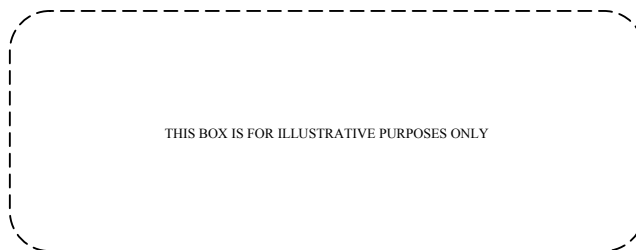




2010 Trial Examination



THIS BOX IS FOR ILLUSTRATIVE PURPOSES ONLY

STUDENT NUMBER

Figures

Words

Letter

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SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 2

Reading time: 15 minutes

Writing time: 2 hours

QUESTION AND ANSWER BOOK

Structure of book

Section	Number of questions	Number of questions to be answered	Number of marks
1	22	22	22
2	5	5	58
			Total 80

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or approved CAS calculator and a scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question book of 22 pages.

Instructions

- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks are **not** deducted for incorrect answers.

If more than 1 answer is completed for any question, no mark will be given.

Take the **acceleration due to gravity**, to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

Question 1

The equation $x^2 - 4y^2 - 4mx - 8my = 4$ represents

- A. an ellipse with centre at the point $(-2m, m)$ and the semi-axes $a = 2m$ and $b = m$
- B. a hyperbola with centre at the point $(2m, -m)$ and the semi-axes $a = 2$ and $b = 1$
- C. an ellipse with centre at the point $(-2m, m)$ and the semi-axes $a = 1$ and $b = 2$
- D. a hyperbola with centre at the point $(2m, -m)$ and the semi-axes $a = 4$ and $b = 1$
- E. a hyperbola with centre at the point $(-2m, m)$ and the semi-axes $a = 1$ and $b = 2$

Question 2

If the function $f(x) = Ax^2 - x + B \ln x$, $x > 0$, $A, B \in \mathbb{R}$ has a non-stationary point of inflection for $x = 1$ and is increasing over its implied domain, then

- A. $A > \frac{1}{4}, B > \frac{1}{2}$
- B. $0 < A < \frac{1}{4}, 0 < B < \frac{1}{2}$
- C. $0 < A < \frac{1}{2}, B > \frac{1}{4}$
- D. $A > \frac{1}{2}, 0 < B < \frac{1}{4}$
- E. $0 < A < \frac{1}{2}, 0 < B < \frac{1}{4}$

SECTION 1 - continued

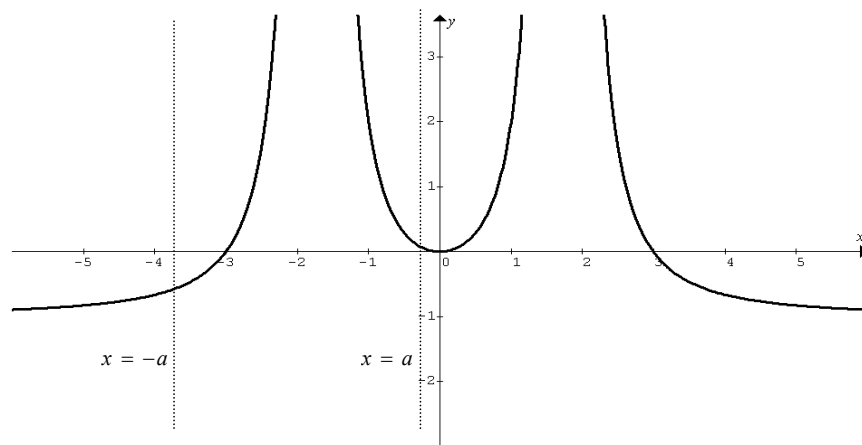
Question 3

The implied domain of $f(x) = \sqrt{4a^2x^3 - x}$, $a \in \mathbb{R}^+$, is

- A. $\left(-\frac{1}{2a}, 0\right) \cup \left(\frac{1}{2a}, +\infty\right)$
- B. $\left(-\frac{1}{4a^2}, 0\right) \cup \left(\frac{1}{4a^2}, +\infty\right)$
- C. $\left[-\frac{1}{2a}, \frac{1}{2a}\right]$
- D. $\left[-\frac{1}{2a}, 0\right] \cup \left[\frac{1}{2a}, +\infty\right)$
- E. The domain depends on the sign of a .

