

Year 2010
VCE
Specialist Mathematics
Solutions
Trial Examination 2



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SECTION 1

ANSWERS

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E

SECTION 1

Question 1**Answer D**

The graph of $y = \frac{1}{3a^2 + 2ax - x^2}$

$$y = \frac{-1}{x^2 - 2ax - 3a^2} = \frac{-1}{(x-3a)(x+a)}$$

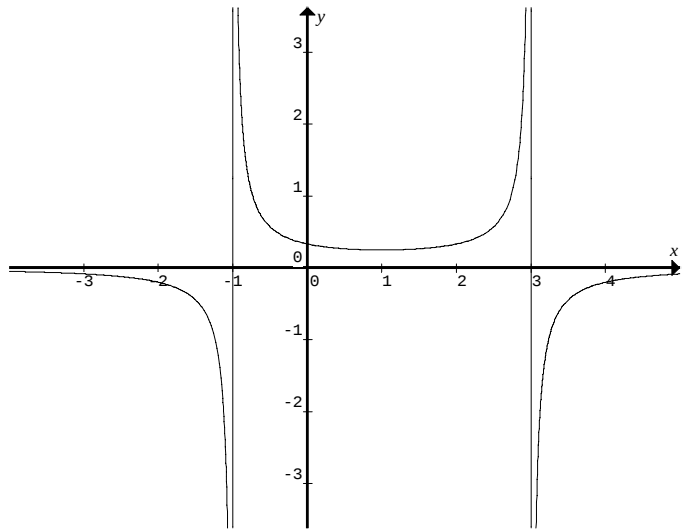
so it has vertical asymptotes at

$$x = 3a \quad \text{and} \quad x = -a$$

$$y = \frac{-1}{x^2 - 2ax - 3a^2} = \frac{-1}{x^2 - 2ax + a^2 - 4a^2}$$

$$y = \frac{-1}{(x-a)^2 - 4a^2} \quad \text{when } x = a \quad y = \frac{1}{4a^2}$$

$\left(a, \frac{1}{4a^2}\right)$ is a minimum, graph with $a = 1$

**Question 2****Answer D**

The hyperbola has its centre at $(-3, 2)$ and the distance from the centre to the point

$(-3, 0)$ is 2, its equation is of the form $\frac{(y-2)^2}{4} - \frac{(x+3)^2}{a^2} = 1$. The asymptotes are

$$\frac{y-2}{2} = \pm \frac{x+3}{a}, \quad \text{one asymptote passes through the origin, } (0, 0)$$

$$x = 0 \quad y = 0 \Rightarrow -1 = \frac{-3}{a} \Rightarrow a = 3, \text{ so the equation is } \frac{(y-2)^2}{4} - \frac{(x+3)^2}{9} = 1$$

Question 3**Answer A**

$x^2 - 2ax + 2y^2 + 8ay = 9 - 10a^2$ completing the square

$$x^2 - 2ax + a^2 + 2(y^2 + 4ay + 4a^2) = 9 - 10a^2 + a^2 + 8a^2 \Rightarrow (x-a)^2 + 2(y+2a)^2 = 9 - a^2$$

this is an ellipse with centre $(a, -2a)$ provided that $9 - a^2 > 0 \Rightarrow a^2 < 9$ or $|a| < 3$

Question 4**Answer A**

The domain and range of $y = \cos^{-1}(x)$ are $[-1, 1]$ and $[0, \pi]$ respectively.

$$\text{The domain of } f(x) = \frac{b}{\pi} \cos^{-1}\left(\frac{x}{b} - 1\right) - b \text{ is } \left|\frac{x}{b} - 1\right| \leq 1 \Rightarrow -1 \leq \frac{x}{b} - 1 \leq 1$$

$$0 \leq \frac{x}{b} \leq 2 \Rightarrow x \in [0, 2b] \text{ and the range is } \frac{b}{\pi} \times [0, \pi] - b = [-b, 0],$$

so the domain and range are respectively $[0, 2b]$ and $[-b, 0]$

Question 5**Answer D**

$$x + yi = r \operatorname{cis}(\theta) = r(\cos(\theta) + i \sin(\theta))$$

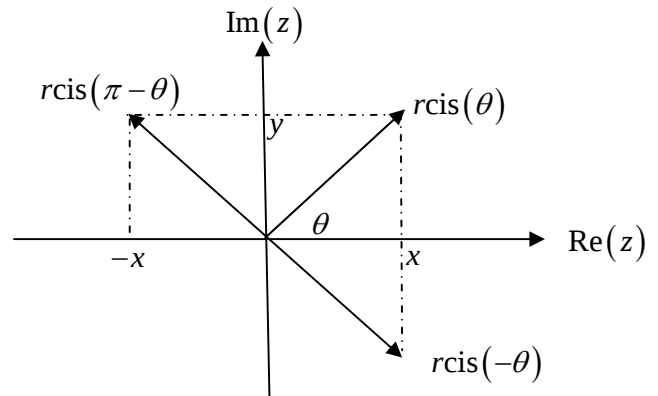
$$x + yi = r \cos(\theta) + ir \sin(\theta)$$

$$x = r \cos(\theta) \quad \text{and} \quad y = r \sin(\theta), \text{ now}$$

$$-x = -r \cos(\theta) = r \cos(\pi - \theta) \quad \text{and}$$

$$y = r \sin(\theta) = r \sin(\pi - \theta)$$

$$\text{so } -x + yi = r \operatorname{cis}(\pi - \theta)$$

**Question 6****Answer B**

$$(\bar{z} + ai)(z - ai) = a^2, \text{ let } z = x + yi \text{ and } \bar{z} = x - yi$$

$$(x + (a - y)i)(x + (y - a)i) = a^2$$

$$x^2 + x(y - a)i + x(a - y)i + i^2(a - y)(y - a) = a^2$$

$$x^2 + (y - a)^2 = a^2$$

this is a circle of radius a , with centre at $(0, a)$

Question 7**Answer E**

$$\underline{a} = (m - 2)\underline{i} - (m + 1)\underline{j} + (m + 1)\underline{k}$$

