



THE SCHOOL FOR EXCELLENCE
UNIT 3 & 4 SPECIALIST MATHEMATICS 2010
COMPLIMENTARY WRITTEN EXAMINATION 2 - SOLUTIONS

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MARKING SCHEME (EXTENDED ANSWER QUESTIONS)

- $(A4 \times \frac{1}{2} \downarrow)$ means four answer half-marks rounded **down** to the next integer.
Rounding occurs at the end of a part of a question.
- M1 = 1 Method mark.
- A1 = 1 Answer mark (it **must** be this or its equivalent).
- H1 = 1 consequential mark (**His/Her** mark...correct answer from incorrect statement or slip).

SECTION 1 – MULTIPLE CHOICE QUESTIONS

1	2	3	4	5	6	7	8	9	10	11
D	E	B	D	C	E	C	E	D	B	B

12	13	14	15	16	17	18	19	20	21	22
B	C	D	B	C	B	D	A	E	C	B

QUESTION 1 Answer is D

$$\text{imaginary part} = r \sin(\theta)i = 2 \sin\left(\frac{4\pi}{3}\right)i = -2 \times \frac{\sqrt{3}}{2}i = -\sqrt{3}i$$

QUESTION 2 Answer is E

$$\text{The vector } \vec{AB} = 4\vec{i} + 8\vec{j} + 16\vec{k} = 2(2\vec{i} + 8\vec{j} + 16\vec{k})$$

$$\text{The vector } \vec{BC} = (x-4)\vec{i} + 4\vec{j} + 8\vec{k}$$

$\therefore$ The A, B and C are collinear.

$$\begin{aligned}\therefore x-4 &= 2 \\ x &= 6\end{aligned}$$

QUESTION 3 Answer is B

$\therefore 6i$ and $2i$ are continues lines

$$\therefore 2 \leq \text{Im}(z) \leq 6$$

$\therefore$ Region enclosed by angle $\frac{3\pi}{4}$ with a continues line and angle $\frac{\pi}{4}$ with doted line

$$\therefore \frac{\pi}{4} < \text{Arg } z \leq \frac{3\pi}{4}$$

$$\therefore \{z : 2 \leq \text{Im}(z) \leq 6, \frac{\pi}{4} < \text{Arg } z \leq \frac{3\pi}{4}\}$$

QUESTION 4 Answer is D

When $b = 0$.

$$\text{Ellipse with equation } \frac{x^2}{\left(\frac{36}{a}\right)} + \frac{y^2}{36} = 1$$

$\therefore$ The ellipse only touches the hyperbola with equation $\frac{x^2}{9} - \frac{y^2}{9} = 1$.

$\therefore$ The ellipse and the hyperbola only intersect at the two x-intersects of the hyperbola (3, 0) and (-3, 0)

$$\therefore \sqrt{\frac{36}{a}} = \sqrt{9}$$

$$\therefore a = 4$$

When $b \in [-3, 3]$

The ellipse and the hyperbola maximum can have 2 points of intersection.

When $b \in [-6, 6]$

The ellipse and the hyperbola can have maximum 3 solutions. Use CAS to solve the equations of the ellipse and the hyperbola when $b = 6$, there are only 3 intersection points shown.

QUESTION 5 Answer is C

$$(1+i\sqrt{3})^{\frac{4}{3}} = \left[(1+i\sqrt{3})^4 \right]^{\frac{1}{3}} = \left[2^4 \operatorname{cis}\left(\frac{4\pi}{3}\right) \right]^{\frac{1}{3}}$$

Three equivalent representations are taken since we need this expression's cube root.

