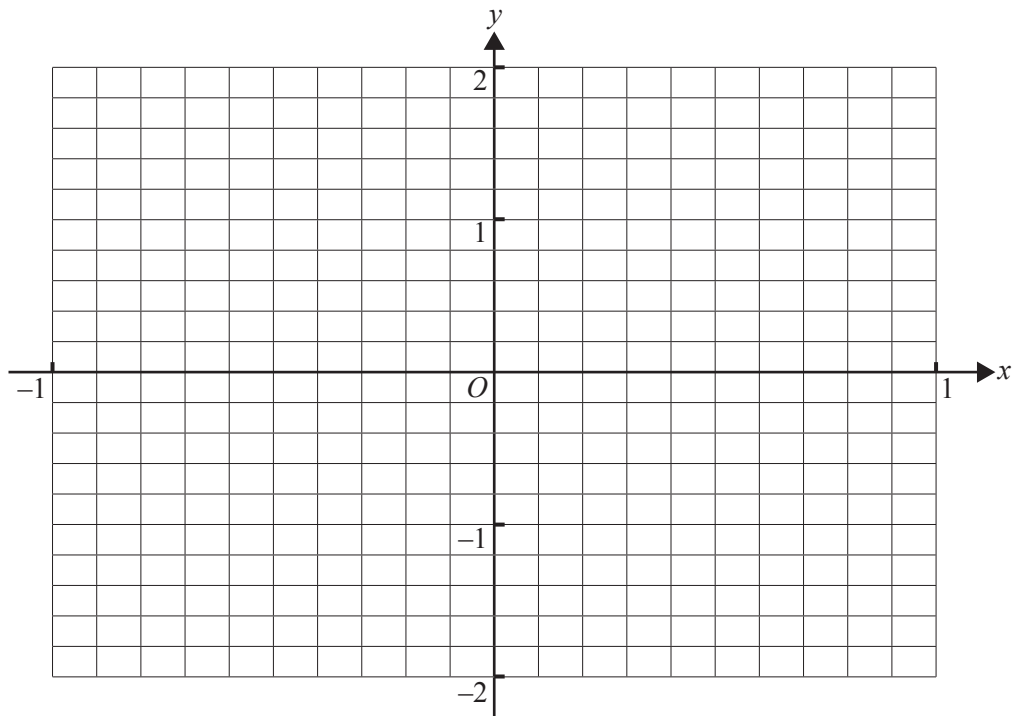
Total 12 marks

Working space

Question 4

Consider the function f with rule $f(x) = \sin^{-1}(2x^2 - 1)$.

- a. Sketch the graph of the relation $y = f(x)$ on the axes below. Label the endpoints with their **exact coordinates**, and label the x and y intercepts with their exact values.



3 marks

- b. i. Write down a **definite integral** in terms of y , which when evaluated will give the volume of the solid of revolution formed by rotating the graph drawn above about the y -axis.

- ii. Find the exact value of the definite integral in **part i**.

2 + 1 = 3 marks

- c. Use calculus to show that

$$f'(x) = \frac{2}{\sqrt{1-x^2}}, \text{ for } x \in (0, a) \text{ and find the value of } a.$$

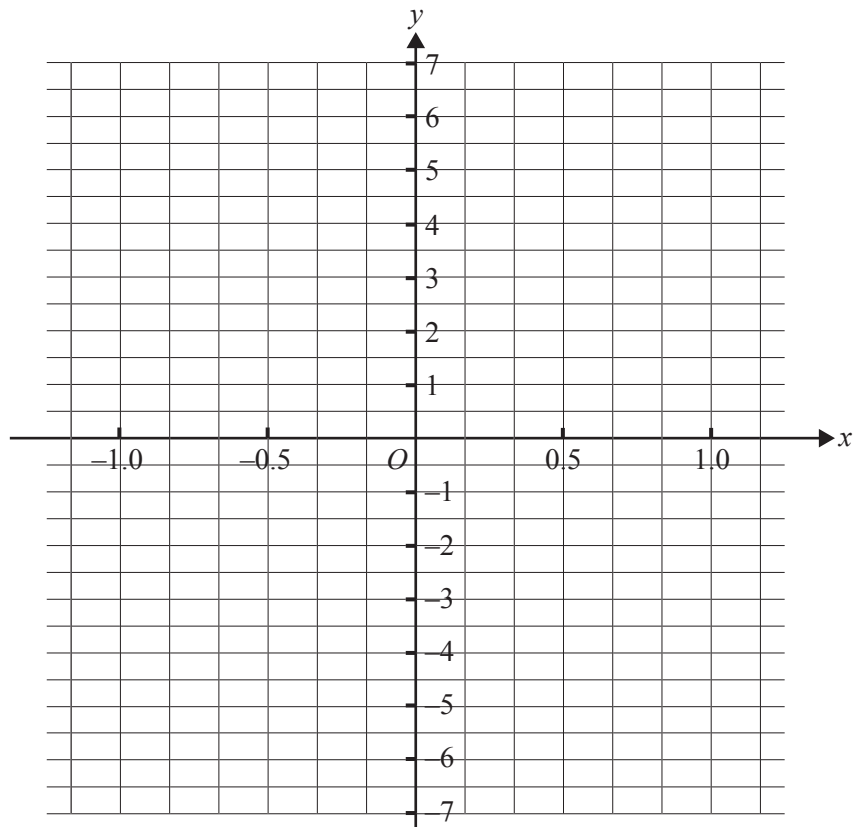
3 marks

- d. Complete the following to specify $f'(x)$ as a hybrid function over the maximal domain of f' .

$$f'(x) = \begin{cases} \text{_____} & \text{for } x \in \text{_____} \\ \text{_____} & \text{for } x \in \text{_____} \end{cases}$$

2 marks

- e. Sketch the graph of the hybrid function f' on the set of axes below, showing any asymptotes.



2 marks

Total 13 marks

SECTION 2 – continued
TURN OVER

Question 5

Let $u = 6 - 2i$ and $w = 1 + 3i$ where $u, w \in \mathbb{C}$.

- a. Given that $z_1 = \frac{(u+w)\bar{u}}{iw}$, show that $|z_1| = 10\sqrt{2}$.

1 mark

- b. The complex number z_1 can be expressed in polar form as

$$z_1 = 200^{\frac{1}{2}} \operatorname{cis}\left(-\frac{3\pi}{4}\right).$$

Find all distinct complex numbers z such that $z^3 = z_1$.

Give your answers in the form $a^{\frac{1}{n}} \operatorname{cis}\left(\frac{b\pi}{c}\right)$, where a, b, c and n are integers.

3 marks

Let the argument of u be given by $\text{Arg}(u) = -\alpha$. (**You are not required to find α .**)

- c. By expressing iw in polar form in terms of α , show that

$$\frac{\bar{u}}{iw} = 2 \text{cis}(2\alpha - \pi).$$

3 marks

- d. Use the relation given in **part a.** to find $\text{Arg}(u + w)$ in terms of α .

3 marks

Total 10 marks

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc \sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$

$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$

$\sin(2x) = 2 \sin(x) \cos(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x-y) = \sin(x) \cos(y) - \cos(x) \sin(y)$

$\cos(x-y) = \cos(x) \cos(y) + \sin(x) \sin(y)$

$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$

$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \quad (\text{de Moivre's theorem})$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), \quad x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } \quad v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

TURN OVER

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$