# Year 2010

# VCE

# Specialist Mathematics

# Trial Examination 2



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|---|---|
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# Victorian Certificate of Education 2010

**STUDENT NUMBER**

| | | | | | | | | | Letter |
|---|---|---|---|---|---|---|---|---|---|
| Figures | | | | | | | | | |
| Words | | | | | | | | | |

# SPECIALIST MATHEMATICS
## Trial Written Examination 2

Reading time: 15 minutes
Total writing time: 2 hours

## QUESTION AND ANSWER BOOK

**Structure of book**

| *Section* | *Number of questions* | *Number of questions to be answered* | *Number of marks* |
|---|---|---|---|
| 1 | 22 | 22 | 22 |
| 2 | 5 | 5 | 58 |
| | | | Total 80 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved graphics calculator or CAS ( memory DOES NOT need to be cleared ) and, if desired, one scientific calculator.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

- Question and answer booklet of 32 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Answer sheet for multiple choice questions.

**Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, and sign your name in the space provided to verify this.
- All written responses must be in English.

**At the end of the examination**

- Place the answer sheet for multiple-choice questions inside the front cover of this book.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

# SECTION 1

**Instructions for Section I**

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
A correct answer scores 1 mark, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No mark will be given if more than one answer is completed for any question.
Take the acceleration due to gravity to have magnitude g m/s$^2$, where $g = 9.8$

**Question 1**

Consider the graph of $y = \dfrac{1}{3a^2 + 2ax - x^2}$ where $a$ is a non-zero real constant. Which of the following is true?

**A.** The lines $x = -3a$ and $x = a$ are vertical asymptotes and the graph crosses the $y$-axis at $\left(0\ ,\ \dfrac{1}{3a^2}\right)$

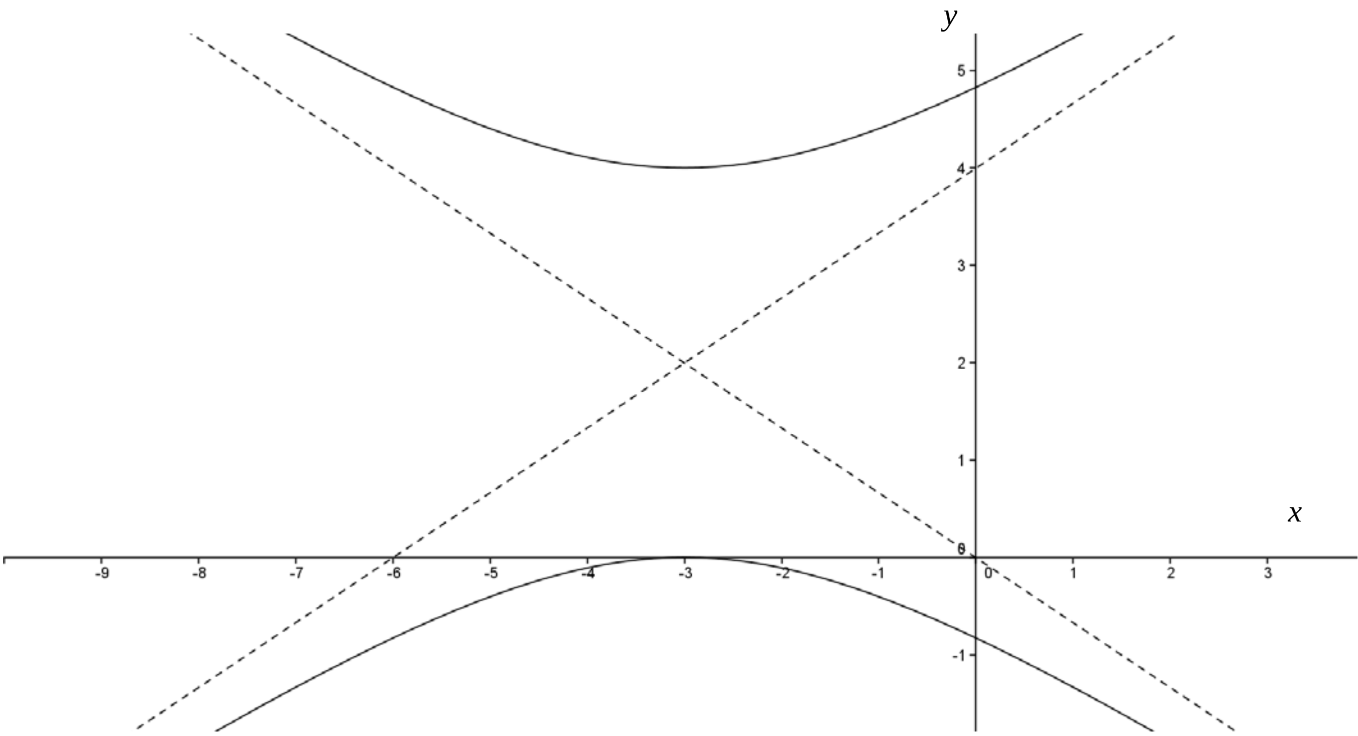
**B.** The lines $x = -3a$ and $x = a$ are vertical asymptotes and the graph has a maximum at $\left(a, \dfrac{1}{4a^2}\right)$

**C.** The lines $x = 3a$ and $x = -a$ are vertical asymptotes and the graph has a maximum at $\left(a, \dfrac{1}{4a^2}\right)$

**D.** The lines $x = 3a$ and $x = -a$ are vertical asymptotes and the graph has a minimum at $\left(a, \dfrac{1}{4a^2}\right)$

**E.** The graph has no vertical asymptotes, the $x$-axis is a horizontal asymptote and the graph crosses the $y$-axis at $\left(0\ ,\ \dfrac{1}{3a^2}\right)$