Therefore:

$$(1+i\sqrt{3})^4 = 16 \operatorname{cis}\left(\frac{4\pi}{3}\right), 16 \operatorname{cis}\left(\frac{10\pi}{3}\right), 16 \operatorname{cis}\left(\frac{16\pi}{3}\right)$$

Therefore:

$$(1+i\sqrt{3})^{\frac{4}{3}} = 16^{\frac{1}{3}} \operatorname{cis}\left(\frac{4\pi}{9}\right), 16^{\frac{1}{3}} \operatorname{cis}\left(\frac{10\pi}{9}\right), 16^{\frac{1}{3}} \operatorname{cis}\left(\frac{16\pi}{9}\right)$$

QUESTION 6 Answer is E

$$\int_{-a}^a \frac{1}{4+9x^2} dx = 1$$

$$\int_{-a}^a \frac{\frac{1}{9}}{\frac{4}{9}+x^2} dx = 1$$

$$\frac{1}{6} \int_{-a}^a \frac{\frac{2}{3}}{\frac{4}{9}+x^2} dx = 1$$

$$\frac{1}{6} \left[\tan^{-1}\left(\frac{3x}{2}\right) \right]_{-a}^a = 1$$

$$\frac{1}{6} \left[\tan^{-1}\left(\frac{3a}{2}\right) - \tan^{-1}\left(\frac{-3a}{2}\right) \right] = 1$$

$$2 \tan^{-1}\left(\frac{3a}{2}\right) = 6$$

$$\tan^{-1}\left(\frac{3a}{2}\right) = 3$$

$$\tan(3) = \frac{3a}{2}$$

$$a = \frac{2 \tan(3)}{3}$$

QUESTION 7 Answer is C

$$z = \text{cis}(\theta)$$

$$z^n - \frac{1}{z^n} = z^n - z^{-n} = \text{cis}(n\theta) - \text{cis}(-n\theta) \\ = 2i \sin(n\theta)$$

QUESTION 8 Answer is E

$$\text{Use CAS: } \int \frac{3x^2 + 9}{(2x-1)(x^2 + 2x + 2)} dx = \frac{3}{2} \ln|2x-1| - 3 \tan^{-1}(x+1)$$

$$\therefore a = \frac{3}{2} \quad b = 3 \quad \frac{d}{dx}(\tan^{-1}(x+1)) = \frac{1}{x^2 + 2x + 2}$$

$$\therefore g(x) = \frac{1}{x^2 + 2x + 2}$$

QUESTION 9 Answer is D

$$\sin(t + 2t) = \sin(t)\cos(2t) + \sin(2t)\cos(t) \\ = \sin(t)[1 - 2\sin^2(t)] + 2\sin(t)\cos^2(t) \\ = \sin(t) - 2\sin^3(t) + 2\sin(t)(1 - \sin^2(t)) \\ = \sin(t) - 2\sin^3(t) + 2\sin(t) - 2\sin^3(t) \\ = -4\sin^3(t) + 3\sin(t)$$

QUESTION 10 Answer is B

$$\frac{dt}{d\theta} = \frac{1}{-k(\theta - 10)}$$

$$t = \frac{1}{-k} L_n(\theta - 10) + c$$

$$\text{Sub } t=0, \theta = 70: c = \frac{L_n(60)}{k}$$

$$\therefore t = -\frac{1}{k} L_n(\theta - 10) + \frac{L_n(60)}{k} = \frac{1}{k} (L_n(60) - L_n(\theta - 10))$$

Sub $t = 10$ $\theta = 40$. Use CAS to find k .

Sub $t = 15$. Use Solve() in CAS to Solve θ

$$\theta = 31.21^\circ \text{C}$$

QUESTION 11**Answer is B**When $y=2$, $x \geq 0$

$$2x^2 = 2$$

$$x = 1$$

$$\therefore \pi \int_0^1 (2 - 2x^2)^2 dx$$

$$\pi \int_0^1 (4 - 8x^2 + 4x^4) dx$$

QUESTION 12**Answer is B**

$$\frac{dt}{dQ} = \frac{1}{4(100 - Q)}$$

$$t = -\frac{1}{4} \ln(100 - Q) + c$$

$$4(c - t) = \ln(100 - Q)$$

$$e^{4(c-t)} = 100 - Q$$

$$Q = -e^{4(c-t)} + 100$$

QUESTION 13**Answer is C**

$$\frac{A \cdot B}{|A|} = \frac{-36 + 4 + 6}{\sqrt{76}} = \frac{-26}{\sqrt{76}} \approx -2.97$$

