



Trial Examination 2010

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Suggested Solutions

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Question 1

Method 1:

Factorising in two groups of two we obtain:

$$z^3 - \sqrt{2}z^2 + z - \sqrt{2} = z^2(z - \sqrt{2}) + 1(z - \sqrt{2})$$

M1

$$\text{So } (z^2 + 1)(z - \sqrt{2}) = 0.$$

$$\text{Hence } z = \pm i, \sqrt{2}.$$

A1

Method 2:

$$\text{Let } P(z) = z^3 - \sqrt{2}z^2 + z - \sqrt{2}.$$

$$\text{Showing that either } P(\sqrt{2}) = 0 \text{ or } P(i) = 0 \text{ or } P(-i) = 0.$$

M1

$$\text{Using a suitable method to obtain } (z^2 + 1)(z - \sqrt{2}) = 0.$$

$$\text{Hence } z = \pm i, \sqrt{2}.$$

A1

Question 2

$$\text{a. Parametric equations are } x(t) = -3 \sin\left(\frac{t}{2}\right) \text{ and } y(t) = 4 \cos\left(\frac{t}{2}\right) - 1.$$

$$-\frac{x}{3} = \sin\left(\frac{t}{2}\right) \text{ and } \frac{y+1}{4} = \cos\left(\frac{t}{2}\right)$$

A1

Squaring both equations we obtain:

$$\frac{x^2}{9} = \sin^2\left(\frac{t}{2}\right) \text{ and } \frac{(y+1)^2}{16} = \cos^2\left(\frac{t}{2}\right)$$

Adding both equations:

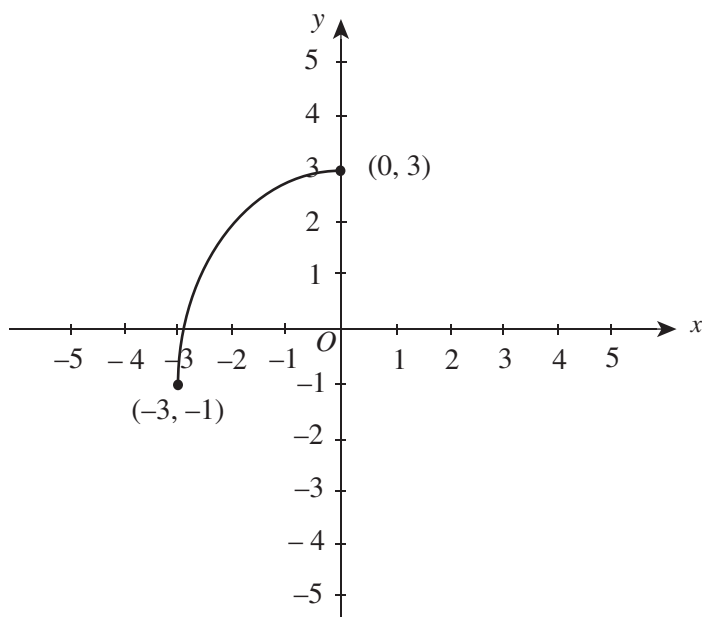
M1

$$\frac{x^2}{9} + \frac{(y+1)^2}{16} = \sin^2\left(\frac{t}{2}\right) + \cos^2\left(\frac{t}{2}\right)$$

$$\text{As } \sin^2\left(\frac{t}{2}\right) + \cos^2\left(\frac{t}{2}\right) = 1, \text{ we obtain } \frac{x^2}{9} + \frac{(y+1)^2}{16} = 1.$$

A1

b.



Correct shape

A1

Correct end points

A1

Question 3Let $u = 1 + x$ and so $du = dx$.When $x = -1$, $u = 0$.When $x = 0$, $u = 1$.

$$\text{So } 15 \int_{-1}^0 x \sqrt{1+x} \, dx = 15 \int_0^1 (u-1) \sqrt{u} \, du.$$

M1

$$= 15 \int_0^1 \left(u^{\frac{3}{2}} - u^{\frac{1}{2}} \right) du$$

$$= 15 \left[\frac{2}{5} u^{\frac{5}{2}} - \frac{2}{3} u^{\frac{3}{2}} \right]_0^1$$

A1

$$= 15 \left(\left(\frac{2}{5} - \frac{2}{3} \right) - 0 \right)$$

$$= -4$$

A1

Question 4

If $m\mathbf{u} + n\mathbf{v} + p\mathbf{w} = \mathbf{0}$ only when $m = n = p = 0$, then $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are linearly independent.

$$m(\mathbf{i} + \mathbf{j} - \mathbf{k}) + n(2\mathbf{i} + \mathbf{j} - 2\mathbf{k}) + p(\mathbf{i} + 2\mathbf{j} + \mathbf{k}) = \mathbf{0} \quad \text{A1}$$

For the above vector equation to be satisfied, the coefficients of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ must all be zero.

$$m + 2n + p = 0 \quad (1)$$

$$m + n + 2p = 0 \quad (2)$$

$$-m - 2n + p = 0 \quad (3)$$

For stating the above three equations

A1

(1) + (3) gives $2p = 0$ i.e. $p = 0$.

