The Mathematical Association of Victoria

SPECIALIST MATHEMATICS 2010

Trial Written Examination 2--SOLUTIONS

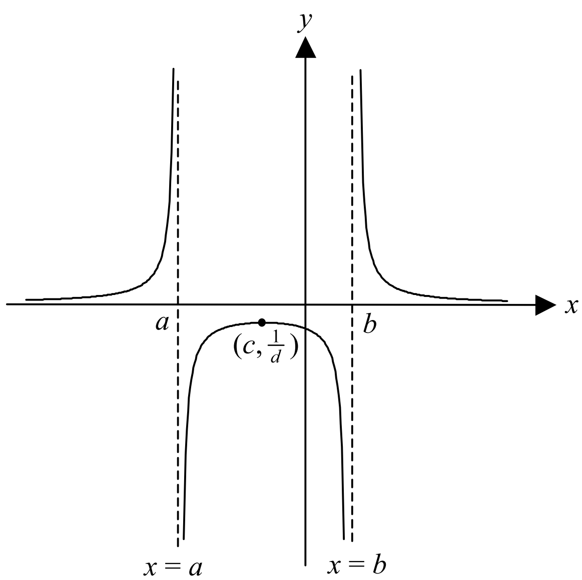
SECTION 1: Multiple Choice

Answers:

1.	C	**2.**	D	**3.**	D	**4.**	B	**5.**	A
6.	C	**7.**	E	**8.**	D	**9.**	E	**10.**	E
11.	A	**12.**	D	**13.**	C	**14.**	D	**15.**	B
16.	A	**17.**	E	**18.**	C	**19.**	B	**20.**	A
21.	C	**22.**	D						

Worked Solutions:

Question 1



The graph has asymptotes at $x = a$ and $x = b$, and a local maximum at $\left(c, \dfrac{1}{d}\right)$

Answer C

Question 2

$y = 3\sin^{-1}(x)$ Domain: $[-1, 1]$

$y = 3\sin^{-1}(ax)$ Domain: $\left[-\frac{1}{a}, \frac{1}{a}\right]$

$y = 3\sin^{-1}(ax+b)$

$= 3\sin^{-1} a\left(x + \frac{b}{a}\right)$ Domain: $\left[-\frac{1}{a} - \frac{b}{a}, \frac{1}{a} - \frac{b}{a}\right]$

Domain: $\left[\frac{-1-b}{a}, \frac{1-b}{\text{a}}\right]$

Answer D

Question 3

$$y = -\text{cosec}\left(\frac{x}{a}\right)$$

$$= \frac{-1}{\sin\left(\frac{\text{x}}{\text{a}}\right)}$$

$$= -\left(\sin\left(\frac{x}{a}\right)\right)^{-1}$$

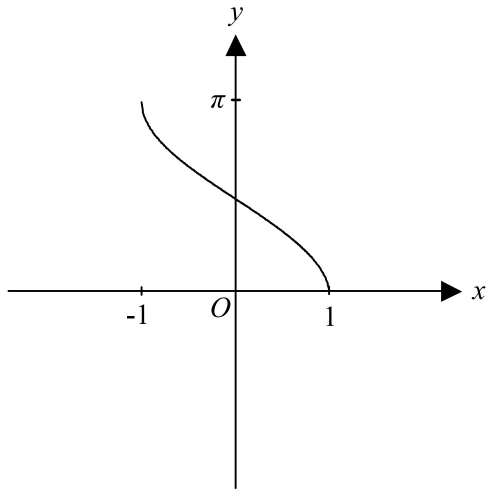
$$\frac{\text{dy}}{\text{dx}} = \left(\sin\left(\frac{x}{a}\right)\right)^{-2} \times \frac{1}{a}\cos\left(\frac{x}{a}\right)$$

$$= \frac{1}{a} \times \frac{\cos\left(\frac{x}{a}\right)}{\sin^2\left(\frac{x}{a}\right)}$$

$$= \frac{1}{a}\text{cosec}\left(\frac{x}{a}\right)\cot\left(\frac{x}{a}\right)$$

Answer D

Question 4



If $-1 \leq x \leq 1$ then $\cos(\arccos(x)) = x$

Answer B

Question 5

$\operatorname{cosec}\theta = -\dfrac{2}{\sqrt{3}}$, where $\pi \leq \theta \leq \dfrac{3\pi}{2}$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\cot^2\theta = \operatorname{cosec}^2\theta - 1$

$= \dfrac{4}{3} - 1$

$= \dfrac{1}{3}$

$\cot\theta = \dfrac{1}{\sqrt{3}}$, since $\pi \leq \theta \leq \dfrac{3\pi}{2}$

$\tan\theta = \sqrt{3}$

Answer A

Question 6

$z = a + bi$,

$iz = -b + ai$

$|iz| = \sqrt{a^2 + b^2}$

$$\frac{|iz|^2}{\bar{z}} = \frac{a^2+b^2}{a-bi}$$

$$= \frac{a^2+b^2}{a-bi} \times \frac{a+bi}{a+bi}$$

$$= \frac{(a^2+b^2)(a+bi)}{a^2+b^2}$$

$$= a + bi$$

$$= z$$

Answer C

Question 7

The line makes an angle of $\frac{3\pi}{4}$ with the positive direction of the x-axis, hence its gradient is $m = \tan\left(\frac{3\pi}{4}\right) = -1$. From the graph, the y-intercept, $c = 2$.