Question 4

This graph represents $y = f'(x)$. $f'(x) = 0$ for $x = -3, x = 0$, and $x = 3$.



Which one of the following statements must be true?

- A. $f(x)$ is increasing for $x \in (-\infty, -a) \cup (0, a)$
- B. $f(x)$ has a local maximum at $x = -3$.
- C. $f''(-4) < 0$
- D. $f(x)$ has a stationary point of inflection at $x = 0$
- E. $f''(x) > 0$ for $x \in (-a, a)$

SECTION 1 - continued
TURN OVER

Question 5

Let $S = \{z : |z| \leq 3\} \cap \{z : |z - 3 - 3i| \leq 3\}$. Which one of the following is correct?

- A. $\text{Arg}(z) \geq \frac{\pi}{2}$
- B. $3(\sqrt{2} - 1) \leq |z| \leq 3$
- C. $\text{Arg}(z) = \frac{\pi}{4}$
- D. $\sqrt{3} - 1 \leq |z| \leq 3$
- E. $\text{Re}(z) - \text{Im}(z) = 3$

Question 6

If $|z| - z = 1 + 2i$, then

- A. $\text{Re}(z) + \text{Im}(z) = \frac{1}{2}$
- B. $|z| = \sqrt{5}$
- C. $\text{Arg}(z) = \tan^{-1}(2)$
- D. $\text{Re}(z) < \text{Im}(z)$
- E. $|z| = \frac{5}{2}$

Question 7

If a polynomial with real coefficients has solutions $z = 1 - ai$ and $z = 2$ over the set of complex numbers, where a is real, then its quadratic factor must be

- A. $z^2 + 4$
- B. $z^2 - 2aiz + 1$
- C. $z^2 - 2z + 1 + a^2$
- D. $z^2 - 4$
- E. $z^2 + 2z + 1 - a^2$

Question 8

Let $\mathbf{a} = 6\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = 3\mathbf{j} - \mathbf{k}$ and $\mathbf{c} = -2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$. The value of n for which vectors $\mathbf{a} + n\mathbf{b}$ and $\mathbf{c}$ are perpendicular is

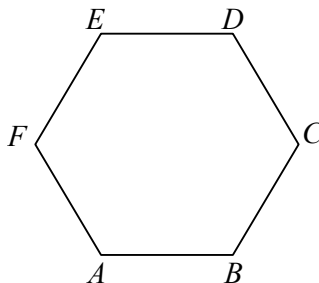
- A. $-\frac{3}{2}$
- B. 4
- C. $\frac{3}{2}$
- D. -1
- E. 1

SECTION 1 - continued

Question 9

The diagram shows a regular hexagon $ABCDEF$. If $\vec{AB} = \mathbf{u}$ and $\vec{BC} = \mathbf{v}$, then vector $\vec{AE}$ can be expressed as

- A. $2\mathbf{u} - \mathbf{v}$
- B. $2\mathbf{u} + \mathbf{v}$
- C. $-\mathbf{u} + 2\mathbf{v}$
- D. $\mathbf{u} + 2\mathbf{v}$
- E. $2\mathbf{u} - 2\mathbf{v}$

**Question 10**

$\tan\left(\cos^{-1}\frac{1}{\sqrt{1+a^2}} - \cos^{-1}\frac{a}{\sqrt{1+a^2}}\right)$, $a > 0$ is equal to

