Neap:

Trial Examination 2010

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
10	10	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 10 pages. The question and answer booklet has a detachable sheet of miscellaneous formulas in the centrefold.
Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this booklet during reading time.
Write **your name** and your **teacher's name** in the space provided above on this page.
All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2010 VCE Specialist Mathematics Units 3 & 4 Written Examination 1.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Find all solutions to the equation $z^3 - \sqrt{2}z^2 + z - \sqrt{2} = 0, \quad z \in C$.

2 marks

Question 2

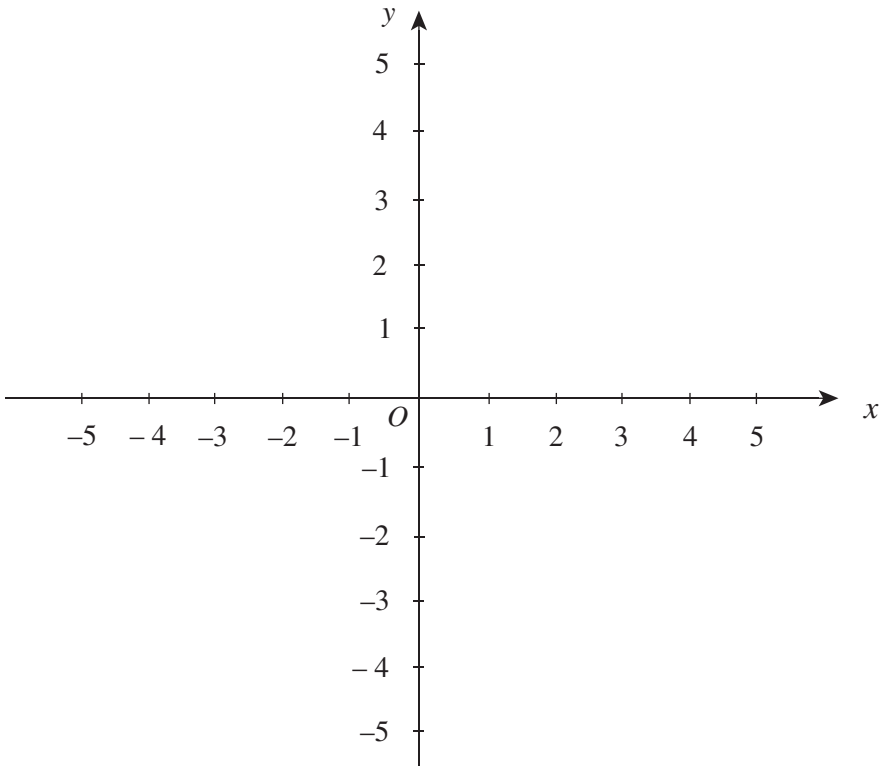
A particle moves so that its position vector at time t is given by

$$\underset{\sim}{r}(t) = -3\sin\left(\frac{t}{2}\right)\underset{\sim}{i} + \left(4\cos\left(\frac{t}{2}\right) - 1\right)\underset{\sim}{j}, \ 0 \le t \le \pi .$$

a. Show that the cartesian equation of the particle's path is given by $\frac{x^2}{9} + \frac{(y+1)^2}{16} = 1$.

3 marks

b. Sketch the path followed by the particle on the axes below. Label the coordinates of any end points.
Note: You are not expected to find any x-axis intercepts.



2 marks

Question 3

Evaluate $15\int_{-1}^{0} x\sqrt{1+x}\ dx$.