QUESTION 14**Answer is D**

$$\int_{\frac{\pi}{2}}^a \left(\frac{\sin(\theta)}{1 - \cos(\theta)} \right) dx = \frac{1}{2}$$

$$\left[L_n(1 - \cos(\theta)) \right]_{\frac{\pi}{2}}^a = \frac{1}{2}$$

$$L_n(1 - \cos(a)) = \frac{1}{2}$$

$$e^{\frac{1}{2}} = 1 - \cos(a)$$

$$\cos(a) = 1 - e^{\frac{1}{2}}$$

$$a = \cos^{-1}(1 - \sqrt{e})$$

QUESTION 15**Answer is B**

$$\because a \perp \text{line } x + y = 1$$

$$\frac{1 - \sin(\theta)}{2 \sin(\theta)} = 1$$

$$1 - \sin(\theta) = 2 \sin(\theta)$$

$$(1 - \sin(\theta))^2 = 4 \sin^2(\theta)$$

$$1 - 2 \sin(\theta) + \sin^2(\theta) = 4 \sin^2(\theta)$$

$$0 = 3 \sin^2(\theta) + 2 \sin(\theta) - 1$$

$$\sin(\theta) = \frac{1}{3} \text{ or } \sin(\theta) = -1$$

$\therefore$ Finding the acute angle

$$\therefore \sin(\theta) = \frac{1}{3}$$

$$\therefore \theta = \sin^{-1}\left(\frac{1}{3}\right)$$

QUESTION 16**Answer is C**

$$\frac{dv}{dt} = -\frac{1}{2}v$$

$$\frac{dt}{dv} = \frac{-2}{v}$$

$$t = -2 \ln(|v|) + c$$

$$\text{Sub } t = 0 \quad v = 40$$

$$c = 2 \ln(40)$$

$$t = -2 \ln(|v|) + 2 \ln(40)$$

$$t = 2 \ln\left(\frac{40}{v}\right)$$

$$0.5t = \ln\left(\frac{40}{v}\right)$$

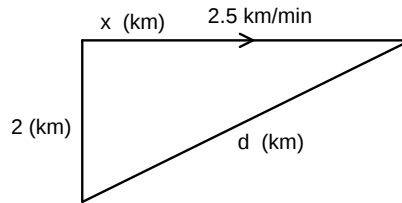
$$e^{0.5t} = \frac{40}{v}$$

$$v = 40e^{-0.5t}$$

QUESTION 17**Answer is B**

$$150\text{km/h} = 2.5\text{km/min}$$

$$\begin{aligned}\frac{dd}{dt} &= \frac{dx}{dt} \times \frac{dd}{dx} \\ &= 2.5 \times \frac{dd}{dx} \\ &= 2.5 \times \frac{x}{\sqrt{4+x^2}}\end{aligned}$$



$$\text{When } t = 1 \quad x = 2.5$$

$$\frac{dd}{dt} = \frac{2.5 \times 2.5}{\sqrt{10.25}} \Rightarrow 1.95\text{km/min}$$

$$\begin{aligned}d &= \sqrt{2^2 + x^2} \\ &= \sqrt{4 + x^2}\end{aligned}$$

$$\frac{dd}{dx} = \frac{x}{\sqrt{4+x^2}}$$

QUESTION 18**Answer is D**

$$r = 4e^{0.5t}i - 4e^{-0.5t}j + c$$

$$0 = 4i - 4j + c$$

$$c = -4i + 4j$$

$$\begin{aligned}r &= (4e^{0.5t} - 4)i - (4e^{-0.5t} - 4)j \\ &= 4(e^{0.5t} - 1)i - 4(e^{-0.5t} - 1)j\end{aligned}$$