**Question 2**




The graph shown above has the equation

**A.** $\dfrac{(x+3)^2}{9}-\dfrac{(y-2)^2}{4}=1$

**B.** $\dfrac{(x+3)^2}{3}-\dfrac{(y-2)^2}{2}=1$

**C.** $\dfrac{(x-3)^2}{9}-\dfrac{(y+2)^2}{4}=1$

**D.** $\dfrac{(y-2)^2}{4}-\dfrac{(x+3)^2}{9}=1$

**E.** $\dfrac{(y+2)^2}{2}-\dfrac{(x-3)^2}{3}=1$

**Question 3**

The expression $x^2 - 2ax + 2y^2 + 8ay = 9 - 10a^2$ will represent an ellipse with centre at

**A.** $(a, -2a)$ if $|a| < 3$

**B.** $(a, -2a)$ if $a \in R$

**C.** $(-a, 2a)$ if $|a| < 3$

**D.** $(-a, 2a)$ if $a \in R$

**E.** $(-a, 2a)$ if $|a| > 3$

**Question 4**

Consider the function with the rule $f(x) = \frac{b}{\pi}\cos^{-1}\left(\frac{x}{b} - 1\right) - b$, where $b$ is a positive real number, then the maximal domain and range respectively are equal to

**A.** $[0, 2b]$ and $[-b, 0]$

**B.** $[0, 2b]$ and $[0, b(\pi - 1)]$

**C.** $[0, 2b]$ and $\left[-\frac{3b}{2}, -\frac{b}{2}\right]$

**D.** $[0, b]$ and $[0, b\pi]$

**E.** $[0, b]$ and $[0, b]$

**Question 5**

If $x + yi = r\,\text{cis}(\theta)$ then $-x + yi$ is equal to

**A.** $-r\,\text{cis}(\theta)$

**B.** $r\,\text{cis}\left(\frac{\pi}{2} - \theta\right)$

**C.** $r\,\text{cis}\left(\frac{\pi}{2} + \theta\right)$

**D.** $r\,\text{cis}(\pi - \theta)$

**E.** $r\,\text{cis}(\pi + \theta)$

**Question 6**

The relation $(\bar{z}+ai)(z-ai)=a^2$, where $a\in R$, when graphed on an Argand diagram, would be

**A.** a circle of radius *a*, with centre at $(a,0)$

**B.** a circle of radius *a*, with centre at $(0,a)$

**C.** a circle of radius $\sqrt{2}a$, with centre at $(-a,0)$

**D.** a circle of radius $\sqrt{2}a$, with centre at $(0,-a)$

**E.** a null set.

**Question 7**

The length of the vector $(m-2)\underset{\sim}{i}-(m+1)\underset{\sim}{j}+(m+1)\underset{\sim}{k}$ where *m* is a real constant is equal to

**A.** $m-2$

**B.** $3m$

**C.** $\sqrt{3}\left(m+\sqrt{2}\right)$

**D.** $\sqrt{3m}$

**E.** $\sqrt{3\left(m^2+2\right)}$

**Question 8**

If *m* and *n* are real constants, then the two vectors
$\sqrt{m}\,\underset{\sim}{i}+n\underset{\sim}{j}-n\underset{\sim}{k}$ and $-2\sqrt{m}\,\underset{\sim}{i}+4\underset{\sim}{j}-\sqrt{m}\underset{\sim}{k}$ are parallel when

**A.** $n=-1$ and $m=1$

**B.** $n=-2$ and $m=4$

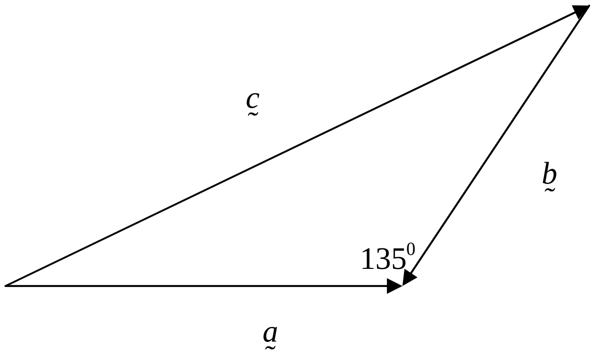
**C.** $n=-2$ and $m=16$

**D.** $n=2$ and $m=-4$

**E.** $n=8$ and $m=16$

**Question 9**

Vectors $\underset{\sim}{a}$ , $\underset{\sim}{b}$ and $\underset{\sim}{c}$ are shown below.




From the diagram it follows that

**A.** $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + \sqrt{2}|\underset{\sim}{a}||\underset{\sim}{b}|$

**B.** $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - \sqrt{2}|\underset{\sim}{a}||\underset{\sim}{b}|$

**C.** $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + |\underset{\sim}{a}||\underset{\sim}{b}|$

**D.** $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - |\underset{\sim}{a}||\underset{\sim}{b}|$

**E.** $|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2$

**Question 10**

If $b$, $c$, $\alpha$ and $\beta$ are all real numbers, and the quadratic $z^2 + bz + c$ has $-\alpha + \beta i$ as a root, then the quadratic, which has $2\alpha - 2\beta i$ as a root, is given by

**A.** $z^2 + 2bz + 2c$

**B.** $z^2 - 2bz + 2c$

**C.** $z^2 + 2bz + 4c$

**D.** $z^2 - 2bz + 4c$

**E.** $z^2 - 4bz + 8c$

**Question 11**

If $|\underset{\sim}{r}| = 6$ and $\underset{\sim}{s} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$ and the vector resolute of $\underset{\sim}{r}$ in the direction of $\underset{\sim}{s}$ is equal to $3\left(-2\underset{\sim}{i} + 3\underset{\sim}{j} - \underset{\sim}{k}\right)$, then the scalar resolute of $\underset{\sim}{s}$ in the direction of $\underset{\sim}{r}$ is equal to