The equation of the straight line is $y = -x + 2$

Hence $\mathrm{Re}(z) + \mathrm{Im}(z) = 2$

Answer E

Question 8

$$\left(a\mathrm{cis}\left(\frac{\pi}{3}\right)\right)^3 \left(b\mathrm{cis}\left(\frac{\pi}{6}\right)\right)^2$$

$$= \left(a^3\mathrm{cis}\left(\frac{3\pi}{3}\right)\right)\left(b^2\mathrm{cis}\left(\frac{2\pi}{6}\right)\right)$$

$$= a^3b^2 cis\left(\pi + \frac{\pi}{3}\right)$$

$$= a^3b^2 cis\left(\frac{4\pi}{3}\right)$$

$$= a^3b^2 cis\left(-\frac{2\pi}{3}\right)$$

Answer D

Question 9

The general equation of an ellipse is $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

Graph shows centre: $(h, k)=(2, 3)$

$a=2$, $b=3$

Equation is $\frac{(x-2)^2}{2^2}+\frac{(y-3)^2}{3^2}=1$

$\frac{(x-2)^2}{4}+\frac{(y-3)^2}{9}=1$

Answer E

Question 10

$\frac{dy}{dx}=x-\sin(y)$, $x_0=2$, $y_0=\frac{\pi}{3}$

x	y
$x_0=2$	$y_0=\frac{\pi}{3}$
2.2	$y_1=y_0+0.2(x_0-\sin(y_0))$
2.4	$y_2=y_1+0.2(x_1-\sin(y_1))$
2.6	$y_3=y_2+0.2(x_2-\sin(y_2))$ $=y_2+0.2(2.4-\sin(y_2))$

Answer E

Question 11

$\frac{dQ}{dt_{in}}=\frac{dV}{dt_{in}}\frac{dQ}{dV_{in}}$ Since pure Oxygen is poured in $\frac{dQ}{dV_{in}}=1$

$=5\times1$

$=5$

$\frac{dQ}{dt_{out}}=\frac{dV}{dt_{out}}\frac{dQ}{dV_{out}}$

$=5\times\frac{Q}{100}$

$\frac{dQ}{dt}=\frac{dQ}{dt_{in}}-\frac{dQ}{dt_{out}}$

$$\frac{dQ}{dt} = 5 - \frac{Q}{20}$$

Answer A

Question 12

$y = e^{kx^2}$ Let $u = kx^2$

$y = e^u$ $\frac{du}{dx} = 2kx$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = e^{kx^2} \times 2kx$$

$$\frac{d^2y}{dx^2} = e^{kx^2}(2k) + 2kx\left(2kx \times e^{kx^2}\right)$$

Answer D

Question 13

$\int \frac{2x}{\sqrt{2x-1}}\,dx$ Let $u = 2x - 1$

$\frac{du}{dx} = 2$ $x = \frac{u+1}{2}$

$$= \frac{1}{2}\int \frac{u+1}{\sqrt{u}} \frac{du}{dx} dx$$

$$= \frac{1}{2}\int \left(u^{\frac{1}{2}} + u^{-\frac{1}{2}}\right) du$$

Answer C

Question 14

Since X, Y and Z are collinear $\overrightarrow{XZ} = \lambda \overrightarrow{XY}$ where λ is a scalar quantity.

Find vectors $\overrightarrow{XY}$ and $\overrightarrow{XZ}$

$\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OY}$ $\overrightarrow{XZ} = \overrightarrow{XO} + \overrightarrow{OZ}$

$\overrightarrow{XY} = -(m\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}) + \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$ $\overrightarrow{XZ} = -(m\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}) - \underset{\sim}{i} + 2\underset{\sim}{j} + 4\underset{\sim}{k}$

$\overrightarrow{XY} = (1-m)\underset{\sim}{i} + \underset{\sim}{k}$ $\overrightarrow{XZ} = (-1-m)\underset{\sim}{i} + 3\underset{\sim}{k}$

From the $\underset{\sim}{k}$ components of both vectors we see $\lambda = 3$, so $\overrightarrow{XZ} = 3\overrightarrow{XY}$

Therefore $3(1-m)=(-1-m)$
$3-3m=-1-m$
$4=2m$
$m=2$
Answer D

Question 15

$|\underset{\sim}{a}+\underset{\sim}{b}|^2=(\underset{\sim}{a}+\underset{\sim}{b}).(\underset{\sim}{a}+\underset{\sim}{b})$

$|\underset{\sim}{a}+\underset{\sim}{b}|^2=\underset{\sim}{a}.\underset{\sim}{a}+\underset{\sim}{a}.\underset{\sim}{b}.+\underset{\sim}{b}.\underset{\sim}{a}+\underset{\sim}{b}.\underset{\sim}{b}$

$|\underset{\sim}{a}+\underset{\sim}{b}|^2=|\underset{\sim}{a}|^2+2\underset{\sim}{a}.\underset{\sim}{b}+|\underset{\sim}{b}|^2$

$|\underset{\sim}{a}+\underset{\sim}{b}|^2=5^2+2\times10+6^2$

$|\underset{\sim}{a}+\underset{\sim}{b}|^2=81$

$|\underset{\sim}{a}+\underset{\sim}{b}|=9$

Answer B

Question 16

Option A defines a square. Both options B and C define a rhombus. Option D defines a rectangle. Option E defines a parallelogram.
Answer A