QUESTION 19**Answer is A**

$$a = \frac{4g - 3g}{4 + 7} = \frac{g}{7}$$

QUESTION 20**Answer is E**

$$\begin{aligned}a &= \frac{F}{m} = \frac{m_2g - \mu N}{m_2 - m_1} \\ &= \frac{m_2g - \mu m_1g}{m_2 - m_1} \\ &= \frac{(m_2 - \mu m_1)g}{m_2 - m_1}\end{aligned}$$

QUESTION 21**Answer is C**

Let mass M_1 moving direction as +

$$\begin{aligned}\text{Total momentum} &= 7M_1 + (-5M_2) \\ &= 7M_1 - 5M_2\end{aligned}$$

QUESTION 22**Answer is B**

$$\frac{dx}{dt} = 10t - 4$$

$$\frac{dy}{dt} = 10t$$

$$\frac{dx}{dy} = \frac{dx}{dt} \times \frac{dt}{dy} = \frac{10t - 4}{10t}$$

$$\text{Let } \frac{dx}{dy} = 0$$

$$10t - 4 = 0$$

$$t = 0.4$$

Sub $t = 0.4$ into $x(t)$ and $y(t)$

$$x(0.4) = -2.8$$

$$y(0.4) = 5.8$$

SECTION 2

QUESTION 1

a. (i) $|1+i| = \sqrt{2} \Rightarrow |(1+i)^k| = (\sqrt{2})^k = \sqrt{2^k}.$

(ii) $x + iy = (1+i)^k$
 $\therefore \sqrt{x^2 + y^2} = \sqrt{2^k}$
 $\therefore x^2 + y^2 = 2^k$ as required.

b. (i) $1+i = \sqrt{2} \operatorname{cis}\left(\frac{\pi}{4}\right)$
 $1-i = \sqrt{2} \operatorname{cis}\left(-\frac{\pi}{4}\right)$

(ii) $(1+i)^{2n} + (1-i)^{2n} = \left(\sqrt{2} \operatorname{cis}\left(\frac{\pi}{4}\right)\right)^{2n} + \left(\sqrt{2} \operatorname{cis}\left(-\frac{\pi}{4}\right)\right)^{2n}$
 $= (\sqrt{2})^{2n} \operatorname{cis}\left(\frac{n\pi}{2}\right) + (\sqrt{2})^{2n} \operatorname{cis}\left(-\frac{n\pi}{2}\right)$

$n=2$: $(1+i)^4 + (1-i)^4 = 4\operatorname{cis}(\pi) + 4\operatorname{cis}(-\pi) = -8$

$n=3$: $(1+i)^6 + (1-i)^6 = 8\operatorname{cis}\left(\frac{3\pi}{2}\right) + 8\operatorname{cis}\left(-\frac{3\pi}{2}\right) = 0$

$n=4$: $(1+i)^8 + (1-i)^8 = 16\operatorname{cis}(2\pi) + 16\operatorname{cis}(-2\pi) = 32$

(iii) $(1+i)^{2n} + (1-i)^{2n} = (\sqrt{2})^{2n} \operatorname{cis}\left(\frac{n\pi}{2}\right) + (\sqrt{2})^{2n} \operatorname{cis}\left(-\frac{n\pi}{2}\right)$
 $= 2^n \cos\left(\frac{n\pi}{2}\right) + i \sin\left(\frac{n\pi}{2}\right) + 2^n \cos\left(-\frac{n\pi}{2}\right) + i \sin\left(-\frac{n\pi}{2}\right)$
 $= 2^n 2 \cos\left(\frac{n\pi}{2}\right) = 2^{n+1} \cos\left(\frac{n\pi}{2}\right)$

n is an odd integer: $\cos\left(\frac{n\pi}{2}\right) = 0$. Therefore $(1+i)^{2n} + (1-i)^{2n} = 0$

$n/2$ is an odd integer: $\cos\left(\frac{n\pi}{2}\right) = -1$. Therefore $(1+i)^{2n} + (1-i)^{2n} = -2^{n+1}$

Note: If $n/2$ is odd then n is a multiple of 2 but not a multiple of 4.