**A.** $-\frac{1}{2}$

**B.** $-7$

**C.** $-3\sqrt{14}$

**D.** $3\sqrt{14}$

**E.** $-\frac{3}{\sqrt{42}}$

**Question 12**

The acceleration of a particle at a time $t$, $t \geq 0$, is given by $\ddot{\underset{\sim}{r}}(t) = 8\sin^2(t)\underset{\sim}{i} + 8\cos^2(t)\underset{\sim}{j}$. The initial position of the particle is given by $\underset{\sim}{r}(0) = \underset{\sim}{0}$ and the initial velocity is given by $\dot{\underset{\sim}{r}}(0) = \underset{\sim}{0}$. The position vector $\underset{\sim}{r}(t)$ is given by

**A.** $\underset{\sim}{r}(t) = \left(2t^2 - \cos(2t)\right)\underset{\sim}{i} + \left(2t^2 + \cos(2t)\right)\underset{\sim}{j}$

**B.** $\underset{\sim}{r}(t) = \left(2t^2 - \cos(2t) + 1\right)\underset{\sim}{i} + \left(2t^2 + \cos(2t) - 1\right)\underset{\sim}{j}$

**C.** $\underset{\sim}{r}(t) = \left(\cos(2t) + 2t^2 - 1\right)\underset{\sim}{i} + \left(2t^2 - \cos(2t) + 1\right)\underset{\sim}{j}$

**D.** $\underset{\sim}{r}(t) = \left(2t^2 - \cos(2t)\right)\underset{\sim}{i} + \left(2t^2 + \cos(2t)\right)\underset{\sim}{j}$

**E.** $\underset{\sim}{r}(t) = \left(\cos(2t) - 2t^2 + 1\right)\underset{\sim}{i} + \left(\cos(2t) - 2t^2 - 1\right)\underset{\sim}{j}$

**Question 13**

A particle of mass 5 kg, initially at rest is acted upon by two forces. One force has a magnitude of $5\sqrt{2}$ newtons acting in the north-west direction, the other force has magnitude of 10 newtons acting in the east direction. After two seconds, the magnitude of the momentum of the particle in $\text{kg ms}^{-1}$ is equal to

**A.** $50\sqrt{2}$

**B.** $25\sqrt{2}$

**C.** $10\sqrt{2}$

**D.** $2\left(2 - \sqrt{2}\right)$

**E.** $2\sqrt{2}$

**Question 14**

A soccer ball is kicked off the ground with an initial velocity of $16 \text{ ms}^{-1}$ at an angle of $30^0$. The maximum height reached in metres is equal to

**A.** $\frac{16}{g}$

**B.** $\frac{32}{g}$

**C.** $\frac{64}{g}$

**D.** $\frac{32\sqrt{3}}{g}$

**E.** $\frac{128\sqrt{3}}{g}$

**Question 15**

Using a suitable substitution, $\int_1^2 \frac{1}{x\sqrt{3x-2}}\,dx$ can be expressed, in terms of $u$ as

**A.** $\frac{1}{3}\int_1^4 \frac{1}{(u+2)\sqrt{u}}\,du$

**B.** $\int_1^2 \frac{1}{(u+2)\sqrt{u}}\,du$

**C.** $3\int_1^4 \frac{1}{(u+2)\sqrt{u}}\,du$

**D.** $2\int_1^2 \frac{1}{u^2+2}\,du$

**E.** $\frac{3}{2}\int_1^2 \frac{1}{u^2\left(u^2+2\right)}\,du$

**Question 16**

The equation of the normal to the curve $y=\cos^{-1}\left(\frac{x}{4}\right)$ at the point where $x=2$, is given by

**A.** $y=2\sqrt{3}\,x+\frac{\pi}{3}-4\sqrt{3}$

**B.** $y=2\sqrt{3}\,x+\frac{\pi}{6}-4\sqrt{3}$

**C.** $y=-\frac{\sqrt{3}\,x}{6}+\frac{\pi}{3}+\frac{\sqrt{3}}{3}$

**D.** $y=\frac{\sqrt{3}\,x}{6}+\frac{\pi}{3}-\frac{\sqrt{3}}{3}$

**E.** $y=-\frac{\sqrt{3}\,x}{6}+\frac{\pi}{6}+\frac{\sqrt{3}}{3}$

**Question 17**

A body of mass *m* kg moves in a straight line. When its displacement is *x* m from the origin, its velocity is *v* ms$^{-1}$ at a time *t* seconds. The force acting on the body is $mf(x)$ newtons. Given that $v=v_0$ when $x=x_0$ and $v=v_1$ when $x=x_1$, it follows that

**A.** $\frac{1}{2}mv_1^2-\frac{1}{2}mv_0^2=m\left[f(x_1)-f(x_0)\right]$

**B.** $\frac{1}{2}mv_1^2-\frac{1}{2}mv_0^2=m\int_{x_0}^{x_1}f(x)\,dx$

**C.** $v_1-v_0=\left[f(x_1)-f(x_0)\right]$

**D.** $v_1-v_0=\int_{x_0}^{x_1}f(x)\,dx$

**E.** $v_1=\sqrt{v_0^2+m\int_{x_0}^{x_1}f(x)\,dx}$

**Question 18**

An object of mass *m* kg is projected downwards from a point *P*, with an initial speed of of $U$ m/s. The object falls under the influence of gravity in a medium which offers resistance proportional to the velocity. Take the initial position as $y=0$ and downwards as the positive direction. If $k$ is a positive constant, which of the following most accurately reflects the situation ?