Question 17

$\underset{\sim}{v}=3e^{-3t}\underset{\sim}{i}+2e^{-2t}\underset{\sim}{j}$

$\underset{\sim}{r}=\int\left(3e^{-3t}\underset{\sim}{i}+2e^{-2t}\underset{\sim}{j}\right)dt$

$\underset{\sim}{r}=3\times-\frac{1}{3}e^{-3t}\underset{\sim}{i}+2\times-\frac{1}{2}e^{-2t}\underset{\sim}{j}+\underset{\sim}{c}$

$\underset{\sim}{r}=-e^{-3t}\underset{\sim}{i}+-e^{-2t}\underset{\sim}{j}+\underset{\sim}{c}$

At $t=0$, $\underset{\sim}{r}=0\underset{\sim}{i}+0\underset{\sim}{j}$ $\quad\Rightarrow\quad 0\underset{\sim}{i}+0\underset{\sim}{j}=-e^{-3\times0}\underset{\sim}{i}+-e^{-2\times0}\underset{\sim}{j}+\underset{\sim}{c}$

$\Rightarrow\quad \underset{\sim}{c}=\underset{\sim}{i}+\underset{\sim}{j}$

$\underset{\sim}{r}=-e^{-3t}\underset{\sim}{i}+-e^{-2t}\underset{\sim}{j}+\underset{\sim}{i}+\underset{\sim}{j}$

$\underset{\sim}{r}=(1-e^{-3t})\underset{\sim}{i}+(1-e^{-2t})\underset{\sim}{j}$

Answer E

Question 18

Distance travelled while decelerating

$s = ut + \frac{1}{2}at^2$

$s = 35\times10 + \frac{1}{2}\times -2\times10^2$

$s = 250$ metres

Velocity after 10 seconds

$u = 35\,\text{m/s} \qquad a = -2\ \text{m/s}^2 \qquad t = 10$

$v = u + at$

$v = 35 - 2\times10$

$v = 15\,\text{m/s}$

Distance travelled in the next 50 seconds

$s = 15\times50$

$s = 750$ metres

Total distance travelled in 60 seconds is 1000 metres

Answer C

Question 19

$|\underset{\sim}{F}| = m|\underset{\sim}{a}|$

$|\underset{\sim}{F}| = 0.5\times10$

$|\underset{\sim}{F}| = 5$ newtons

The resultant of the two forces must be of magnitude 5 newtons.

$|\underset{\sim}{F}| = \sqrt{1^2+4^2} = \sqrt{17}$

$|\underset{\sim}{F}| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

$|\underset{\sim}{F}| = \sqrt{6^2+4^2} = \sqrt{52} = 2\sqrt{13}$

$|\underset{\sim}{F}| = \sqrt{5^2+5^2} = \sqrt{50} = 2\sqrt{5}$

$|\underset{\sim}{F}| = \sqrt{2.5^2+2.5^2} = \sqrt{12.5}$

Answer B

Question 20
Using Lami's Theorem

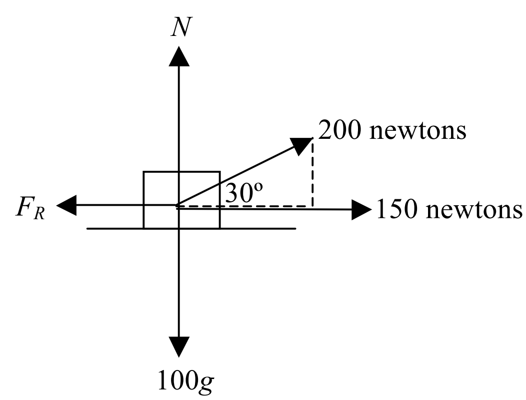
$$\frac{P}{\sin(150°)} = \frac{12}{\sin(120°)}$$

$$P = \frac{12\sin(150)}{\sin(120°)}$$

$P = 4\sqrt{3}$ newtons
Answer A

Question 21
$F = ma$
$4t + 2 = 2a$
$a = 2t + 1$
$\frac{dv}{dt} = 2t + 1$
$v = \int (2t+1)dt$
$v = t^2 + t + c$
When $t = 0$, $v = 1$ m/s $\Rightarrow c = 1$
$v = t^2 + t + 1$
After 3 seconds the velocity of the particle is $v = 3^2 + 3 + 1 = 13$ m/s
Answer C

Question 22
Resolving vertical forces to find the normal reaction, N.



$N + 200\sin(30°) - 100g = 0$
$N = 100g - 100$

Calculating the friction, F_R, acting to oppose motion
$F_R = \mu N$
$F_R = 0.2(100g - 100)$
$F_R = 20g - 20$

Equation of motion
$150 + 200\cos(30°) - F_R = 100a$

$150 + 100\sqrt{3} - (20g - 20) = 100a$

$$a = \frac{170 + 100\sqrt{3} - 20g}{100}$$

$a = 1.47$ m/s^2
Answer D

SECTION 2
Question 1
a.

$g(x) = (\cos(2x))^{-1}$

$g'(x) = -1(\cos(2x))^{-2} \times (-2\sin(2x))$ [M1]

$= \dfrac{2\sin(2x)}{\cos^2(2x)}$

$= \dfrac{2\sin(2x)}{\cos(2x)} \times \dfrac{1}{\cos(2x)}$ [A1]

$= 2\tan(2x)\sec(2x)$

b.
Using either a solve function on calculator or graphing and finding points of intersection:
$x = -0.543,\ 0.319$ [A2]

c. i.