Substitute $p = 0$ into (1) and (2) to obtain:

$$m + 2n = 0 \quad (4)$$

$$m + n = 0 \quad (5)$$

(4) – (5) gives $n = 0$ and so $m = 0$.

Attempting to solve the above system of equations.

M1

So $m = n = p = 0$ and thus $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are linearly independent.

A1

Question 5

Let A be the area of the shaded region.

$$A = \int_0^{\frac{\pi}{4}} \sin(2x) - 2\sin^2(x) \, dx \quad \text{A1}$$

Using $\cos(2x) = 1 - 2\sin^2(x)$ i.e. $-2\sin^2(x) = \cos(2x) - 1$ we obtain:

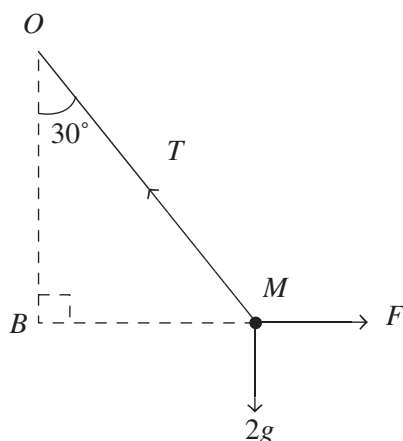
$$\text{Area} = \int_0^{\frac{\pi}{4}} \sin(2x) \, dx + \int_0^{\frac{\pi}{4}} \cos(2x) - 1 \, dx \quad (\text{or equivalent}) \quad \text{M1}$$

$$= \left[-\frac{1}{2} \cos(2x) \right]_0^{\frac{\pi}{4}} + \left[\frac{1}{2} \sin(2x) \right]_0^{\frac{\pi}{4}} - [x]_0^{\frac{\pi}{4}} \quad \text{A1}$$

$$= -\frac{1}{2} \cos\left(\frac{\pi}{2}\right) + \frac{1}{2} \cos(0) + \frac{1}{2} \sin\left(\frac{\pi}{2}\right) - \frac{1}{2} \sin(0) - \frac{\pi}{4} + 0$$

$$= 0 + \frac{1}{2} + \frac{1}{2} - 0 - \frac{\pi}{4}$$

$$= 1 - \frac{\pi}{4} \quad (\text{square units}) \quad \text{A1}$$

Question 6

The forces are in equilibrium i.e. $\Sigma \vec{F} = \vec{0}$.

$OM = 4$ (m) and $BM = 2$ (m) so that $\sin(\hat{BOM}) = \frac{1}{2}$ and $\hat{BOM} = 30^\circ$.

Horizontally: $F - T \sin(30^\circ) = 0$

A1

Vertically: $T \cos(30^\circ) - 2g = 0$

A1

Substituting $T = \frac{2g}{\cos(30^\circ)}$ into $F = T \sin(30^\circ)$ gives $F = \frac{2\sqrt{3}g}{3}$ (newtons).

A1

Question 7

a. $\frac{d}{dx} \left(\frac{1}{2} v^2 \right) = 3x^2 - 12x$

$$\frac{1}{2} v^2 = \int (3x^2 - 12x) dx$$

$$\frac{1}{2} v^2 = x^3 - 6x^2 + c$$

M1

At $x = 0$, $v = 4\sqrt{2}$ and so $c = 16$.

Hence $\frac{1}{2} v^2 = x^3 - 6x^2 + 16$ and so $v^2 = 2(x^3 - 6x^2 + 16)$.

A1

b. At $x = 2$, $v = 2(2^3 - 6 \times 2^2 + 16) = 0$ i.e. the particle's velocity is zero.

At $x = 2$, $a = -12$ i.e. the particle's acceleration is -12 m/s^2 .

At $x = 2$, $v = 0$ and $a = -12$.

A1

Hence the particle moves towards O from $x = 2$.

A1

Question 8

If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$.

Here $x_0 = 2$, $y_0 = 1$ and $h = 0.1$.

For $x = 2.1$, using $y_1 = y_0 + hf(x_0)$, we obtain $y_1 = 1 + 0.1e^{-2}$. A1

For $x = 2.2$, using $y_2 = y_1 + hf(x_1)$, we obtain $y_2 = y_1 + 0.1e^{-2.1}$.