$$|\underline{a}| = \sqrt{(m - 2)^2 + (m + 1)^2 + (m + 1)^2}$$

$$|\underline{a}| = \sqrt{m^2 - 4m + 4 + m^2 + 2m + 1 + m^2 + 2m + 1}$$

$$|\underline{a}| = \sqrt{3m^2 + 6} = \sqrt{3(m^2 + 2)}$$

Question 8**Answer C**

$$\underline{a} = \sqrt{m}\underline{i} + n\underline{j} - n\underline{k} \text{ and } \underline{b} = -2\sqrt{m}\underline{i} + 4\underline{j} - \sqrt{m}\underline{k}, \text{ if } \underline{a} \text{ and } \underline{b} \text{ are parallel, then}$$

$$\underline{b} = \lambda \underline{a} \text{ and } \lambda = -2, \underline{b} = -2\underline{a}, \text{ from the } \underline{j} \text{ and } \underline{k} \text{ components}$$

$$4 = -2n \text{ and } -\sqrt{m} = 2n \Rightarrow n = -2 \text{ and } m = 16$$

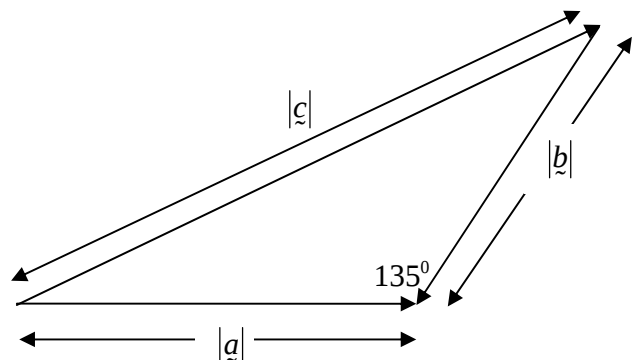
Question 9**Answer A**

Using the Cosine Rule

$$|\underline{c}|^2 = |\underline{a}|^2 + |\underline{b}|^2 - 2|\underline{a}||\underline{b}|\cos(135^\circ)$$

$$|\underline{c}|^2 = |\underline{a}|^2 + |\underline{b}|^2 - 2|\underline{a}||\underline{b}|\frac{-\sqrt{2}}{2}$$

$$|\underline{c}|^2 = |\underline{a}|^2 + |\underline{b}|^2 + \sqrt{2}|\underline{a}||\underline{b}|$$



Question 10**Answer D**

Since b, c, α and β are all real, the roots of

$z^2 + bz + c$ occur in conjugate pairs and are $u = -\alpha + \beta i$ and $\bar{u} = -\alpha - \beta i$

now $u + \bar{u} = -2\alpha$ and $u\bar{u} = \alpha^2 - \beta^2 i^2 = \alpha^2 + \beta^2$, so that

$$z^2 + bz + c = z^2 + 2\alpha z + \alpha^2 + \beta^2 \Rightarrow 2\alpha = b \text{ and } \alpha^2 + \beta^2 = c$$

Let $v = 2\alpha - 2\beta i$ and $\bar{v} = 2\alpha + 2\beta i$, now

$v + \bar{v} = 4\alpha$ and $v\bar{v} = 4\alpha^2 - 4\beta^2 i^2 = 4(\alpha^2 + \beta^2)$, so the quadratic is

$$z^2 - 4\alpha z + 4(\alpha^2 + \beta^2) = z^2 - 2bz + 4c$$

Question 11**Answer B**

$$\underline{s} = 2\underline{i} - 3\underline{j} + \underline{k} \quad |\underline{s}| = \sqrt{4+9+1} = \sqrt{14} \Rightarrow \hat{\underline{s}} = \frac{1}{\sqrt{14}}(2\underline{i} - 3\underline{j} + \underline{k})$$

The vector resolute of $\underline{r}$ in the direction of $\underline{s}$ is equal to

$$(\underline{r} \cdot \hat{\underline{s}})\hat{\underline{s}} = (\underline{r} \cdot \hat{\underline{s}})\frac{1}{\sqrt{14}}(2\underline{i} - 3\underline{j} + \underline{k}) = 3(-2\underline{i} + 3\underline{j} - \underline{k}) = -3(2\underline{i} - 3\underline{j} + \underline{k})$$

$$\Rightarrow \underline{r} \cdot \hat{\underline{s}} = -3\sqrt{14} = \frac{\underline{r} \cdot \underline{s}}{|\underline{s}|} = \frac{\underline{r} \cdot \underline{s}}{\sqrt{14}} \Rightarrow \underline{r} \cdot \underline{s} = -3 \times 14$$

The scalar resolute of $\underline{s}$ in the direction of $\underline{r}$ is equal to $\underline{s} \cdot \hat{\underline{r}} = \frac{\underline{s} \cdot \underline{r}}{|\underline{r}|} = \frac{-3 \times 14}{6} = -7$

Question 12**Answer C**

$$\ddot{\underline{r}}(t) = 8\sin^2(t)\underline{i} + 8\cos^2(t)\underline{j}$$

$$\dot{\underline{r}}(t) = \int 8\sin^2(t) dt \underline{i} + \int 8\cos^2(t) dt \underline{j}$$

$$\dot{\underline{r}}(t) = \int 4(1 - \cos(2t)) dt \underline{i} + \int 4(1 + \cos(2t)) dt \underline{j}$$

$$\dot{\underline{r}}(t) = 2(2t - \sin(2t))\underline{i} + 2(2t + \sin(2t))\underline{j} + \underline{C}_1 \text{ now } \dot{\underline{r}}(0) = \underline{0} \Rightarrow \underline{C}_1 = \underline{0}$$

$$\underline{r}(t) = \int (4t - 2\sin(2t)) dt \underline{i} + \int (4t + 2\sin(2t)) dt \underline{j}$$

$$\underline{r}(t) = (2t^2 + \cos(2t))\underline{i} + (2t^2 - \cos(2t))\underline{j} + \underline{C}_2$$

$$\text{now } \underline{r}(0) = \underline{0} \Rightarrow \underline{i} - \underline{j} + \underline{C}_2 = \underline{0} \Rightarrow \underline{C}_2 = -\underline{i} + \underline{j}$$

$$\underline{r}(t) = (\cos(2t) + 2t^2 - 1)\underline{i} + (2t^2 - \cos(2t) + 1)\underline{j}$$