$f'(x) = \dfrac{-1}{\sqrt{1-x^2}}$

$2\tan(2x)\sec(2x) = \dfrac{-1}{\sqrt{1-x^2}}$

Solving on calculator $x = -0.996$, $x = -0.217$ [A1]

c. ii.

$f'(x)$ and $g'(x)$ represent the gradients of $f(x)$ and $g(x)$ respectively.
For $x > 0$, $f'(x) < 0$ and $g'(x) > 0$ hence there is no solution for $x > 0$. [A1]

d. i.

Area = $\displaystyle\int_{-0.543}^{0.319} \left(\cos^{-1}(x) - \sec(2x)\right) dx$ [A1]

d. ii.

Area = $\displaystyle\int_{-0.543}^{0.319} \left(\cos^{-1}(x) - \frac{1}{\cos(2x)}\right) dx = 0.412$ [A1]

e.

$$V = \int_{-0.542}^{0.319} \left(\cos^{-1}(x)\right)^2 dx - \int_{-0.542}^{0.319} \left(\sec(2x)\right)^2 dx$$ [A1]

$$V = \int_{-0.542}^{0.319} \left(\cos^{-1}(x)\right)^2 dx - \int_{-0.542}^{0.319} \left(\frac{1}{\cos(2x)}\right)^2 dx$$ [M1]

$= 7.8688 - 4.1324$

$= 3.736$ cubic units [A1]

Total 11 marks

Question 2

a.

$|z+4|^2$

$=|x+iy+4|^2$

$=|(x+4)+iy|^2$

$=\left(\sqrt{(x+4)^2+y^2}\right)^2$

$=(x+4)^2+y^2$ [A1]

$=x^2+y^2+8x+16$

b.

$(|z|+2)^2$

$=(|x+iy|+2)^2$

$=\left(\sqrt{x^2+y^2}+2\right)^2$

$=\left(\sqrt{x^2+y^2}\right)^2+2\times\sqrt{x^2+y^2}\times 2+4$

$=x^2+y^2+4\sqrt{x^2+y^2}+4$ [A1]

$=x^2+y^2+4+4\sqrt{x^2+y^2}$

c.

$|z+4|=|z|+2$

$\Rightarrow \quad |z+4|^2=(|z|+2)^2$

$x^2+y^2+8x+16=x^2+y^2+4+4\sqrt{x^2+y^2}$ [A1]

$8x+12=4\sqrt{x^2+y^2}$

$2x+3=\sqrt{x^2+y^2}$

$(2x+3)^2=x^2+y^2$ [M1]

$3x^2+12x+9-y^2=0$

$3(x^2+4x+4)+9-12-y^2=0$

$3(x+2)^2 - y^2 = 3$ [A1]

$(x+2)^2 - \frac{y^2}{3} = 1$ Graph is a hyperbola

d.
Asymptotes of hyperbola are given by:

$y - k = \pm\frac{b}{a}(x-h)$

$y - 0 = \pm\frac{\sqrt{3}}{1}(x+2)$

$y = \sqrt{3}(x+2)$ and $y = -\sqrt{3}(x+2)$ [A1] [A1]

e.

Sketching hyperbola $(x+2)^2 - \frac{y^2}{3} = 1$

x-intercepts, $y = 0$

$(x+2)^2 - \frac{0^2}{3} = 1$

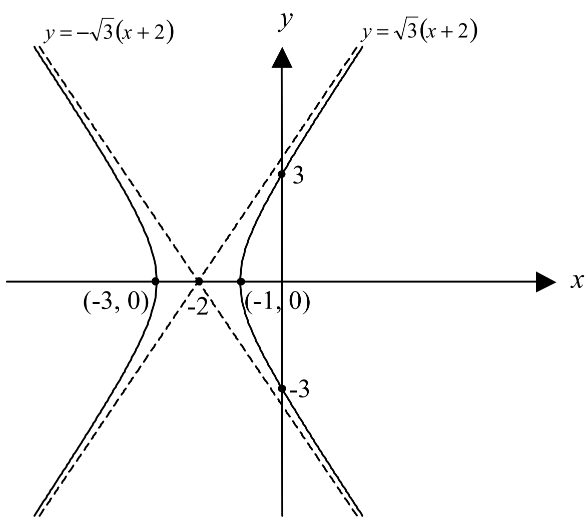
$(x+2) = \pm 1$

$x = -1, \text{ or } -3$

y-intercepts, $x = 0$

$(0+2)^2 - \frac{y^2}{3} = 1$

$y = \pm 3$



Shape [A1]
Intercepts [A1]
Asymptotes [A1]

Total 10 marks

Question 3
a.

$\frac{dQ}{dt} = -kQ$

$\frac{dt}{dQ} = -\frac{1}{kQ}$

$t = -\frac{1}{k}\int \frac{1}{Q}\, dQ$ [M1]

$-kt = \log_e |Q| + c$

$-kt - c = \log_e |Q|$

$Q = e^{-kt-c}$ [A1]

$Q = e^{-kt}e^{-c}$

$Q = Ae^{-kt}$, where $A = e^{-c}$ [A1]

When t = 0, $Q = Q_0$

Hence $Q = Q_0 e^{-kt}$ as required

b.