**A.** $\ddot{y}-k\dot{y}=mg \quad y(0)=0 \quad \dot{y}(0)=U$

**B.** $\ddot{y}-k\dot{y}=g \quad y(0)=0 \quad \dot{y}(0)=-U$

**C.** $\ddot{y}+k\dot{y}=mg \quad y(0)=0 \quad \dot{y}(0)=U$

**D.** $\ddot{y}+k\dot{y}=mg \quad y(0)=0 \quad \dot{y}(0)=-U$

**E.** $\ddot{y}+k\dot{y}=g \quad y(0)=0 \quad \dot{y}(0)=U$

**Question 19**

When Euler's method, with a step size of $\frac{1}{3}$, is used to solve the differential equation $\frac{dy}{dx}=e^{x+y}$ with $x_0=0$ and $y_0=0$, the value of $y_2$ would be given as

**A.** 0.798

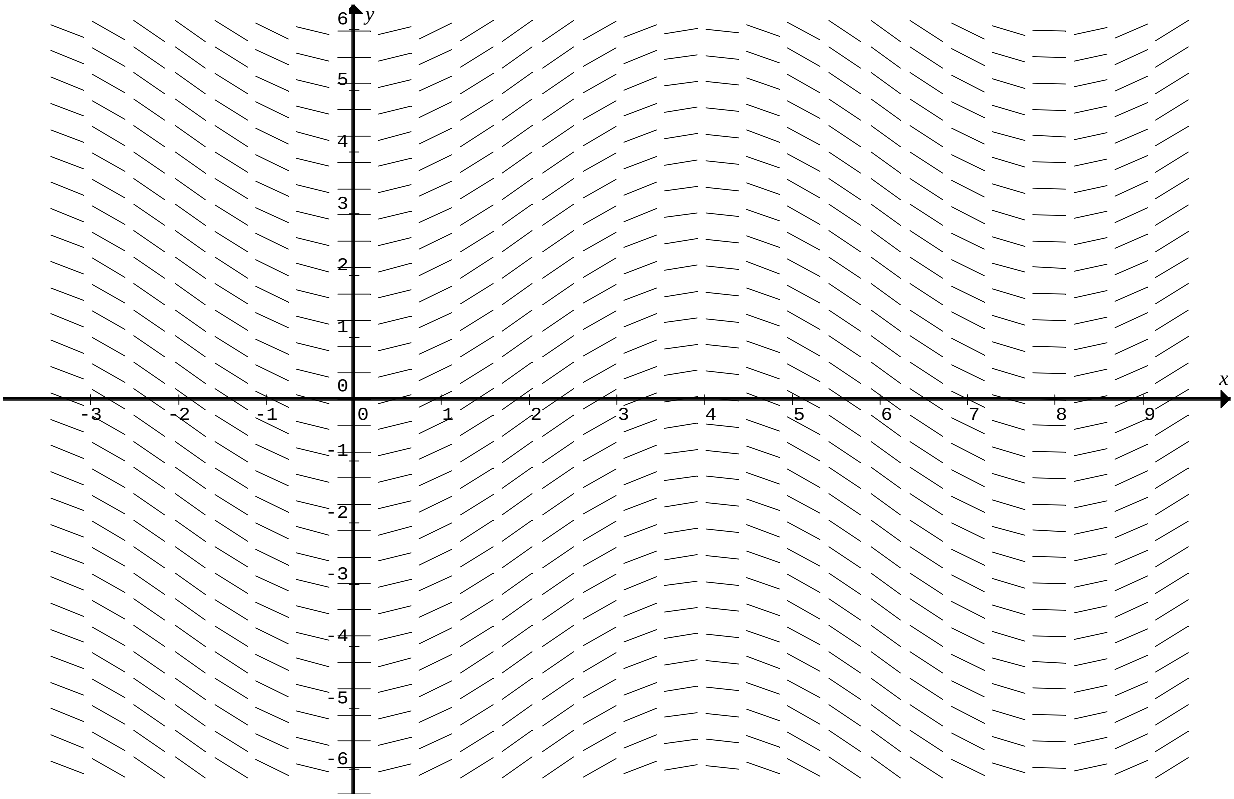
**B.** $-\log_e(e-2)$

**C.** $\frac{1}{3}\left(1+e^{\frac{1}{3}}\right)$

**D.** $\frac{1}{3}\left(1+e^{\frac{1}{3}}+e^{\frac{2}{3}}\right)$

**E.** $\frac{1}{3}\left(1+e^{\frac{2}{3}}\right)$

**Question 20**




The direction ( slope ) field for a certain first order differential equation is shown above.
The differential equation could be

**A.** $\frac{dy}{dx} = \sin\left(\frac{\pi x}{4}\right)$

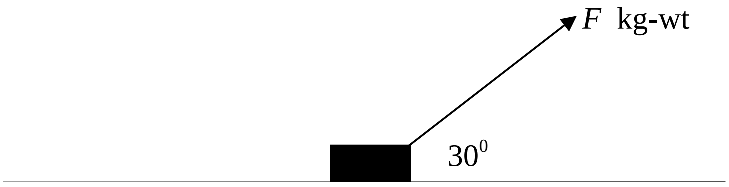
**B.** $\frac{dy}{dx} = \cos\left(\frac{\pi x}{4}\right)$

**C.** $\frac{dy}{dx} = -\cos\left(\frac{\pi x}{4}\right)$

**D.** $\frac{dy}{dx} = \sin(4\pi x)$

**E.** $\frac{dy}{dx} = -\sin(4\pi x)$

**Question 21**




A box of mass 3 kg is on a horizontal plane. A force of magnitude $F$ kg-wt acting at an angle of $30^0$ to the horizontal is applied to the box. The coefficient of friction between the box and the plane is $\frac{\sqrt{3}}{2}$. Which of the following is true?

**A.** If $F < 2g$ the box is not on the point of moving.

**B.** If $F < 2g$ the box moves with constant velocity.

**C.** If $F = 2$ the box moves with constant acceleration.

**D.** If $F > 2$ the box moves with constant velocity.

**E.** If $F > 2$ the box moves with constant acceleration.

**Question 22**

The velocity $v$ ms$^{-1}$ of a body moving in a straight line after a time $t$ seconds, is given by $v(t) = \begin{cases} 10 - 2t & \text{for } 0 \le t \le 8 \\ 2t - 22 & \text{for } 8 \le t \le 16 \end{cases}$

After 16 seconds, the distance in metres of the body from its starting point is

**A.** 0

**B.** 18

**C.** 32

**D.** 50

**E.** 68

**END OF SECTION 1**

# SECTION 2

**Instructions for Section 2**

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are not drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s$^2$, where $g = 9.8$

**Question 1**

A plank of wood can be raised and lowered as shown in the diagram.