$n/2$ is an even integer: $\cos\left(\frac{n\pi}{2}\right) = 1$. Therefore $(1+i)^{2n} + (1-i)^{2n} = 2^{n+1}$

Note: If $n/2$ is even then n is a multiple of 4.

c. (i) From De Moivre's Theorem $k = 2$.

$$(ii) \frac{\text{cis}(3\theta)}{\text{cis}(2\theta)} = \text{cis}(3\theta - 2\theta) = \text{cis}(\theta).$$

$$(iii) A + iB = \text{cis}\left(\frac{\pi}{n}\right) + \text{cis}\left(\frac{2\pi}{n}\right) + \text{cis}\left(\frac{3\pi}{n}\right) + \cdots + \text{cis}\left(\frac{2n\pi}{n}\right)$$

(iv) $A + iB$ is a geometric series with $a = \text{cis}\left(\frac{\pi}{n}\right)$, $r = \text{cis}\left(\frac{\pi}{n}\right)$ and number of terms equal to $2n$.

From the given formula:

$$\begin{aligned} A + iB &= \frac{\text{cis}\left(\frac{\pi}{n}\right)\left(1 - \left[\text{cis}\left(\frac{\pi}{n}\right)\right]^{2n}\right)}{1 - \text{cis}\left(\frac{\pi}{n}\right)} \\ &= \frac{\text{cis}\left(\frac{\pi}{n}\right)(1 - \text{cis}(2\pi))}{1 - \text{cis}\left(\frac{\pi}{n}\right)} \\ &= \frac{\text{cis}\left(\frac{\pi}{n}\right)(1 - 1)}{1 - \text{cis}\left(\frac{\pi}{n}\right)} = 0 \end{aligned}$$

Therefore $A = 0$ and $B = 0$.

QUESTION 2

a. (i) $\frac{dt}{dv} = \frac{-50}{v(1+v^2)}$

$$\therefore t = - \int_{10}^v \frac{50}{u(1+u^2)} du + 0$$

Substitute $v = 5$: $\therefore t = - \int_{10}^5 \frac{50}{u(1+u^2)} du = \int_{10}^5 \frac{50}{v(1+v^2)} dv$

(ii) $t = 0.7318$ seconds.

b. (i) $v \frac{dv}{dx} = \frac{-v(1+v^2)}{50}$

$$\frac{dv}{dx} = \frac{-(1+v^2)}{50}$$

(ii) $\frac{dx}{dv} = \frac{-50}{1+v^2}$

$$x = - \int \frac{50}{1+v^2} dv = -50 \tan^{-1}(v) + C.$$

Substitute $v = 10$ when $x = 0$: $C = 50 \tan^{-1}(10)$.

Therefore $x = 50 \tan^{-1}(10) - 50 \tan^{-1}(v)$.

(iii) $\frac{x}{50} = \tan^{-1}(10) - \tan^{-1}(v)$

$$\therefore \tan\left(\frac{x}{50}\right) = \tan\left(\tan^{-1}(10) - \tan^{-1}(v)\right).$$

Apply the double angle formula $\tan(A - B) = \frac{\tan(A) - \tan(B)}{1 + \tan(A)\tan(B)}$:

$$\tan\left(\frac{x}{50}\right) = \frac{10 - v}{1 + 10v}$$

$$\Rightarrow \tan\left(\frac{x}{50}\right) + 10v \tan\left(\frac{x}{50}\right) = 10 - v$$

$$\Rightarrow 10v \tan\left(\frac{x}{50}\right) + v = 10 - \tan\left(\frac{x}{50}\right)$$

$$\Rightarrow v = \frac{10 - \tan\left(\frac{x}{50}\right)}{1 + 10 \tan\left(\frac{x}{50}\right)} \text{ as required.}$$

$$\text{c. (i) } 0 = \frac{10 - \tan\left(\frac{x}{50}\right)}{1 + 10 \tan\left(\frac{x}{50}\right)} \Rightarrow 10 - \tan\left(\frac{x}{50}\right) = 0 \Rightarrow x = 50 \tan^{-1}(10) \approx 73.56.$$

Therefore the particle is stationary at 73.56 metres to the right of O .

$$\text{(ii) Substitute } v=0 \text{ into } a = \frac{-v(1+v^2)}{50}: a=0.$$

Therefore the velocity does not change.