Question 13**Answer C**

$$m = 5 \text{ kg} \quad \underline{F}_1 = 10\hat{i} \quad |\underline{F}_1| = 10$$

$$\text{and } \underline{F}_2 = -5\hat{i} + 5\hat{j} \quad |\underline{F}_2| = 5\sqrt{2}$$

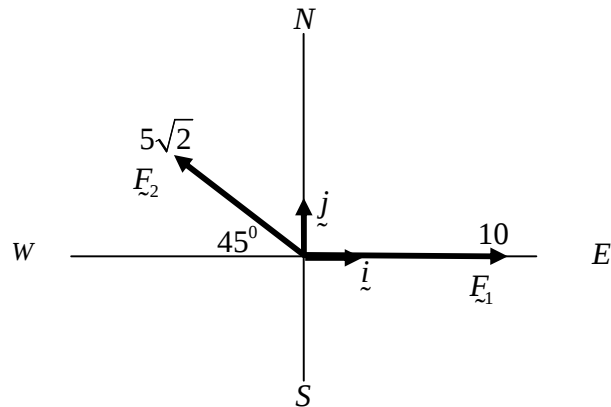
$$\underline{F}_1 + \underline{F}_2 = 5\hat{i} + 5\hat{j}$$

$$|\underline{F}_1 + \underline{F}_2| = 5\sqrt{2} = ma = 5a$$

$$a = \sqrt{2} \quad u = 0 \quad t = 2$$

$$\text{using } v = u + at \Rightarrow v = 2\sqrt{2}$$

$$\text{momentum } mv = 10\sqrt{2} \text{ kg ms}^{-1}$$

**Question 14****Answer B**

$$V = 16 \text{ ms}^{-1} \quad \alpha = 30^\circ$$

$$\text{The maximum height of a projectile is given by } H = \frac{V^2 \sin^2(\alpha)}{2g}$$

$$H = \frac{16^2 \sin^2(30^\circ)}{2g} = \frac{16^2}{2g} \times \frac{1}{4} = \frac{32}{g} \text{ metres}$$

Question 15**Answer D**

$$\text{let } u = \sqrt{3x-2}$$

$$\text{terminals, when } x=2 \quad u = \sqrt{4} = 2 \quad \text{and when } x=1 \quad u = \sqrt{1} = 1$$

$$u^2 = 3x-2 \Rightarrow 2u \frac{du}{dx} = 3 \quad \frac{dx}{du} = \frac{2u}{3} \quad \text{and } x = \frac{1}{3}(u^2 + 2)$$

$$\int_1^2 \frac{1}{x\sqrt{3x-2}} \frac{dx}{du} du = \int_1^2 \frac{3}{u^2+2} \times \frac{1}{u} \times \frac{2u}{3} du = 2 \int_1^2 \frac{1}{u^2+2} du$$

Question 16**Answer A**

$$y = \cos^{-1}\left(\frac{x}{4}\right) \quad \text{when } x=2 \quad y = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \quad P\left(2, \frac{\pi}{3}\right)$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{16-x^2}} \quad \text{when } x=2 \quad m_T = \frac{-1}{\sqrt{12}} \quad m_N = \sqrt{12} = 2\sqrt{3}$$

$$\text{equation of the normal is } y - \frac{\pi}{3} = 2\sqrt{3}(x-2) \quad \text{or } y = 2\sqrt{3}x + \frac{\pi}{3} - 4\sqrt{3}$$

Question 17**Answer B**

$$m\ddot{x} = mf(x)$$

$$m \frac{d}{dx} \left(\frac{1}{2} v^2 \right) = mf(x)$$

$$m \left[\frac{1}{2} v^2 \right]_{v_0}^{v_1} = m \int_{x_0}^{x_1} f(x) dx$$

$$\frac{1}{2} m v_1^2 - \frac{1}{2} m v_0^2 = m \int_{x_0}^{x_1} f(x) dx$$

Question 18**Answer E**

motion is downwards, positive direction

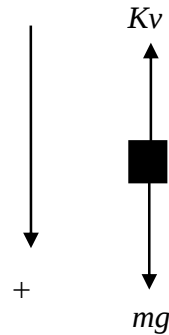
$$m\ddot{y} = mg - R \quad \text{where} \quad R = Kv = K\dot{y}$$

$$m\ddot{y} = mg - Kv$$

$$m\ddot{y} = mg - K\dot{y}$$

$$m\ddot{y} + K\dot{y} = mg \quad \text{let} \quad k = \frac{K}{m}$$

$$\ddot{y} + ky = g \quad y(0) = 0 \quad \dot{y}(0) = U$$

**Question 19****Answer E**

Using Euler's method, with $x_0 = 0$, $y_0 = 0$ $h = \frac{1}{3}$ $\frac{dy}{dx} = f(x, y) = e^{x+y}$

so that $x_1 = \frac{1}{3}$ and $x_2 = \frac{2}{3}$

$$y_1 = y_0 + hf(x_0, y_0) = 0 + \frac{1}{3} e^{0+0} = \frac{1}{3}$$

$$y_2 = y_1 + hf(x_1, y_1) = \frac{1}{3} + \frac{1}{3} e^{\frac{1}{3} + \frac{1}{3}} = \frac{1}{3} \left(1 + e^{\frac{2}{3}} \right)$$

Question 20**Answer A**

The solution curves are of the form $y = -a \cos\left(\frac{\pi x}{4}\right) + c$ with $a > 0$, since the period

of the solution curves is $T = \frac{2\pi}{\frac{\pi}{4}} = 8$, or $\frac{dy}{dx} = \sin\left(\frac{\pi x}{4}\right)$ with amplitude one.