$Q = Q_0 e^{-kt}$ When $t = 6$, $Q = \frac{1}{2}Q_0$

$\frac{1}{2}Q_0 = Q_0 e^{-6k}$

$2^{-1} = e^{-6k}$ [A1]

$\log_e 2^{-1} = \log_e e^{-6k}$

$-\log_e 2 = -6k$

$k = \frac{1}{6}\log_e 2$ [A1]

Hence $Q = Q_0 e^{-\frac{t}{6}\log_e 2}$

c.

$Q = Q_0 e^{-\frac{t}{6}\log_e 2}$

Want t, when $Q = \frac{Q_0}{5}$

$\frac{Q_0}{5} = Q_0 e^{\left(-\frac{t}{6}\log_e 2\right)}$ [M1]

$\frac{1}{5} = e^{\left(-\frac{t}{6}\log_e 2\right)}$

$\log_e\left(\frac{1}{5}\right) = -\frac{t}{6}\log_e 2$ Can solve on calculator from here.

$-\log_e 5 = -\frac{t}{6}\log_e 2$

$t = \frac{6\log_e 5}{\log_e 2}$

$t = 13.93$ hours or 13 hours, 56 minutes [A1]

d.

$Q = Q_0 e^{-\frac{t}{6}\log_e 2}$

$Q = 0.002e^{-\frac{30}{60}\times\frac{1}{6}\log_e 2}$ [M1]

$= 0.002e^{-\frac{1}{12}\log_e 2}$

$= 0.00189$ grams [A1]

Total 9 marks

Question 4

a.

At $t = 0$, $\quad \underset{\sim}{r}(0) = \cos(0)\underset{\sim}{i} + \sin(2\times 0)\underset{\sim}{j}$

$\underset{\sim}{r}(0) = \underset{\sim}{i} + 0\underset{\sim}{j}$

Initially the particle is at the point (1, 0). It is 1 metre to the right of *O*. [A1]

b.

$\underset{\sim}{r}(t) = \cos(t)\underset{\sim}{i} + \sin(2t)\underset{\sim}{j}$

$x = \cos(t) \qquad y = \sin(2t)$

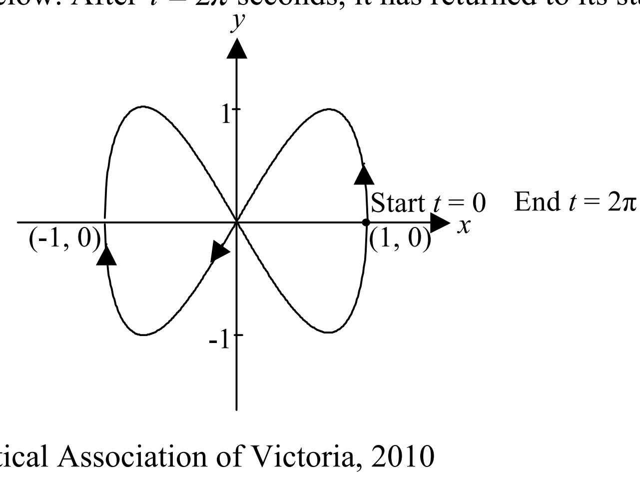
$y = 2\sin(t)\cos(t)$ [M1]

$y = 2x\sin(t)$ where $\sin(t) = \pm\sqrt{1-\cos^2(t)} = \pm\sqrt{1-x^2}$

$y = \pm 2x\sqrt{1-x^2}$ [A1]

c.

Use parametric mode on calculator to sketch graph.
The particle starts at (1, 0) and moves in an anti-clockwise direction as shown by the arrows on the graph below. After $t = 2\pi$ seconds, it has returned to its starting position.



Shape [A1]
Direction of motion [A1]

Alternatively: Sketch the graph using the Cartesian equations
$y = 2x\sqrt{1-x^2}$ for path when $t \in [0, \pi]$ and $y = -2x\sqrt{1-x^2}$ for path when $t \in [\pi, 2\pi]$

d.
Differentiate to find the velocity vector
$\dot{\underset{\sim}{r}}(t) = -\sin(t)\,\underset{\sim}{i} + 2\cos(2t)\,\underset{\sim}{j}$ [A1]

Speed is given by $s = |\dot{\underset{\sim}{r}}(t)|$

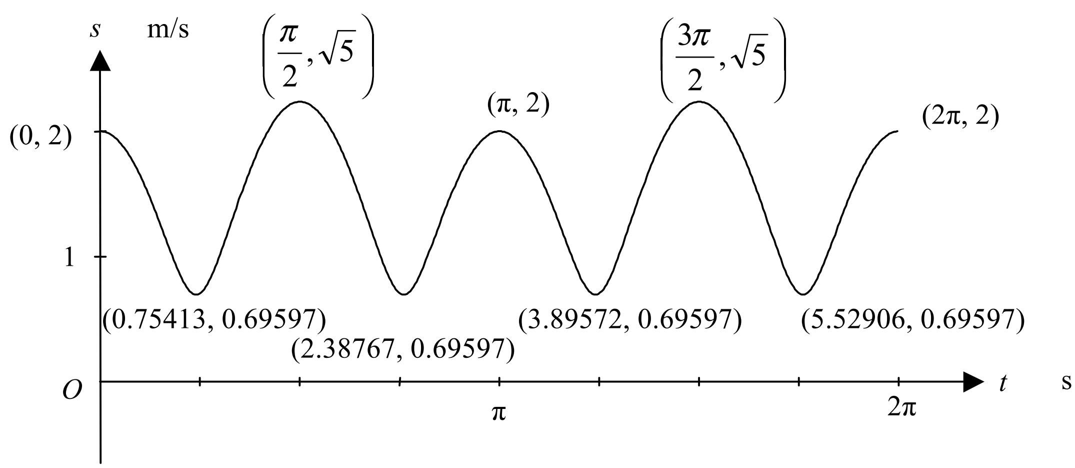
$s = |\dot{\underset{\sim}{r}}(t)| = \sqrt{(-\sin(t))^2 + (2\cos(2t))^2}$ [A1]