**a.** A stone is placed on the plank, when it is inclined at an angle of $\theta$ where $\theta < 30^0$ to the horizontal. The stone is just on the point of moving down the plank. The co-efficient of friction between the plank and the stone is $\mu$.
On the diagram above mark in all the forces acting on the stone and show that $\mu = \tan(\theta)$.

2 marks

**b.** The plank is now raised so that it makes an angle of $2\theta$ to the horizontal. The stone moves from rest down the plank. Show that after moving a distance of *D*, down the slope, its speed is given by $V_1 = \sqrt{2gD\tan(\theta)}$ and the time taken to move this distance is given by $T_1 = \sqrt{\dfrac{2D}{g\tan(\theta)}}$.

4 marks

**c.** The plank is raised again so that it now makes an angle of $3\theta$ to the horizontal. The stone is placed on the plank again and it moves down the plank. Show that its acceleration while moving down the plank is given by $2g\sin(\theta)$.

Let $V_2$ and $T_2$ be its speed and time taken respectively to move the same distance $D$ from rest down the plank. Express $V_2$ and $T_2$ in terms of $D$, $g$ and $\theta$.

4 marks

**d.** If the ratio $T_2 : T_1$ is $\dfrac{3}{4}$ find the value of $\theta$ giving your answer in degrees and minutes.

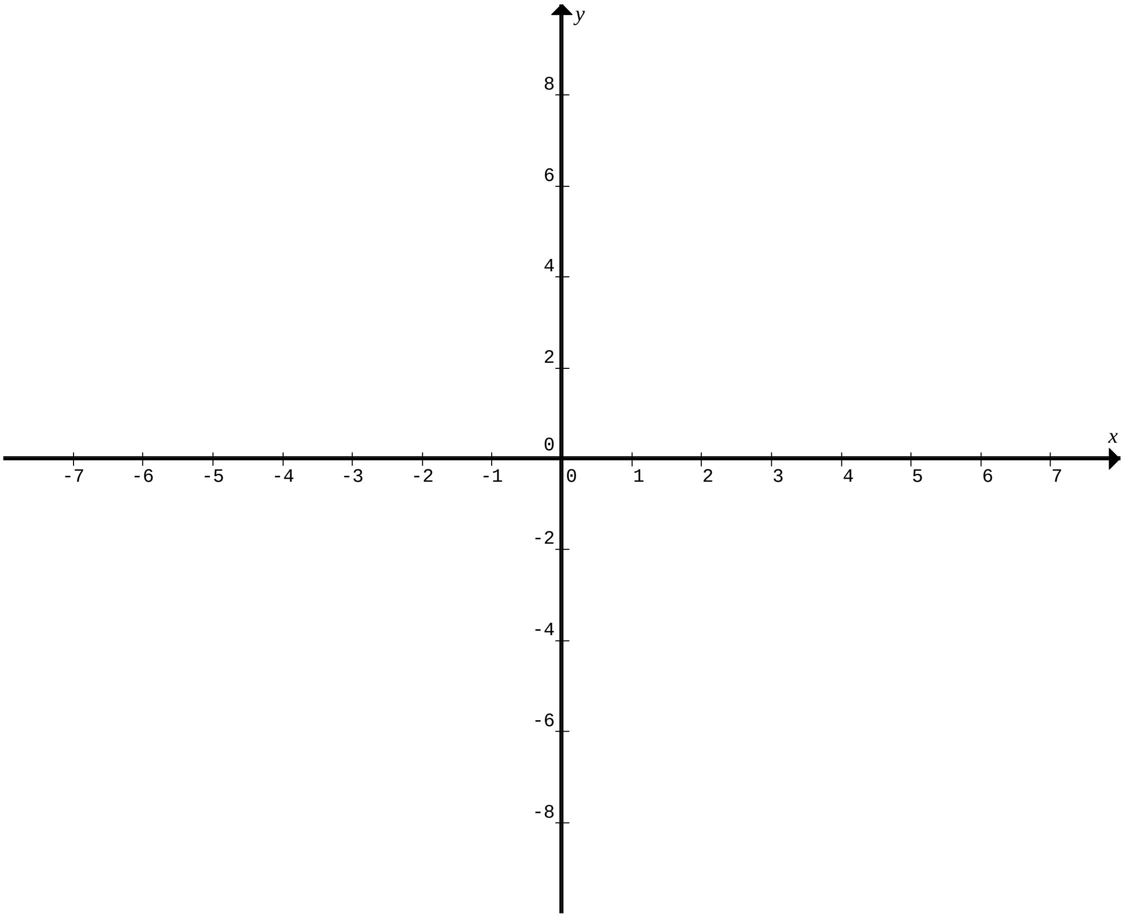
2 marks

**e.** Find the ratio $V_2 : V_1$

1 marks
Total 13 marks

**Question 2**

**a.** Sketch the graph of $\frac{x^2}{9}-\frac{y^2}{36}=1$ on the axes below. Give the coordinates of any axial intercepts and state the equations of all straight line asymptotes.




2 marks

**b.** A vase is formed, when part of the curve $\frac{x^2}{9}-\frac{y^2}{36}=1$ for $3 \le x \le 5$ , $y \ge 0$, is rotated about the $y$-axis to form a volume of revolution. The $x$ and $y$ coordinates are measured in centimetres.