- A. $\frac{a^2 - 1}{2a}$
- B. a
- C. $\frac{1 - a^2}{2a}$
- D. 1
- E. undefined

Question 11

The value of p for which the function $y = \ln\frac{1}{1+x}$ satisfies the differential equation

$$x \frac{dy}{dx} + p = e^y \text{ is}$$

- A. e
- B. $-e$
- C. -1
- D. 1
- E. $\frac{1}{e}$

SECTION 1 – continued
TURN OVER

Question 12

The gradient of the normal to the curve defined parametrically by the equations

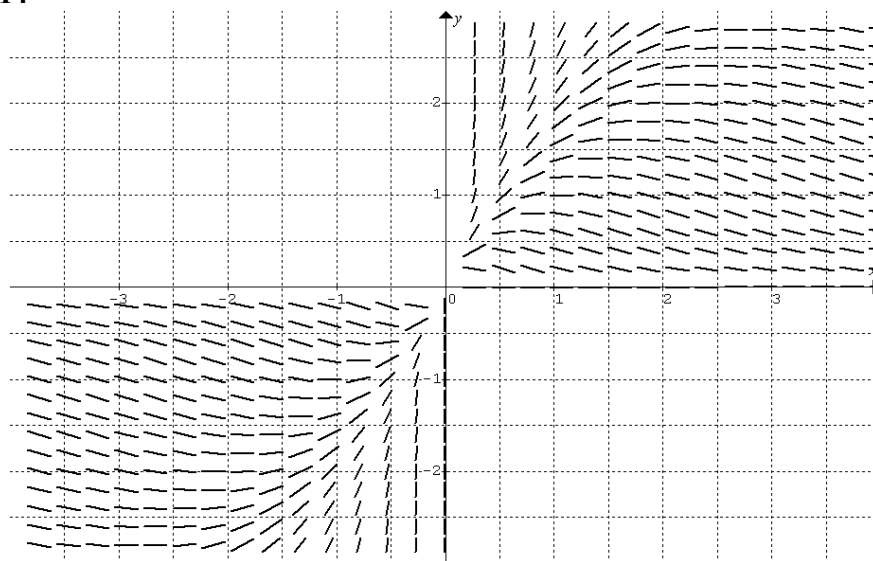
$x = a \cos t$, $y = b \sin t$, $0 \leq t \leq 2\pi$, at the point where $t = \frac{3\pi}{4}$ is

- A. $-\frac{a}{b}$
- B. $\frac{a}{b}$
- C. ab
- D. $-\frac{b}{a}$
- E. $\frac{b}{a}$

Question 13

Using a suitable substitution, $\int_0^3 \frac{2x-1}{\sqrt{x+2}} dx$ is

- A. $\frac{1}{2} \int_0^3 u \sqrt{\frac{2}{u+5}} du$
- B. $\int_2^5 \left(2\sqrt{u} - \frac{5}{\sqrt{u}} \right) du$
- C. $\int_2^5 \left(5\sqrt{u} - \frac{2}{\sqrt{u}} \right) du$
- D. $\int_{-1}^5 \left(\frac{u^2-1}{\sqrt{u}} \right) du$
- E. $\int_{-1}^5 \left(\frac{u^2}{\sqrt{u-1}} \right) du$

Question 14

The differential equation with the slope field shown above could be

- A. $\frac{dy}{dx} = x \ln y$
- B. $\frac{dy}{dx} = y \ln x$
- C. $\frac{dy}{dx} = ye^x$
- D. $\frac{dy}{dx} = \frac{y}{x} e^{\frac{y}{x}}$
- E. $\frac{dy}{dx} = \frac{y}{x} \ln\left(\frac{y}{x}\right)$

Question 15

The area enclosed by the curve $y^2 = -x^4 + 5x^2 - 4$, correct to three decimal places, is

- A. 1.162
- B. 2.323
- C. 0.581
- D. 4.646
- E. 3.485

SECTION 1 - continued
TURN OVER

Question 16

$\int \frac{\sin^2 x \cos^5 x}{1 + \cos 2x} dx$, after the substitution $\sin x = u$, is the same as

- A. $\frac{1}{2} \int (u^4 - u^2) du$
- B. $-\int (1 - u) du$
- C. $\frac{1}{2} \int (u^2 - u^4) du$
- D. $-\int \frac{1 - u^4}{2u^2} du$
- E. $\int (u^2 - 1) du$