Question 21**Answer E**

All forces must be in newtons, $m = 3\text{kg}$ $\mu = \frac{\sqrt{3}}{2}$

resolving perpendicular to the plane

$$(1) \quad N + Fg \sin(30^\circ) - mg = 0$$

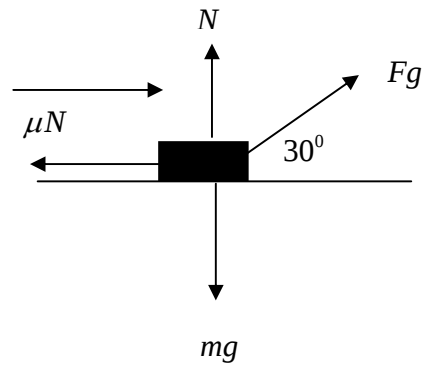
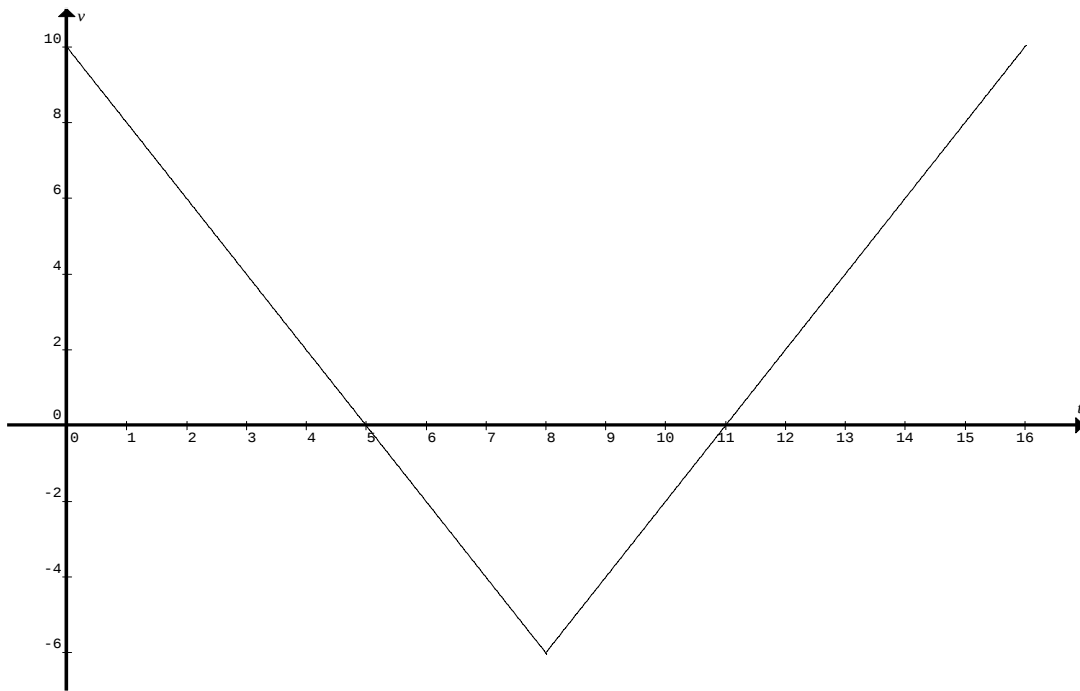
$$\Rightarrow N = 3g - \frac{Fg}{2}$$

resolving parallel to the plane

$$(2) \quad Fg \cos(30^\circ) - \mu N = ma$$

$$\frac{Fg\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \left(3g - \frac{Fg}{2} \right) = 3a$$

$$\Rightarrow \frac{3g\sqrt{3}}{4} (F - 2) = 3a \quad \text{so if } F > 2 \Rightarrow a > 0$$

**Question 22****Answer C**

The distance from the start, is the signed area of the triangles, or displacement

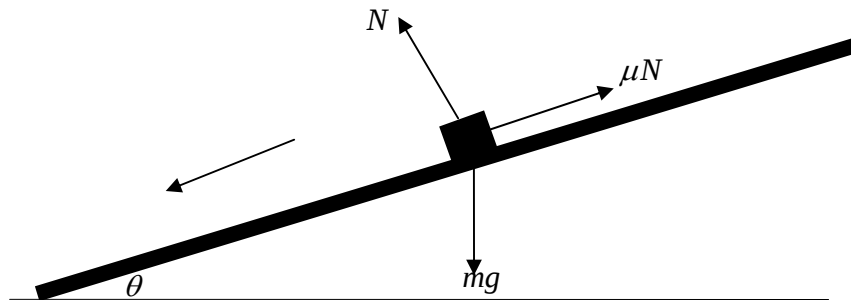
$$\begin{aligned} & \frac{1}{2} \times 10 \times 5 - \frac{1}{2} \times 6 \times 6 + \frac{1}{2} \times 5 \times 10 \\ &= 25 - 18 + 25 \\ &= 32 \end{aligned}$$

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2

Question 1

let the mass of the stone be m kg.



resolving perpendicular to the plank

$$N - mg \cos(\theta) = 0 \Rightarrow N = mg \cos(\theta) \quad \text{M1}$$

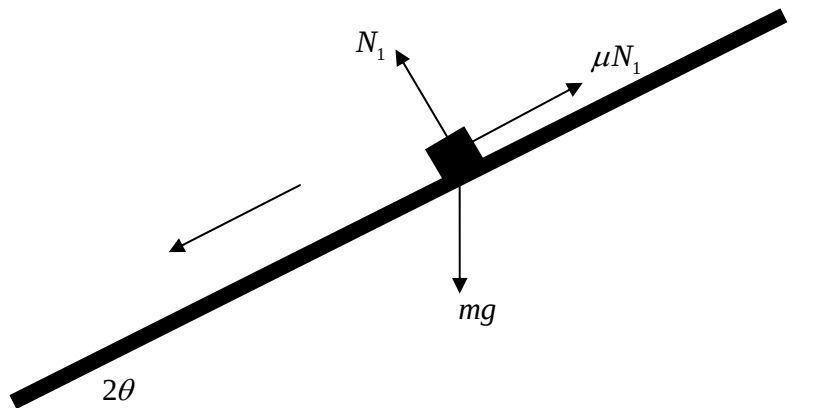
resolving parallel to the plank

$$mg \sin(\theta) - \mu N = 0 \Rightarrow mg \sin(\theta) = \mu N \quad \text{A1}$$

$$mg \sin(\theta) = \mu mg \cos(\theta)$$

$$\mu = \tan(\theta)$$

b.