**i.** If $H$ is the height of the vase, find the value of $H$, and draw the shape of the vase on the diagram above.

______________________________________________________________________

______________________________________________________________________

______________________________________________________________________

1 mark

**ii.** Write down a definite integral, which when evaluated gives the total volume of the vase.

2 marks

**iii.** Using calculus, show that when the vase is filled to a height of $h$ cm, where $0 \le h \le H$, the volume $V$ of the vase is given by $V = \dfrac{\pi h}{12}\left(h^2 + 108\right)$ cm$^3$.

2 marks

**iv.** If liquid is poured into an empty vase, at a rate of $18\sqrt{h}$ cm$^3$/s where $h$ is the height of the liquid in the vase, find the exact time in seconds required to fill the vase.

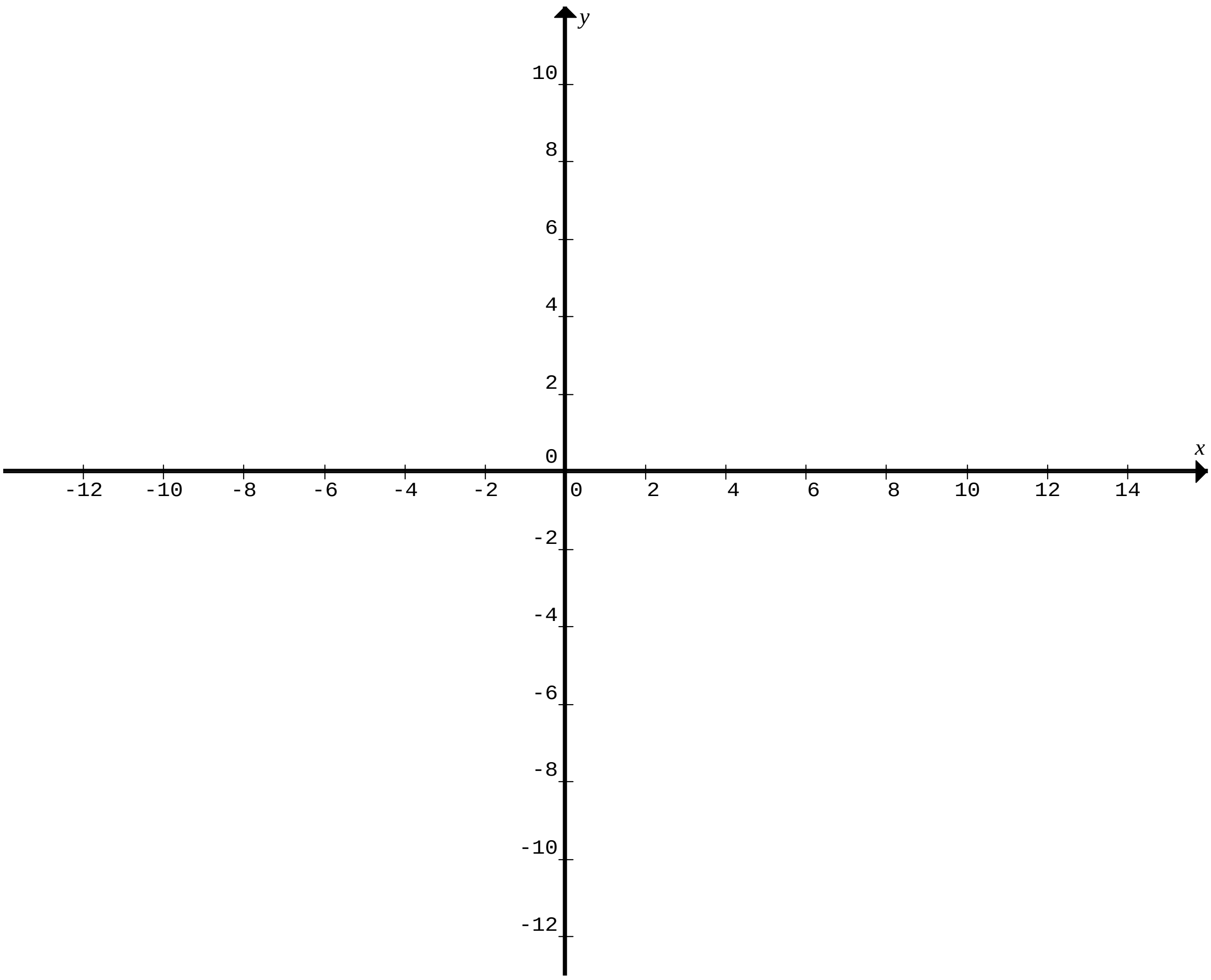
4 marks
Total 11 marks

**Question 3**

The vector equation for a certain type of curve is given by

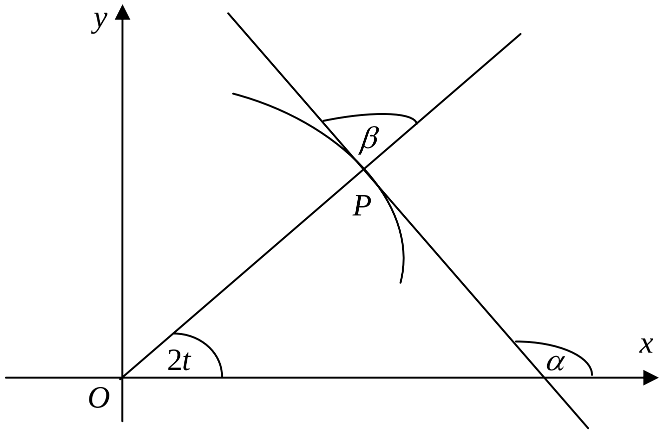
$$\underset{\sim}{r}(t) = 2e^{0.3t}\cos(2t)\underset{\sim}{i} + 2e^{0.3t}\sin(2t)\underset{\sim}{j} \quad \text{for} \quad 0 \le t \le 2\pi$$

**a.** Sketch the Cartesian equation of the curve on the axes below, clearly showing axial cuts.




2 marks

In the diagram, part of the curve is shown. $P$ is a point on the curve. The line from the origin $O$ through $P$ makes an angle of $2t$ with the positive $x$-axis. The tangent to the curve at $P$ makes an angle of $\alpha$ with the positive $x$-axis. $\beta$ is the angle between the tangent line and the line from the origin $O$ through $P$.