Question 17

A tank contains 50 litres of sugar mixture at a concentration of 10 grams of sugar per litre. Three litres of sugar mixture at a concentration 8 grams per litre flow into the tank each minute and two litres of the resultant mixture flow out each minute. The amount of sugar in the tank, x grams, at any time, t minutes, can be found by solving the differential equation

- A. $\frac{dx}{dt} = 24 + \frac{2x}{50 - t}, t = 0, x = 50$
- B. $\frac{dx}{dt} = 24 - \frac{x}{25}, t = 0, x = 50$
- C. $\frac{dx}{dt} = 24 - \frac{2x}{50 + t}, t = 0, x = 500$
- D. $\frac{dx}{dt} = 6 + \frac{8x}{50 - t}, t = 0, x = 500$
- E. $\frac{dx}{dt} = 6 - \frac{8x}{50 - t}, t = 0, x = 500$

Question 18

An approximation to the solution of the differential equation $\frac{dy}{dx} + \log_e(x + y) = x$, given that point (1,0) lies on the curve, is found by using Euler's method with $h = 0.1$. The value of y obtained after two applications of Euler's method is

- A. $0.21 - 0.1 \log_e 1.2$
- B. 0.21
- C. $1.2 - 0.1 \log_e 1.2$
- D. 2.1
- E. $0.12 - 1.1 \log_e 1.2$

SECTION 1 - continued

Question 19

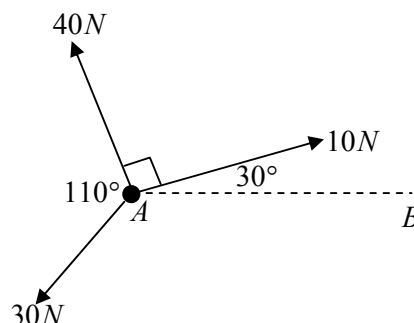
Two cars leave the same point at the same time. The velocity of the first car is given by $v_1 = 3t^2 \text{ ms}^{-1}$, while the velocity of the second car is given by $v_2 = (6t^2 - 10) \text{ ms}^{-1}$. The distance between the cars after 10 seconds is

- A. 900m
- B. 1000m
- C. 100m
- D. 1100m
- E. 800m

Question 20

The sum of the resolved parts of the forces in the direction perpendicular to AB , accurate to the nearest Newton, is

- A. 31N
- B. 63N
- C. 40N
- D. 17N
- E. 15N

**Question 21**

A block of mass m kg rests on a rough inclined plane. It is kept stationary by a minimum force F Newtons acting parallel to the plane. If μ is the coefficient of friction between the plane and the block, α is the angle of inclination, R is the normal reaction force and a is the acceleration of the block, which one of the following is correct?

- A. $F = mg \sin \alpha + \mu R$
- B. $F = mg(\sin \alpha - \mu \cos \alpha)$
- C. $F = mg(\sin \alpha + \mu \cos \alpha)$
- D. $F = m(a + g \sin \alpha - \mu g \cos \alpha)$
- E. $R - mg + F + \mu R = ma$

SECTION 1 - continued
TURN OVER

Question 22

The velocity of a particle moving in a horizontal line is given by

$v = \sqrt[3]{2 - x^2} \text{ ms}^{-1}, -\sqrt{2} \leq x \leq 2$. If the position of the particle is $x \text{ m}$, then its acceleration is

- A. $a = \frac{1}{3v}$
- B. $a = -\frac{2x}{3v}$
- C. $a = \frac{v}{x}$
- D. $a = -\frac{x}{v}$
- E. $a = \frac{2x}{3v}$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions.

A decimal approximation will not be accepted if the question specifically asks for an **exact** answer.

For questions worth more than one mark, appropriate working **must** be shown.