resolving perpendicular to the plank

$$N_1 - mg \cos(2\theta) = 0 \Rightarrow N_1 = mg \cos(2\theta)$$

resolving parallel to the plank

$$ma = mg \sin(2\theta) - \mu N_1 \quad \text{M1}$$

$$ma = mg \sin(2\theta) - \mu mg \cos(2\theta) \quad \text{but } \mu = \tan(\theta)$$

$$ma = mg (\sin(2\theta) - \tan(\theta) \cos(2\theta))$$

$$a = g \left(\sin(2\theta) - \frac{\sin(\theta) \cos(2\theta)}{\cos(\theta)} \right) \quad \text{M1}$$

$$a = g \left(\frac{\sin(2\theta) \cos(\theta) - \sin(\theta) \cos(2\theta)}{\cos(\theta)} \right)$$

$$a = \frac{g \sin(2\theta - \theta)}{\cos(\theta)} = g \tan(\theta) \quad \text{A1}$$

Now using constant acceleration formulae with $u = 0$ $v = V_1$ $s = D$

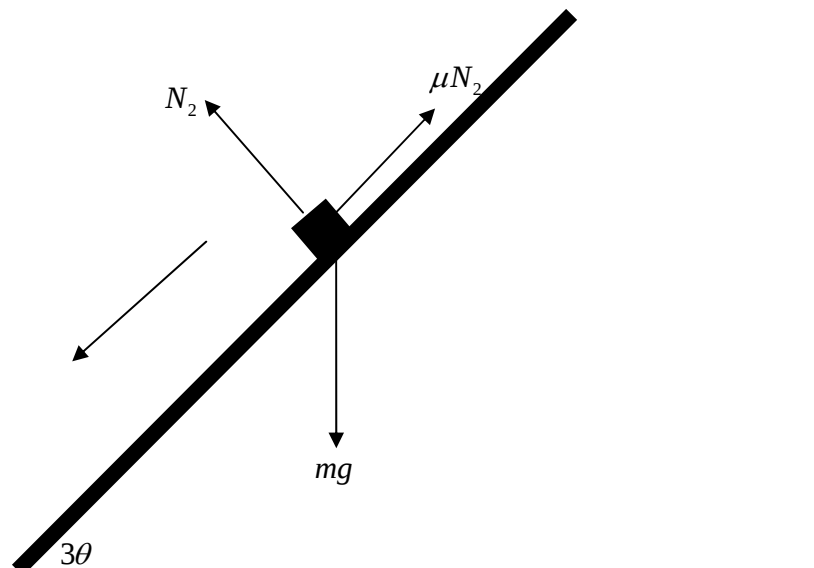
$$v^2 = u^2 + 2as \quad \text{A1}$$

$$V_1 = \sqrt{2gD \tan(\theta)}$$

$$s = ut + \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2s}{a}}$$

$$T_1 = \sqrt{\frac{2D}{g \tan(\theta)}}$$

c.



resolving perpendicular to the plank

$$N_2 - mg \cos(3\theta) = 0 \Rightarrow N_2 = mg \cos(3\theta)$$

resolving parallel to the plank

$$ma = mg \sin(3\theta) - \mu N_2$$

$$ma = mg \sin(3\theta) - \mu mg \cos(3\theta) \quad \text{but } \mu = \tan(\theta)$$

$$ma = mg (\sin(3\theta) - \tan(\theta) \cos(3\theta))$$

$$a = g \left(\sin(3\theta) - \frac{\sin(\theta) \cos(3\theta)}{\cos(\theta)} \right) \quad \text{M1}$$

$$a = g \left(\frac{\sin(3\theta) \cos(\theta) - \sin(\theta) \cos(3\theta)}{\cos(\theta)} \right)$$

$$a = \frac{g \sin(3\theta - \theta)}{\cos(\theta)} = \frac{g \sin(2\theta)}{\cos(\theta)}$$

$$a = \frac{2g \sin(\theta) \cos(\theta)}{\cos(\theta)} \quad \text{A1}$$

$$a = 2g \sin(\theta)$$

Now using constant acceleration formulae with $u = 0$ $v = V_2$ $s = D$

$$v^2 = u^2 + 2as$$

$$V_2 = 2\sqrt{gD \sin(\theta)} \quad \text{A1}$$

$$s = ut + \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2s}{a}}$$

$$T_2 = \sqrt{\frac{2D}{2g \sin(\theta)}}$$

$$T_2 = \sqrt{\frac{D}{g \sin(\theta)}} \quad \text{A1}$$

d. $\frac{T_2}{T_1} = \sqrt{\frac{D}{g \sin(\theta)}} \times \sqrt{\frac{g \tan(\theta)}{2D}} = \sqrt{\frac{Dg \sin(\theta)}{2Dg \sin(\theta) \cos(\theta)}} \quad \text{M1}$

$$\frac{T_2}{T_1} = \sqrt{\frac{1}{2 \cos(\theta)}} = \frac{3}{4} \Rightarrow \cos(\theta) = \frac{8}{9}$$

$$\theta = \cos^{-1}\left(\frac{8}{9}\right)$$

$$\theta = 27^\circ 16' \quad \text{A1}$$

e.
$$\frac{V_2}{V_1} = \frac{2\sqrt{gD \sin(\theta)}}{\sqrt{2gD \tan(\theta)}} = \frac{2\sqrt{gD} \sqrt{\sin(\theta) \cos \theta}}{\sqrt{2gD} \sqrt{\sin(\theta)}}$$

$$\frac{V_2}{V_1} = \frac{2\sqrt{\cos(\theta)}}{\sqrt{2}} = \frac{2}{\sqrt{2}} \times \sqrt{\frac{8}{9}}$$

$$\frac{V_2}{V_1} = \frac{4}{3}$$

A1

Question 2

a. $\frac{x^2}{9} - \frac{y^2}{36} = 1$ crosses the x-axis at $y = 0 \Rightarrow x^2 = 9 \quad x = \pm 3$