**b.** Find an expression for $\frac{dy}{dx}$ in terms of $t$, and hence show that

$$\tan(\alpha) = \frac{0.3\tan(2t)+2}{0.3-2\tan(2t)}.$$

3 marks

**c.** Show that $\tan(\beta)$ is constant and find its exact value.

3 marks

**d.** A particle moves along the curve, show that its speed can be written in the form $ae^{kt}$ and find the values of $a$ and $k$.

3 marks

**e.** Hence setup a definite integral which gives the total length of the curve and find this length, giving your answer correct to three decimal places.

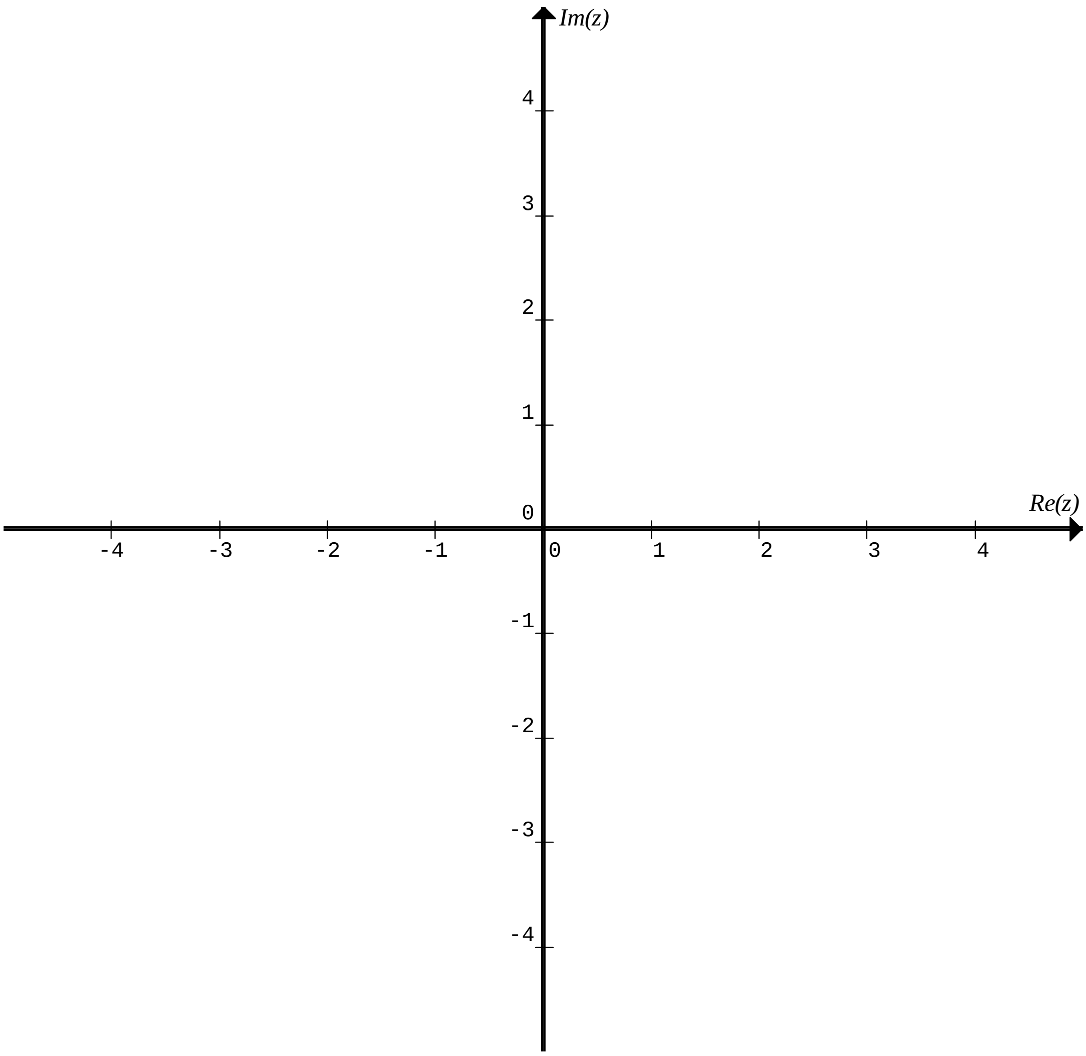
2 marks
Total 13 marks

**Question 4**

Let $S$ and $R$ be defined by

$S = \{ z : |z + 2 - 2i| = 2, z \in C \}$ and $R = \{ z : \text{Arg}(z + 2 - 2i) = \frac{\pi}{6}, z \in C \}$

**a.** Describe $S$ and $R$ and sketch both on the Argand diagram below.




3 marks

**b.** If $z \in S$, find the maximum value of $|z|$.

2 marks

**c.** If $p \in S \cap R$ and $p = a + bi$ find the values of $a$ and $b$.

3 marks

**d.** A straight line $T$ passes through $p$ and is a tangent to $S$. $T$ can be defined by $T = \{ z : |z + 2 - 2i| = |z - d|, z \in C \}$ . Find the complex number $d$.

2 marks

**e.** The locus of $T$ can also be written in the form $\operatorname{Im}(z) = m\operatorname{Re}(z) + k$ , find the values of $m$ and $k$.