Therefore the velocity remains stationary at 73.56 metres to the right of O .

QUESTION 3

$$\text{a. Require } 9 - x^2 \geq 0 \text{ and } -1 \leq \frac{x}{3} \leq 1. \text{ Therefore } -3 \leq x \leq 3 \Rightarrow D = [-3, 3].$$

$$\text{b. (i) } V = \pi \int y^2 dx = \pi \int_0^{2.8} \left(x\sqrt{9-x^2} + 2 \arcsin\left(\frac{x}{3}\right) \right)^2 dx.$$

$$\text{(ii) } V \approx 180.5 \text{ cubic units.}$$

$$\begin{aligned} \text{c. (i) } f'(x) &= \sqrt{9-x^2} - \frac{x^2}{\sqrt{9-x^2}} + \frac{2}{\sqrt{9-x^2}} \\ &= \sqrt{9-x^2} + \frac{2-x^2}{\sqrt{9-x^2}} \end{aligned}$$

$$= \frac{9-x^2+2-x^2}{\sqrt{9-x^2}}$$

$$= \frac{11-2x^2}{\sqrt{9-x^2}}$$

$$\text{(ii) } f''(x) = \frac{(-4x)\sqrt{9-x^2} - \left(\frac{-x}{\sqrt{9-x^2}}\right)(11-2x^2)}{9-x^2}$$

$$= \frac{(-4x)(9-x^2) + x(11-2x^2)}{(9-x^2)^{3/2}}$$

$$= \frac{2x^3 - 25x}{(9-x^2)^{3/2}}$$

$$= \frac{x(2x^2 - 25)}{(9-x^2)^{3/2}} \text{ as required.}$$

d.
$$\int_{-p}^p \frac{11-2x^2}{\sqrt{9-x^2}} dx = \left[x\sqrt{9-x^2} + 2 \arcsin\left(\frac{x}{3}\right) \right]_{-p}^p$$

$$= \left(p\sqrt{9-p^2} + 2 \arcsin\left(\frac{p}{3}\right) \right) - \left(-p\sqrt{9-p^2} + 2 \arcsin\left(\frac{-p}{3}\right) \right)$$

$$= 2p\sqrt{9-p^2} + 4 \arcsin\left(\frac{p}{3}\right) = 2 \left(p\sqrt{9-p^2} + 2 \arcsin\left(\frac{p}{3}\right) \right).$$

e. The maximum value of $I = p\sqrt{9-p^2} + 2 \arcsin\left(\frac{p}{3}\right)$ is required.

From part (c):
$$\frac{dI}{dp} = \frac{11-2p^2}{\sqrt{9-p^2}}$$

$$\frac{dI}{dp} = 0 \Rightarrow \frac{11-2p^2}{\sqrt{9-p^2}} = 0 \Rightarrow p = \pm \sqrt{\frac{11}{2}}.$$

From the sign test there is a maximum turning point when $p = \sqrt{\frac{11}{2}}$.

Therefore the maximum value occurs when $p = \sqrt{\frac{11}{2}}$.

f. Inflection points of $f(x)$ occur at values of x for which $f'(x)$ has turning points.

Stationary points of $f'(x)$: $f''(x) = \frac{x(2x^2-25)}{(9-x^2)^{3/2}} = 0$ when $x=0$ or $x = \pm \frac{5}{\sqrt{2}}$.