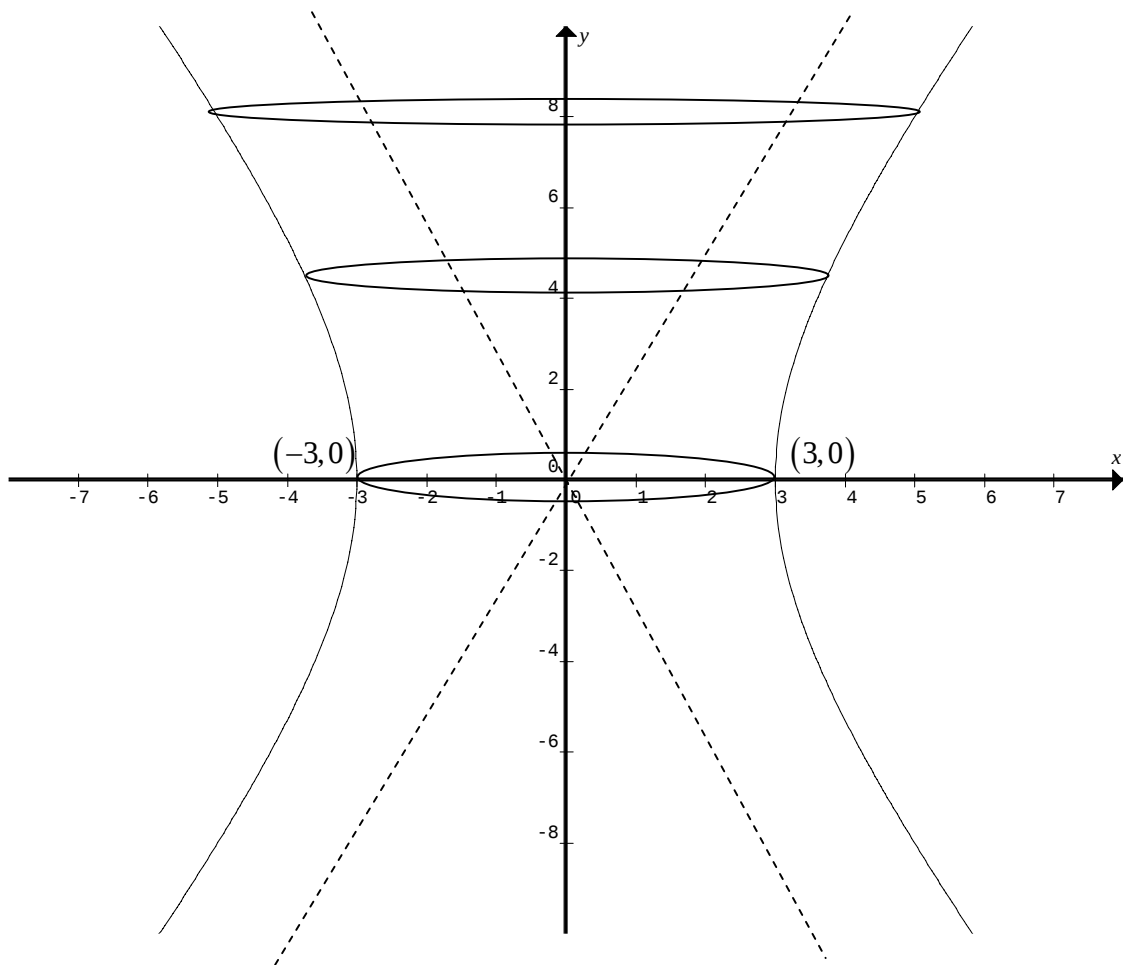
$(3,0) \quad (-3,0)$

A1

the asymptotes are $y = \pm 2x$

correct graph, shape of vase, intercepts

G1



i. when $x = 5$, $y = H$ $\frac{H^2}{36} = \frac{25}{9} - 1 = \frac{16}{9} \Rightarrow H = \sqrt{64}$
 $H = 8 \text{ cm}$

A1

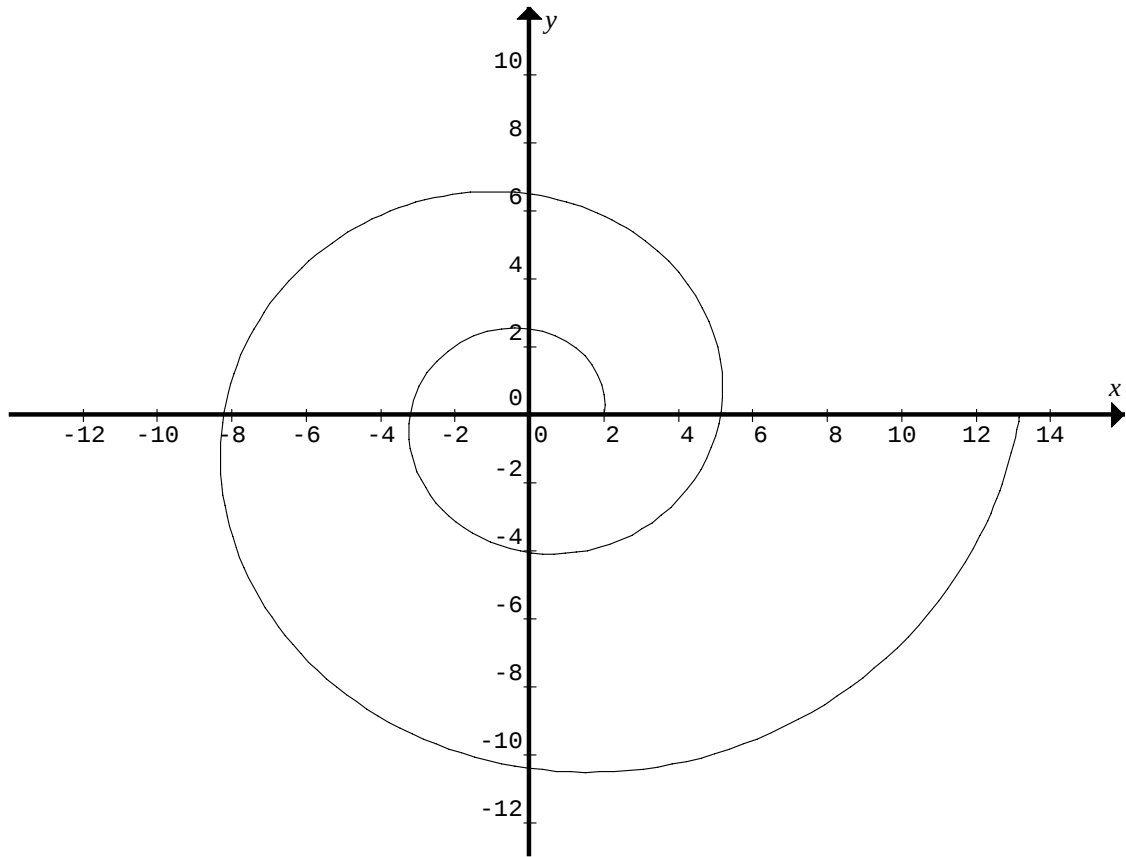
ii. $V = \pi \int_a^b x^2 dy$
 $\frac{x^2}{9} = 1 + \frac{y^2}{36} \Rightarrow x^2 = \frac{9}{36}(36 + y^2)$ M1
 $V = \frac{\pi}{4} \int_0^8 (y^2 + 36) dy \text{ cm}^3$ A1