$s = \sqrt{\sin^2(t) + 4\cos^2(2t)}$ m/s

e.
Maximum speed is $\sqrt{1+4} = \sqrt{5}$ m/s. [A1]

f.
The particle is moving with a maximum speed when both $\sin^2(t) = 1$ and $\cos^2(2t) = 1$
Solve $\{t : \sin^2(t) = 1 \quad \cap \quad \cos^2(2t) = 1\}$ over $t \in [0, 2\pi]$
$\Rightarrow \qquad t = \frac{\pi}{2}, \frac{3\pi}{2}$ seconds [A1]
At $t = \frac{\pi}{2}$ seconds, $\underset{\sim}{r}\left(\frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2}\right)\underset{\sim}{i} + \sin\left(2 \times \frac{\pi}{2}\right)\underset{\sim}{j} = 0\,\underset{\sim}{i} + 0\,\underset{\sim}{j}$
At $t = \frac{3\pi}{2}$ seconds, $\underset{\sim}{r}\left(\frac{3\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right)\underset{\sim}{i} + \sin\left(2 \times \frac{3\pi}{2}\right)\underset{\sim}{j} = 0\,\underset{\sim}{i} + 0\,\underset{\sim}{j}$
The particle is moving at its maximum speed when it passes through the origin (0, 0). [A1]

g.
The minimum speed can be found from the graph of $s = \sqrt{\sin^2(t) + 4\cos^2(2t)}, \quad t \in [0, 2\pi]$



The minimum speed is 0.6960 m/s (correct to four decimal places) [A1]

Finding the position coordinates when the particle is first moving with a minimum speed.
Use position vector $\underset{\sim}{r}(t)=\cos(t)\,\underset{\sim}{i}+\sin(2t)\,\underset{\sim}{j}$

At $t=0.754128$, $\underset{\sim}{r}(0.754128)=\cos(0.754128)\,\underset{\sim}{i}+\sin(2\times 0.754128)\,\underset{\sim}{j}$

$$\underset{\sim}{r}(0.754128)=0.72886898\,\underset{\sim}{i}+0.99804496\,\underset{\sim}{j}$$

The particle first reaches its minimum speed at the point (0.7289, 0.9980) [A1]

Alternatively, the times for minimum speed can be found by solving $\frac{ds}{dt}=0$

$$\frac{d}{dt}\left(\sqrt{\sin^2(t)+4\cos^2(2t)}\right)=0, \qquad t\in[0,\,2\pi]$$

$$\frac{-8\sin(2t)\cos(2t)+\sin(t)\cos(t)}{\sqrt{\sin^2(t)+4\cos^2(2t)}}=0$$

$$\frac{1}{2}\sin(2t)-8\sin(2t)\cos(2t)=0$$

$$\frac{1}{2}\sin(2t)\left(1-16\cos(2t)\right)=0$$

$\sin(2t)=0$ or $1-16\cos(2t)=0$ for $t\in[0,\,2\pi]$

$t=0,\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi$ seconds $\qquad t=0.754128,\ 2.387465,\ 3.895720,\ 5.529058$ seconds

(times for minimum speed)

Minimum speed is $\sqrt{\sin^2(0.754128)+4\cos^2(2\times 0.754128)}=0.6960$ m/s

h.