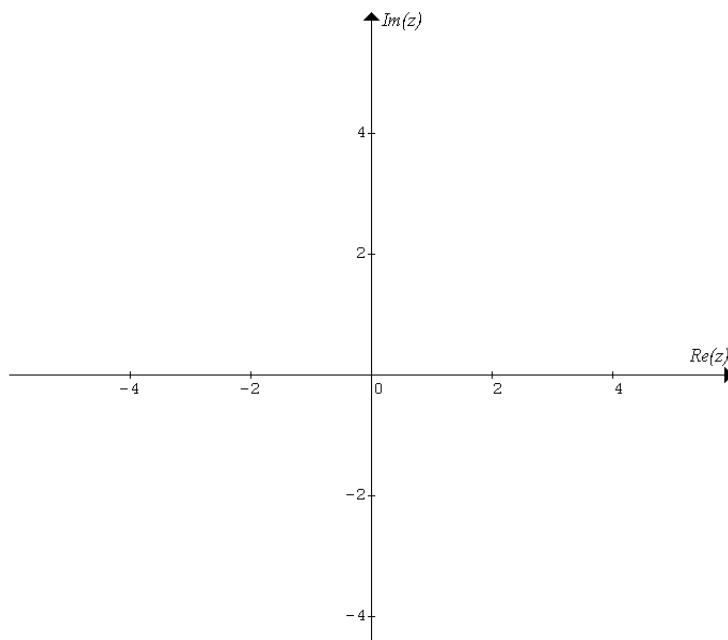
Unless otherwise indicated, the diagrams are **not** drawn to scale.

Take the **acceleration due to gravity**, to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

Let $A = \left\{ z : \text{Arg}(1 + z) = \frac{\pi}{4} \right\}$ and $B = \{ z : |1 + z| = 2 \}$.

a. Sketch the complex regions A and B on the Argand diagram below.



4 marks

SECTION 2 – continued
TURN OVER

- b.** Write down the Cartesian equations of the regions A and B .

2 marks

- c.** Show that $z_0 = (\sqrt{2} - 1) + i\sqrt{2}$ is the point of intersection of the regions A and B .

2 marks

- d. i.** Find $(1 + z_0)^6$ in Cartesian form.

2 marks

SECTION 2 – continued

ii. Find both values of $\sqrt{1 + z_0}$ in polar form.

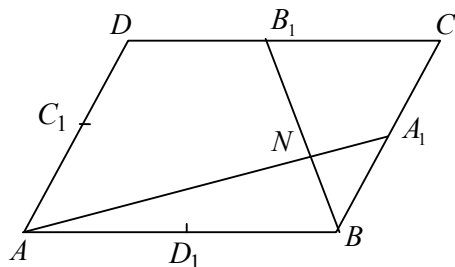
2 marks
Total 12 marks

SECTION 2 – continued
TURN OVER

Question 2

In the diagram below, $ABCD$ is a parallelogram. Points A_1 , B_1 , C_1 and D_1 are the midpoints of BC , CD , DA and AB respectively. Point N is the point of intersection of AA_1 and BB_1 .

Let $\vec{AB} = \mathbf{a}$ and $\vec{BC} = \mathbf{b}$.



- a. Express $\vec{AA_1}$ and $\vec{BB_1}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

2 marks

- b. Use $\vec{AN} = n \vec{AA_1}$ and $\vec{BN} = m \vec{BB_1}$ to find the ratio $AN : NA_1$.

4 marks

SECTION 2 – continued

- c. The angle between the vectors $\vec{AB}$ and $\vec{BC}$ is 60° and $|\mathbf{a}| = \frac{2}{3}|\mathbf{b}|$. If $|\mathbf{a}| = k$, find

 $\overrightarrow{AA_1} \bullet \overrightarrow{BB_1}$ in terms of k .[illegible]