iii. $V = \frac{\pi}{4} \int_0^h (y^2 + 36) dy \quad 0 \leq h \leq 8$
 $V = \frac{\pi}{4} \left[\frac{1}{3} y^3 + 36y \right]_0^h$ M1
 $V = \frac{\pi}{4} \left(\frac{1}{3} h^3 + 36h \right) = \frac{\pi}{4} \left(\frac{h^3 + 108h}{3} \right)$ A1
 $V = \frac{\pi h}{12} (h^2 + 108) \quad \text{for } 0 \leq h \leq 8$

iv. $\frac{dV}{dt} = 18\sqrt{h} \text{ cm}^3/\text{s}$
 $\frac{dV}{dh} = \frac{\pi}{4} (h^2 + 36)$ A1
 $\frac{dh}{dt} = \frac{dh}{dV} \frac{dV}{dt} = \frac{72\sqrt{h}}{\pi(h^2 + 36)}$ M1
 $\frac{dt}{dh} = \frac{\pi(h^2 + 36)}{72\sqrt{h}}$ M1
 $t = \frac{\pi}{72} \int_0^8 \frac{h^2 + 36}{\sqrt{h}} dh$
 $t = \frac{122\pi\sqrt{2}}{45} \text{ sec}$ A1

Question 3

- a. correct graph shape and correct x and y intercepts, correct cycle G2



b. $x(t) = 2e^{0.3t} \cos(2t)$

$$\dot{x} = \frac{dx}{dt} = 2(0.3e^{0.3t} \cos(2t) - 2e^{0.3t} \sin(2t))$$

M1

$$\dot{x} = \frac{dx}{dt} = 2e^{0.3t} (0.3 \cos(2t) - 2 \sin(2t))$$

$$y(t) = 2e^{0.3t} \sin(2t)$$

$$\dot{y} = \frac{dy}{dt} = 2(0.3e^{0.3t} \sin(2t) + 2e^{0.3t} \cos(2t))$$

A1

$$\dot{y} = \frac{dy}{dt} = 2e^{0.3t} (0.3 \sin(2t) + 2 \cos(2t))$$

$$\text{Now } \frac{dy}{dx} = \tan(\alpha) = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{\dot{y}}{\dot{x}}$$

$$\tan(\alpha) = \frac{2e^{0.3t}(0.3\sin(2t) + 2\cos(2t))}{2e^{0.3t}(0.3\cos(2t) - 2\sin(2t))}$$

M1

$$\tan(\alpha) = \frac{0.3\sin(2t) + 2\cos(2t)}{0.3\cos(2t) - 2\sin(2t)} \quad \text{divide by } \cos(2t)$$

$$\tan(\alpha) = \frac{0.3\tan(2t) + 2}{0.3 - 2\tan(2t)}$$

c. In the triangle $2t + (180 - \alpha) + \beta = 180$

$$\beta = \alpha - 2t$$

A1

$$\tan(\beta) = \tan(\alpha - 2t)$$

$$\tan(\beta) = \frac{\tan(\alpha) - \tan(2t)}{1 + \tan(\alpha)\tan(2t)}$$

$$\tan(\beta) = \frac{\left(\frac{0.3\tan(2t) + 2}{0.3 - 2\tan(2t)}\right) - \tan(2t)}{1 + \left(\frac{0.3\tan(2t) + 2}{0.3 - 2\tan(2t)}\right)\tan(2t)}$$

M1

$$\tan(\beta) = \frac{\frac{0.3\tan(2t) + 2 - \tan(2t)(0.3 - 2\tan(2t))}{0.3 - 2\tan(2t)}}{\frac{0.3 - 2\tan(2t) + \tan(2t)(0.3\tan(2t) + 2)}{0.3 - 2\tan(2t)}}$$

$$\tan(\beta) = \frac{0.3\tan(2t) + 2 - 0.3\tan(2t) + 2\tan^2(2t)}{0.3 - 2\tan(2t) + 0.3\tan^2(2t) + 2\tan(2t)}$$

$$\tan(\beta) = \frac{2 + 2\tan^2(2t)}{0.3 + 0.3\tan^2(2t)} = \frac{2(1 + \tan^2(2t))}{0.3(1 + \tan^2(2t))}$$

$$\tan(\beta) = \frac{20}{3}$$

A1

d. Now the speed $|\dot{r}(t)| = \sqrt{\dot{x}^2 + \dot{y}^2}$

$$\dot{x} = 2e^{0.3t} (0.3 \cos(2t) - 2 \sin(2t))$$

M1

$$\dot{x}^2 = (2e^{0.3t} (0.3 \cos(2t) - 2 \sin(2t)))^2$$

$$\dot{x}^2 = 4e^{0.6t} (0.3^2 \cos^2(2t) - 1.2 \sin(2t) \cos(2t) + 4 \sin^2(2t))$$

$$\dot{y} = 2e^{0.3t} (0.3 \sin(2t) + 2 \cos(2t))$$

M1

$$\dot{y}^2 = (2e^{0.3t} (0.3 \sin(2t) + 2 \cos(2t)))^2$$

$$\dot{y}^2 = 4e^{0.6t} (0.3^2 \sin^2(2t) + 1.2 \cos(2t) \sin(2t) + 4 \cos^2(2t))$$