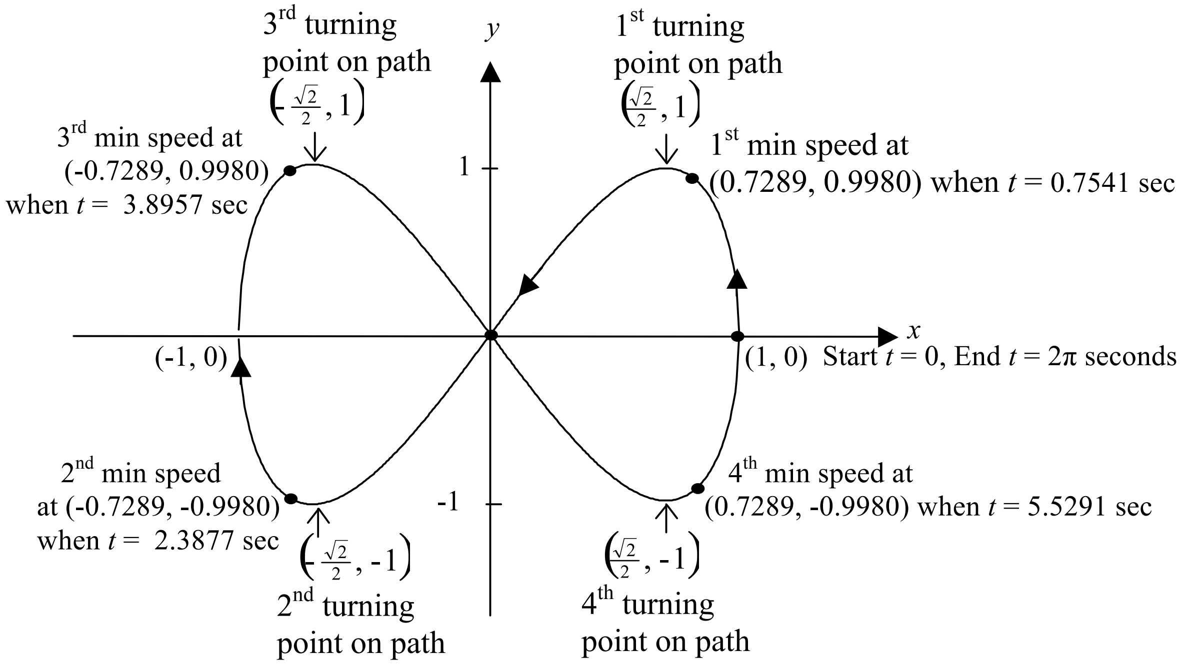
The particle starts at the point (1, 0) with a speed of 2 m/s. It slows down to reach its minimum speed of 0.6960 m/s at the point (0.7289, 0.9980). This occurs just before it reaches the maximum turning point on its path at $\left(\frac{\sqrt{2}}{2},1\right)$. It then increases its speed to a maximum of $\sqrt{5}$ m/s at $\frac{\pi}{2}$ seconds when passing through the origin for the first time. After it passes the origin, the particle slows down. The second place it reaches its minimum speed is at (-0.7289, -0.9980) and this occurs just after it passes the minimum turning point on its path at $\left(-\frac{\sqrt{2}}{2},-1\right)$. This point may be obtained by symmetry and verified by substituting $t=2.387465$ the into the position vector:

$$\underset{\sim}{r}(2.387465)=\cos(2.387465)\,\underset{\sim}{i}+\sin(2\times 2.387465)\,\underset{\sim}{j}$$

$$\underset{\sim}{r}(2.387465)=-0.728869\,\underset{\sim}{i}-0.998045\,\underset{\sim}{j}$$

The particle continues on and increases its speed to 2 m/s when passing through (-1, 0). It then slows down again to its minimum speed at the point (-0.7289, 0.9980). At $\frac{3\pi}{2}$ seconds, it has returned to the origin and is travelling at its maximum speed. It then slows down again to its minimum speed at the point (0.7289, -0.9980). After 2π seconds, the particle has returned to its initial starting position at (1, 0) and is travelling at 2 m/s.

The points where the particle is travelling at its minimum speed are not at the turning points on its path, as shown in the graph below.

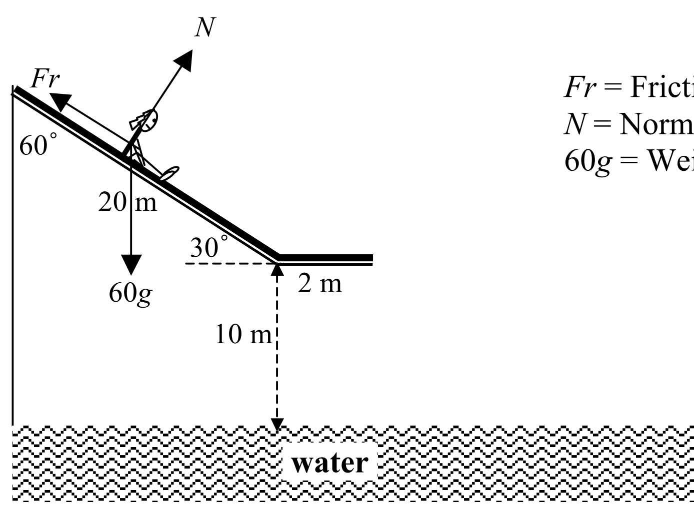


Reasonable description [A1]
Complete description [A1]

Total 14 marks

Question 5

a.



Fr = Frictional force
N = Normal reaction
$60g$ = Weight force

All forces shown [A1]

b.

Resolving forces parallel to plane

$mg\sin(30°) - Fr = ma$

$60g\sin(30°) - \mu N = ma \ldots(1)$ where $\mu = 0.3$

Resolving forces perpendicular to plane

$N = 60g\cos(30°) \ \ldots(2)$

Resolution of forces [M2]

Substitute (2) into (1)

$60g\sin(30°) - 0.3\times 60g\cos(30°) = 60a$

$a = \dfrac{60g\sin(30°) - 0.3\times 60g\cos(30°)}{60}$ [A1]

$a = 2.35\ \text{m/s}^2$

c.