So $y_2 = y_1 + 0.1e^{-2.1}$. A1

Question 9

Attempting implicit differentiation i.e. $\frac{2x}{a^2} - \frac{2y}{b^2} \frac{dy}{dx} = 0$. M1

So, $\frac{dy}{dx} = \frac{b^2 x}{a^2 y}$. A1

The equation of the tangent line at the point $P(x_1, y_1)$ is given by $y - y_1 = \frac{b^2 x_1}{a^2 y_1}(x - x_1)$. A1

$$\frac{y_1}{b^2}(y - y_1) = \frac{x_1}{a^2}(x - x_1)$$

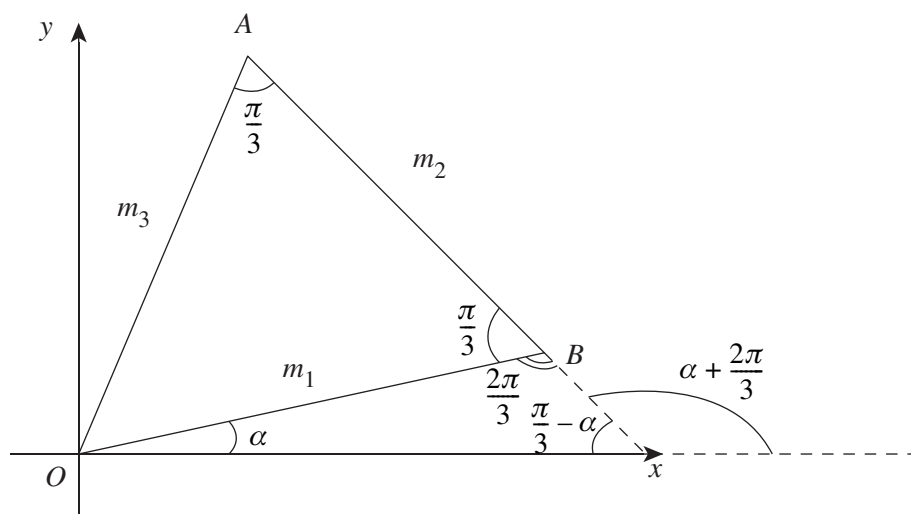
$$\frac{y_1 y}{b^2} - \frac{(y_1)^2}{b^2} = \frac{x_1 x}{a^2} - \frac{(x_1)^2}{a^2}$$
A1

$$\frac{(x_1)^2}{a^2} - \frac{(y_1)^2}{b^2} = \frac{x_1 x}{a^2} - \frac{y_1 y}{b^2}$$

Given that $\frac{(x_1)^2}{a^2} - \frac{(y_1)^2}{b^2} = 1$, the equation of the tangent line becomes $\frac{x_1 x}{a^2} - \frac{y_1 y}{b^2} = 1$. A1

Question 10

a.



$$m_2 = \tan\left(\alpha + \frac{2\pi}{3}\right) \quad \text{A1}$$

Using $\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$ we obtain: M1

$$m_2 = \tan\left(\alpha + \frac{2\pi}{3}\right) = \frac{\tan(\alpha) + \tan\left(\frac{2\pi}{3}\right)}{1 - \tan(\alpha)\tan\left(\frac{2\pi}{3}\right)} \quad \text{A1}$$

As $\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$, $m_2 = \frac{\tan(\alpha) - \sqrt{3}}{1 + \sqrt{3}\tan(\alpha)}$. A1

b. $m_1 m_2 = \frac{\tan(\alpha)(\tan(\alpha) - \sqrt{3})}{1 + \sqrt{3}\tan(\alpha)}$, $m_2 m_3 = \frac{\tan^2(\alpha) - 3}{1 - 3\tan^2(\alpha)}$, $m_3 m_1 = \frac{\tan(\alpha)(\tan(\alpha) + \sqrt{3})}{1 - \sqrt{3}\tan(\alpha)}$ A1

$$m_1 m_2 + m_2 m_3 + m_3 m_1 = \frac{\tan(\alpha)(1 - \sqrt{3}\tan(\alpha))(\tan(\alpha) - \sqrt{3}) + (\tan^2(\alpha) - 3) + \tan(\alpha)(1 + \sqrt{3}\tan(\alpha))(\tan(\alpha) + \sqrt{3})}{1 - 3\tan^2(\alpha)} \quad \text{M1}$$

$$= \frac{-\sqrt{3}\tan^3(\alpha) + 4\tan^2(\alpha) - \sqrt{3}\tan(\alpha) + \tan^2(\alpha) - 3 + \sqrt{3}\tan^3(\alpha) + 4\tan^2(\alpha) + \sqrt{3}\tan(\alpha)}{1 - 3\tan^2(\alpha)}$$

$$= \frac{9\tan^2(\alpha) - 3}{1 - 3\tan^2(\alpha)} \quad \text{A1}$$

$$= \frac{-3(1 - 3\tan^2(\alpha))}{1 - 3\tan^2(\alpha)}$$

$$= -3 \quad \text{A1}$$