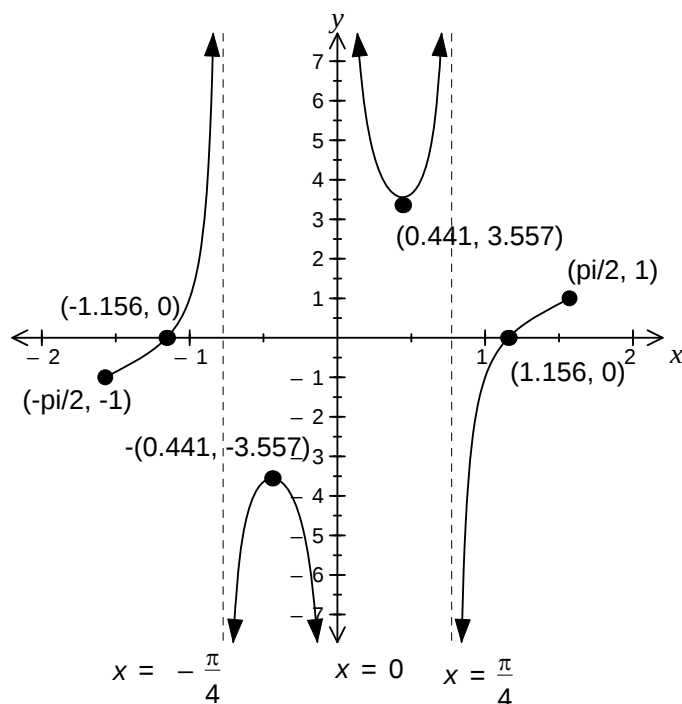
From the sign test $f'(x)$ has a turning point when $x=0$.

Therefore $f(x)$ has an inflection point at $x=0$.

$f(x)$ has no inflection point at $x = \pm \frac{5}{\sqrt{2}}$ because these values of x lie outside the domain D (see part (a)).

QUESTION 4

a.



b.

$$\frac{1}{\sin(x)} + \frac{\sin(2x)}{\cos(2x)} = 0$$

$$\Rightarrow \frac{1}{\sin(x)} + \frac{2\sin(x)\cos(x)}{2\cos^2(x) - 1} = 0$$

$$\Rightarrow \frac{2\cos^2(x) - 1 + 2\sin(x)\cos(x)}{2\sin(x)(\cos^2(x) - 1)} = 0$$

$$\Rightarrow 2\cos^2(x) + 2\sin^2(x)\cos(x) = 0$$

$$\Rightarrow 2\cos^2(x) + 2(1 - \cos^2(x))\cos(x) = 0$$

$$\Rightarrow 2\cos^2(x) + 2\cos(x) - 2\cos^3(x) = 0$$

$$\Rightarrow 2\cos^3(x) - 2\cos^2(x) - 2\cos(x) + 1 = 0 \text{ as required.}$$

$$\text{c. } \frac{dy}{dx} = \frac{-\cos(x)}{\sin^2(x)} + \frac{2}{\cos^2(2x)} = 0$$

$$\Rightarrow \frac{-\cos(x)\cos^2(2x) + 2\sin^2(x)}{\sin^2(x)\cos^2(2x)} = 0$$

$$\Rightarrow -\cos(x)\cos^2(2x) + 2\sin^2(x) = 0$$

$$\Rightarrow -\cos(x)(2\cos^2(x) - 1)^2 + 2(1 - \cos^2(x)) = 0$$

$$\Rightarrow -4\cos^5(x) + 4\cos^3(x) - \cos(x) + 2 - 2\cos^2(x) = 0$$

$$\Rightarrow 4\cos^5(x) - 4\cos^3(x) + 2\cos^2(x) + \cos(x) - 2 = 0 \quad \text{as required.}$$

$$\text{d. } f(\pi - x) = \frac{1}{\sin(\pi - x)} + \frac{\sin(\pi - 2x)}{\cos(\pi - 2x)} = \frac{1}{\sin(x)} + \frac{\sin(2x)}{-\cos(2x)} = \frac{1}{\sin(x)} - \frac{\sin(2x)}{\cos(2x)}.$$

$$f(\pi + x) = \frac{1}{\sin(\pi + x)} + \frac{\sin(\pi + 2x)}{\cos(\pi + 2x)} = \frac{1}{-\sin(x)} + \frac{-\sin(2x)}{-\cos(2x)} = \frac{-1}{\sin(x)} + \frac{\sin(2x)}{\cos(2x)}.$$

$$\text{Therefore } f(\pi - x) + f(\pi + x) = 0.$$

QUESTION 5

a. At $t = -1$:

Position of satellite: $\vec{r}(-1) = -\vec{i} - 10\vec{j} - 2\vec{k}$.