4 marks

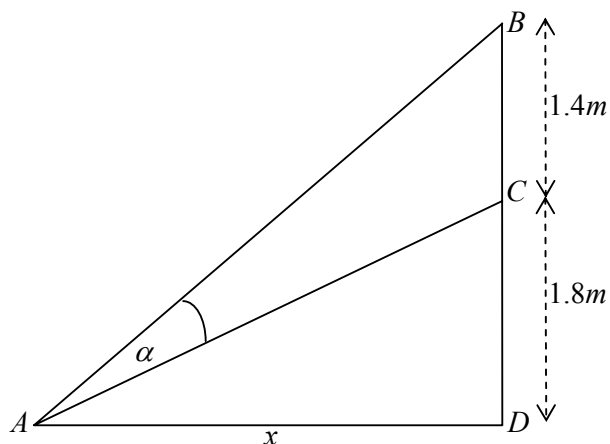
Total 10 marks

SECTION 2 – continued
TURN OVER

Question 3

A monitor showing flight information hangs on a vertical wall inside the airport terminal. Its bottom end is 1.8 metres above the eyes of a traveller, which are at point A, as shown in the diagram. The monitor is 1.4 metres high.

Let α be the angle, in degrees, through which the monitor is seen by the traveller and x the distance, in metres, from the traveller to the wall.



- a. Show that $\tan \alpha = \frac{1.4x}{x^2 + 5.76}$

4 marks

SECTION 2 – continued

b. Let $f(x) = \frac{1.4x}{x^2 + 5.76}$.

i. Show that $f'(x) = \frac{1.4(5.76 - x^2)}{(x^2 + 5.76)^2}$.

ii. Hence, find the maximum value of angle α , in degrees, correct to two decimal places.

2 + 3 = 5 marks

- c. The traveller moves along the line segment AD , towards the wall, with a speed of 1.2 ms^{-1} . Find the rate of change of angle α , in degrees per second, when the traveller is 4 metres away from the wall. Give your answer correct to one decimal place.

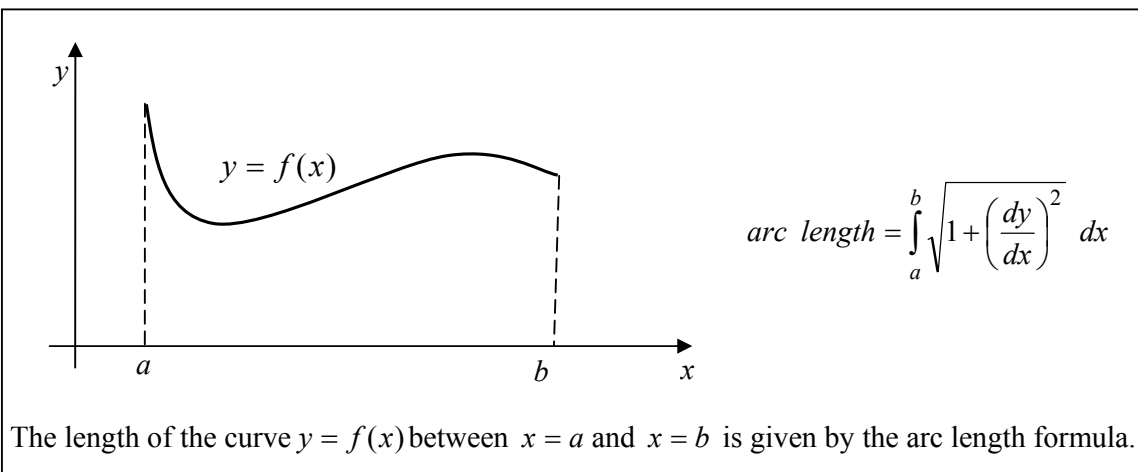
3 marks

Total 12 marks

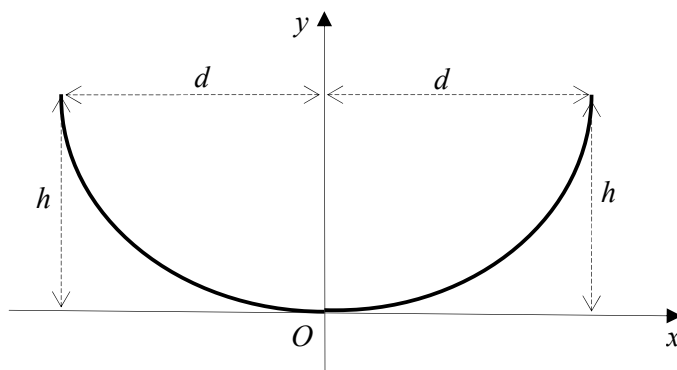
SECTION 2 – continued
TURN OVER

Question 4

In this question, the arc length formula will be used.