2 marks

Total 12 marks

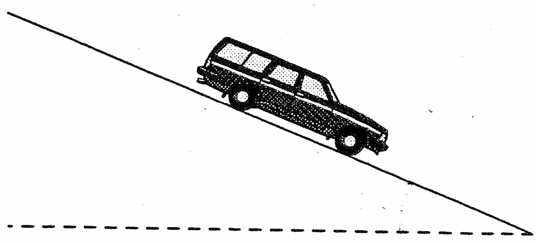
**Question 5**

**a.** Using calculus, show that $\frac{d}{dx}\left[14\log_e\left(14-\sqrt{x}\right)+\sqrt{x}\right]=\frac{1}{2\left(\sqrt{x}-14\right)}$

for $0<x<b$, and specify the value of $b$.

2 marks

**b.** A station-wagon of mass 1400 kg is on a hill.



The hill is inclined at an angle of $\sin^{-1}\left(\dfrac{5}{7}\right)$ to the horizontal, and the station-wagon moves down the hill from rest. The total resistance to the motion of the station-wagon is $700\sqrt{v}$ newtons where $v$ m/s is its speed at a time $t$ s.

**i.** On the diagram above, mark in all the forces acting on the station-wagon, and show that $a = 7 - \dfrac{\sqrt{v}}{2}$ where $a$ m/s$^2$ is the acceleration of the station-wagon.

2 marks

**ii.** Using calculus, hence find the exact time in seconds for the station-wagon to reach a speed of 16 m/s.

2 marks

**iii.** Find the speed of the station-wagon after it has travelled a distance of 20 metres from rest down the hill. Give your answer in m/s correct to two decimal places.

3 marks
Total 9 marks

**END OF EXAMINATION**

**EXTRA WORKING SPACE**

**END OF QUESTION AND ANSWER BOOKLET**

# SPECIALIST MATHEMATICS

## Written examination 2

## FORMULA SHEET

### Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

# Specialist Mathematics Formulas

## Mensuration

| | |
|---|---|
| area of a trapezium: | $\frac{1}{2}(a+b)h$ |
| curved surface area of a cylinder: | $2\pi rh$ |
| volume of a cylinder: | $\pi r^2 h$ |
| volume of a cone: | $\frac{1}{3}\pi r^2 h$ |
| volume of a pyramid: | $\frac{1}{3}Ah$ |
| volume of a sphere: | $\frac{4}{3}\pi r^3$ |
| area of triangle: | $\frac{1}{2}bc\sin(A)$ |
| sine rule: | $\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$ |
| cosine rule: | $c^2=a^2+b^2-2ab\cos(C)$ |

## Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$  hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

## Circular ( trigonometric ) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$  $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$  $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$  $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$  $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$  $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

| function | $\sin^{-1}$ | $\cos^{-1}$ | $\tan^{-1}$ |
|---|---|---|---|
| domain | $[-1,1]$ | $[-1,1]$ | $R$ |
| range | $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ | $[0,\pi]$ | $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ |

## Algebra ( Complex Numbers )

$z = x + yi = r\left(\cos(\theta) + i\sin(\theta)\right) = r\,\text{cis}(\theta)$

$|z| = \sqrt{x^2 + y^2} = r$ $\qquad -\pi < \text{Arg}(z) \le \pi$

$z_1 z_2 = r_1 r_2 \,\text{cis}(\theta_1 + \theta_2)$ $\qquad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\,\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n \text{cis}(n\theta)$ (de Moivre's theorem )

## Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$ $\qquad \underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

## Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

sliding friction: $F \le \mu N$

constant ( uniform ) acceleration:

$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \frac{1}{2}(u+v)t$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right)$

## Calculus

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$$

$$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$$

$$\frac{d}{dx}\left(\sin(ax)\right)=a\cos(ax)$$

$$\frac{d}{dx}\left(\cos(ax)\right)=-a\sin(ax)$$

$$\frac{d}{dx}\left(\tan(ax)\right)=a\sec^2(ax)$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right)=\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right)=\frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right)=\frac{1}{1+x^2}$$

$$\int x^n\,dx=\frac{1}{n+1}x^{n+1}+c\ ,\ n\neq -1$$

$$\int e^{ax}\,dx=\frac{1}{a}e^{ax}+c$$

$$\int \frac{1}{x}\,dx=\log_e(x)+c,\ \text{ for }\ x>0$$

$$\int \sin(ax)\,dx=-\frac{1}{a}\cos(ax)+c$$

$$\int \cos(ax)\,dx=\frac{1}{a}\sin(ax)+c$$

$$\int \sec^2(ax)\,dx=\frac{1}{a}\tan(ax)+c$$

$$\int \frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\left(\frac{x}{a}\right)+c,\ a>0$$

$$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx=\cos^{-1}\left(\frac{x}{a}\right)+c,\ a>0$$

$$\int \frac{a}{a^2+x^2}\,dx=\tan^{-1}\left(\frac{x}{a}\right)+c$$

product rule: $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$

Euler's method If $\frac{dy}{dx}=f(x),\ x_0=a$ and $y_0=b,$ then $x_{n+1}=x_n+h$ and $y_{n+1}=y_n+hf(x_n)$

**END OF FORMULA SHEET**

# ANSWER SHEET

**STUDENT NUMBER**

| | | | | | | | | | Letter |
|---|---|---|---|---|---|---|---|---|---|
| Figures | | | | | | | | | |
| Words | | | | | | | | | |

**SIGNATURE** ______________________________

## SECTION 1

| | | | | | |
|---|---|---|---|---|---|
| **1** | **A** | **B** | **C** | **D** | **E** |
| **2** | **A** | **B** | **C** | **D** | **E** |
| **3** | **A** | **B** | **C** | **D** | **E** |
| **4** | **A** | **B** | **C** | **D** | **E** |
| **5** | **A** | **B** | **C** | **D** | **E** |
| **6** | **A** | **B** | **C** | **D** | **E** |
| **7** | **A** | **B** | **C** | **D** | **E** |
| **8** | **A** | **B** | **C** | **D** | **E** |
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| **22** | **A** | **B** | **C** | **D** | **E** |