Position of island: $\vec{L} = \vec{i} - 12\vec{j} - 2\vec{k}$

Position of satellite relative to island: $\vec{r}(-1) - \vec{L} = -2\vec{i} + 2\vec{j}$.

Distance: $|-2\vec{i} + 2\vec{j}| = \sqrt{(-2)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$.

b. Relative position of satellite at time t : $\vec{r} - \vec{L} = (2t)\vec{i} + (t^2 + 1)\vec{j} + (t^2 - 1)\vec{k}$.

$$\begin{aligned} \text{Distance: } \left| (2t)\vec{i} + (t^2 + 1)\vec{j} + (t^2 - 1)\vec{k} \right| &= \sqrt{(2t)^2 + (t^2 + 1)^2 + (t^2 - 1)^2} \\ &= \sqrt{4t^2 + t^4 + 2t^2 + 1 + t^4 - 2t^2 + 1} \\ &= \sqrt{2(t^4 + 2t^2 + 1)} \\ &= \sqrt{2}\sqrt{(t^2 + 1)^2} \\ &= \sqrt{2}|t^2 + 1| \\ &= \sqrt{2}(t^2 + 1) \text{ as required.} \end{aligned}$$

c. From part (b) the minimum distance occurs when $t = 0$ and is equal to $\sqrt{2}$.

d. At $t = -1$:

Position of satellite: $\vec{r}(-1) = -\vec{i} - 10\vec{j} - 2\vec{k}$.

Position of satellite relative to island: $-2\vec{i} + 2\vec{j}$.

At $t = -2$:

Position of satellite: $\vec{r}(-2) = -4\vec{i} + 5\vec{j} + 3\vec{k}$.

Position of satellite relative to island: $-4\vec{i} - 5\vec{j} + \vec{k}$.

From the dot product:

Angle between vectors $-2\vec{i} + 2\vec{j}$ and $-4\vec{i} - 5\vec{j} + \vec{k}$ is

$$\cos^{-1}\left(\frac{8 + 10}{\sqrt{8}\sqrt{50}}\right) = \cos^{-1}\left(\frac{18}{20}\right) \approx 25.84^\circ.$$

Angle to nearest degree is 26° .

QUESTION 6

a. Net force perpendicular to slope: $0 = N - 1200g \cos(8^\circ)$ (1)

Net force parallel to slope: $(1200)(0.25) = T - 1200g \sin(8^\circ) - 0.09N$ (2)

Substitute (1) into (2): $300 = T - 1200g \sin(8^\circ) - (0.09)(1200g \cos(8^\circ))$
 $\Rightarrow T = 300 + 1200g(\sin(8^\circ) + 0.09 \cos(8^\circ))$
 $\Rightarrow T \approx 2985 \text{ newtons}$

b. Net force parallel to slope: $0 = T + F - 1200g \sin(8^\circ)$
 $\Rightarrow T = 1200g \sin(8^\circ) - F$

c. (i) $T = 1200g \sin(8^\circ) - (0.09)1200g \cos(8^\circ) \approx 589 \text{ newtons.}$

(ii) Maximum friction: $\mu N = (0.15)1200g \cos(8^\circ) \approx 1746.8 \text{ newtons.}$

Component of weight force of log down the slope: $1200g \sin(8^\circ) \approx 1636.7 \text{ newtons.}$

Therefore the log is not about to slide down the slope.

Therefore $T = 0$.