A flexible chain of length l hangs loosely between two poles of equal height a distance $2d$ metres apart, so that it sags a distance h metres in the centre.



The curve formed by the chain is called a **catenary**. Using a coordinate system with the lowest point at the origin, the catenary can be described by the equation $y = \frac{e^{mx} + e^{-mx} - 2}{2m}$, where m is a constant.

a. Show that $\left(\frac{dy}{dx}\right)^2 = \frac{e^{2mx} + e^{-2mx} - 2}{4}$.

2 marks

SECTION 2 – continued

- b.** Use the arc length formula to show that the length of the chain is given

$$\text{by } l = \frac{e^{md} - e^{-md}}{m}.$$

[illegible]

4 marks

- c. In a particular catenary, where $m = \frac{1}{32}$, it is required that the height, h , is 20 metres.

Find, correct to two decimal places

- i. the distance between the poles.

SECTION 2 – continued
TURN OVER

- ii. the length of the cable.

1+1= 2 marks

- d. If the length of the cable is 100 metres and the distance between the poles is 80 metres, find the value of m , correct to two decimal places.

2 marks

Total 10 marks

SECTION 2 - continued

Question 5

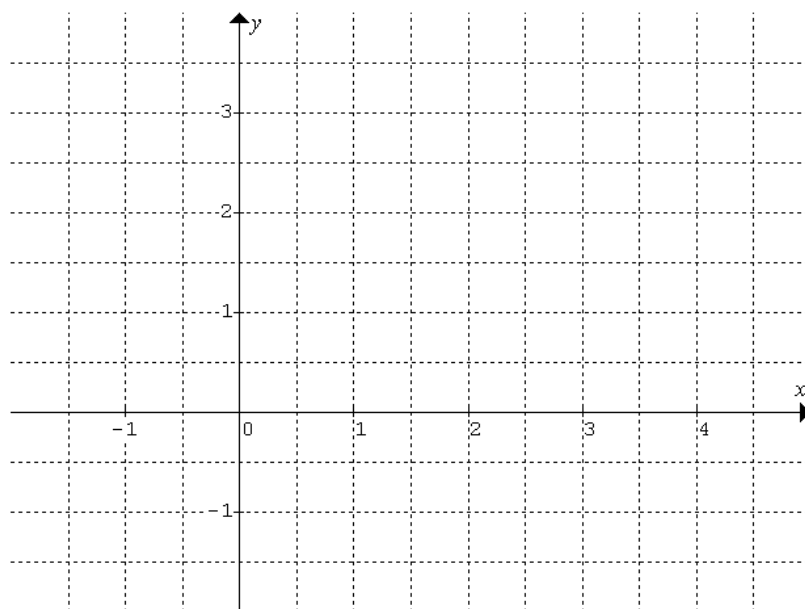
The position vectors of two particles, A and B , with respect to the origin, are given by $\mathbf{r}_A = (1 + \cos(kt))\mathbf{i} + (2 + 2\sin(kt))\mathbf{j}$ and $\mathbf{r}_B = t\mathbf{i} + (2t^3 - 3t^2)\mathbf{j}$, where $t \geq 0$ is the time in seconds and k is a positive constant. The distance of the particles from the origin is measured in metres.

- a. i. Show that the Cartesian equation of the path of particle A is $(x-1)^2 + \frac{(y-2)^2}{4} = 1$.

- ii. Find the Cartesian equation of the path of particle B .

2+1 = 3 marks

- b. On the axes below, sketch the paths of A and B .



2 marks

SECTION 2 – continued
TURN OVER

- c. Find the maximum speed of particle A in terms of k .

4 marks

- d. Find, correct to three decimal places, the coordinates of the points where the paths of A and B intersect.

2 marks

- e. Find the minimum value of k for which the particles will collide, correct to three decimal places.

3 marks

Total 14 marks

END OF QUESTION AND ANSWER BOOK