Harriet slides down the inclined plane with constant acceleration. Her velocity after travelling 20 metres may be found using

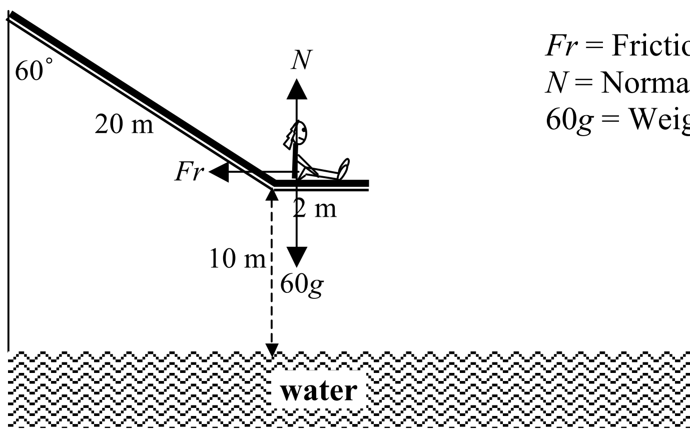
$v^2 = u^2 + 2as$ where $u = 0,\quad s = 20,\quad a = 2.35$ [M1]

$v^2 = 0 + 2\times 2.35\times 20$ [A1]

$v = 9.7$ m/s

d.

The forces acting on Harriet in the horizontal plane are shown in the diagram.



Fr = Frictional force
N = Normal reaction
$60g$ = Weight force

Finding the frictional force

$F_R = 0.3N$ where $N = 60g$

$F_R = 0.3\times 60g = 18g$ [A1]

Equation of motion

$-F_R = ma$

$-18g = 60a$

$a = -0.3g$

$a = -0.3\times 9.8$

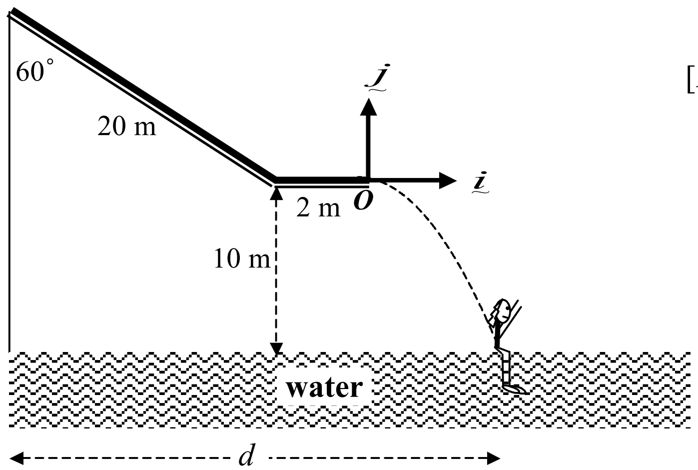
$a = -2.94\ \text{m/s}^2$ [A1]

Acceleration is constant

$u = 9.7, \quad a = -2.94, \quad s = 2$
$v^2 = u^2 + 2as$
$v^2 = 9.7^2 + 2\times(-2.94)\times 2$ [M1]
$v = 9.1\,\text{m/s}$

e.
When Harriet leaves the slide she is in freefall, moving under the force of gravity.
Using vectors to find an expression for her position, $\underset{\sim}{r}$, at any time t seconds.

Define the end of the water slide as the origin O for the $\underset{\sim}{i}$ - $\underset{\sim}{j}$ coordinate system as shown below.



$\underset{\sim}{a} = -9.8\,\underset{\sim}{j}$

$\dfrac{dv}{dt} = -9.8\,\underset{\sim}{j}$ [A1]

$\underset{\sim}{v} = \displaystyle\int\left(-9.8\,\underset{\sim}{j}\right)dt$

$\underset{\sim}{v} = -9.8t\,\underset{\sim}{j} + \underset{\sim}{c}$

When $t = 0$ $\underset{\sim}{v} = 9.1\,\underset{\sim}{i}$

$9.1\,\underset{\sim}{i} = -9.8\times 0\,\underset{\sim}{j} + \underset{\sim}{c}$

$\underset{\sim}{c} = 9.1\,\underset{\sim}{i}$

$\underset{\sim}{v} = \dfrac{dr}{dt} = 9.1\,\underset{\sim}{i} - 9.8t\,\underset{\sim}{j}$ [A1]

$\underset{\sim}{r} = \displaystyle\int\left(9.1\,\underset{\sim}{i} - 9.8t\,\underset{\sim}{j}\right)dt$

$\underset{\sim}{r} = 9.1t\,\underset{\sim}{i} - 4.9t^2\,\underset{\sim}{j} + \underset{\sim}{c}_2$

When $t = 0$ $\underset{\sim}{r} = 0\,\underset{\sim}{i} + 0\,\underset{\sim}{j} \quad \Rightarrow \underset{\sim}{c}_2 = 0\,\underset{\sim}{i} + 0\,\underset{\sim}{j}$

$\underset{\sim}{r} = 9.1t\,\underset{\sim}{i} - 4.9t^2\,\underset{\sim}{j}$ [A1]

Let $\underset{\sim}{r} = 9.1t\,\underset{\sim}{i} \; -10\,\underset{\sim}{j}$ be Harriet's position in relation to O when she enters the water.

Equating $\underset{\sim}{j}$ components of $\underset{\sim}{r}$ to find the time t seconds when Harriet enters the water.

$-4.9t^2 = -10$
$t = 0.7$ seconds [A1]
Harriet's position is $\underset{\sim}{r} = 9.1\times 0.7\,\underset{\sim}{i} \; -10\,\underset{\sim}{j} = 6.4\,\underset{\sim}{i} - 10\,\underset{\sim}{j}$

The horizontal distance travelled, correct to one decimal place
$d = 20\sin(60°) + 2 + 6.4$
$d = 25.7$ metres [A1]
Total 14 marks