3 marks

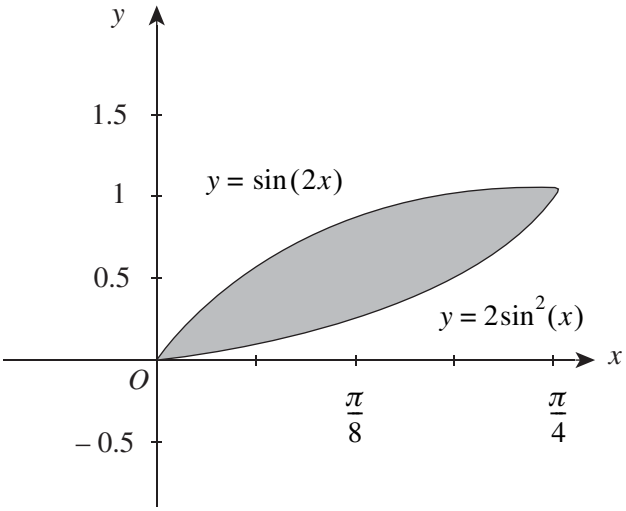
Question 4

Show that the three vectors $\underset{\sim}{u} = \underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$, $\underset{\sim}{v} = 2\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ and $\underset{\sim}{w} = \underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$ are linearly independent.

4 marks

Question 5

The graphs of $y = \sin(2x)$ and $y = 2\sin^2(x)$ for $0 \le x \le \frac{\pi}{4}$ are shown below.



Find the area of the shaded region.

4 marks

Question 6

A mass of 2 kg is attached to the end of a light string *OM* of length 4 metres.

The string is fixed at *O* and the tension in the string is *T* newtons.

Find *F*, the magnitude of the horizontal force required to hold the mass 2 metres from the vertical through *O*. Give your answer in the form $\frac{a\sqrt{b}g}{b}$ where *a* and *b* are integers and *g* (m/s^2) is the acceleration due to gravity.

3 marks

Question 7

The acceleration of a particle moving in a horizontal straight line relative to a fixed origin *O* is given by $a = 3x^2 - 12x$ where *x* metres is the displacement from *O*. Initially the particle is at *O* and its velocity, *v*, is $4\sqrt{2}$ m/s.

a. Show that $v^2 = 2(x^3 - 6x^2 + 16)$.

2 marks

b. Calculate the particle's velocity and acceleration at $x = 2$ and hence state the direction of the particle's motion as it moves from $x = 2$.

2 marks

Question 8

Suppose Euler's method is used to solve the differential equation $\frac{dy}{dx} = e^{-x}$, with a step size of 0.1 and initial condition $y = 1$ at $x = 2$.

Use Euler's method to express y_2 in terms of y_1.

2 marks

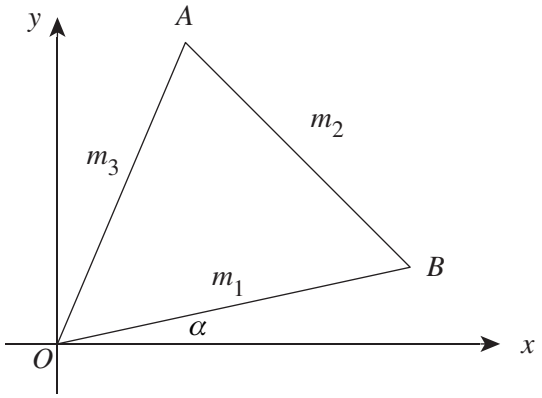
Question 9

Show that the equation of the tangent line to the graph of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $P(x_1, y_1)$ is given by $\frac{x_1 x}{a^2} - \frac{y_1 y}{b^2} = 1$.

5 marks

Question 10

Let m_1, m_2 and m_3 be the gradients of the three sides of an equilateral triangle *OAB* as shown below.



No side of triangle *OAB* is parallel to the *y*-axis. *OB* makes an angle α with the positive direction of the *x*-axis.

a. Show that $m_2 = \dfrac{\tan(\alpha) - \sqrt{3}}{1 + \sqrt{3}\tan(\alpha)}$.

4 marks

b. Given that $m_1 = \tan(\alpha)$ and $m_3 = \dfrac{\tan(\alpha) + \sqrt{3}}{1 - \sqrt{3}\tan(\alpha)}$, find the value of $m_1m_2 + m_2m_3 + m_3m_1$.

4 marks

END OF QUESTION AND ANSWER BOOKLET