$$\dot{x}^2 + \dot{y}^2 = 4e^{0.6t} (0.3^2 (\sin^2(2t) + \cos^2(2t)) + 4 (\sin^2(2t) + \cos^2(2t)))$$

$$\dot{x}^2 + \dot{y}^2 = 4e^{0.6t} (4 + 0.3^2)$$

Now the speed $|\dot{r}(t)| = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{4e^{0.6t} (0.3^2 + 4)} = 2\sqrt{4 + 0.3^2} e^{0.3t}$

$$a = 2\sqrt{4.09} \quad k = 0.3$$

A1

e. the total distance is $s = \int_a^b |\dot{r}(t)| dt$

from **d.** it follows that

$$s = \int_0^{2\pi} 2\sqrt{4 + 0.3^2} e^{0.3t} dt$$

$$s = \frac{2\sqrt{4 + 0.3^2}}{0.3} [e^{0.3t}]_0^{2\pi}$$

A1

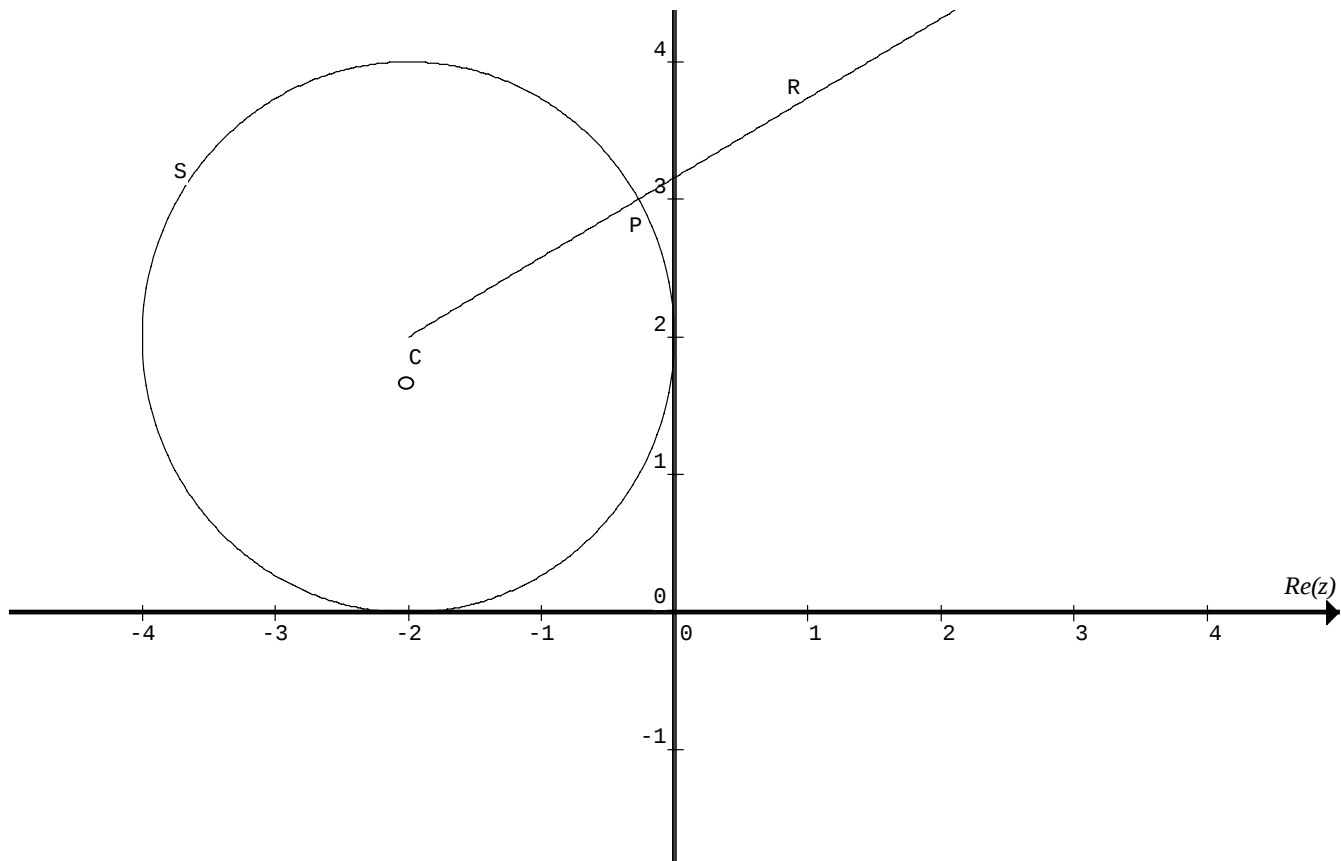
$$s = \frac{20\sqrt{4.09}}{3} [e^{0.6\pi} - 1]$$

$$s = 75.314$$

A1

Question 4

- a. S is a circle, $(x+2)^2 + (y-2)^2 = 4$ centre at $(-2, 2)$ radius 2 A1
 R is a ray, starting from $(-2, 2)$, an open circle, since the point is not included, making an angle of 30° with the line $y = 2$. A1
 correct circle and ray on the diagram A1



- b. maximum value is the furthest point from the origin O , let $C(-2, 2)$ M1
 now the distance from $d(OC) = 2\sqrt{2}$ and the radius of the circle is 2, so
 $z \in S \quad |z|_{\max} = 2\sqrt{2} + 2 = 2(1 + \sqrt{2})$ A1

c. $R \quad \text{Arg}(z + 2 - 2i) = \text{Arg}((x+2) + (y-2)i) = \frac{\pi}{6} \Rightarrow \tan^{-1}\left(\frac{y-2}{x+2}\right) = \frac{\pi}{6}$

$$y - 2 = \frac{1}{\sqrt{3}}(x + 2) \quad \text{for } x > -2$$

$$x + 2 = \sqrt{3}(y - 2) \quad \text{substitute into } (x + 2)^2 + (y - 2)^2 = 4 \quad \text{M1}$$

$$4(y - 2)^2 = 4 \Rightarrow y - 2 = \pm 1 \quad \text{but } y > 2$$

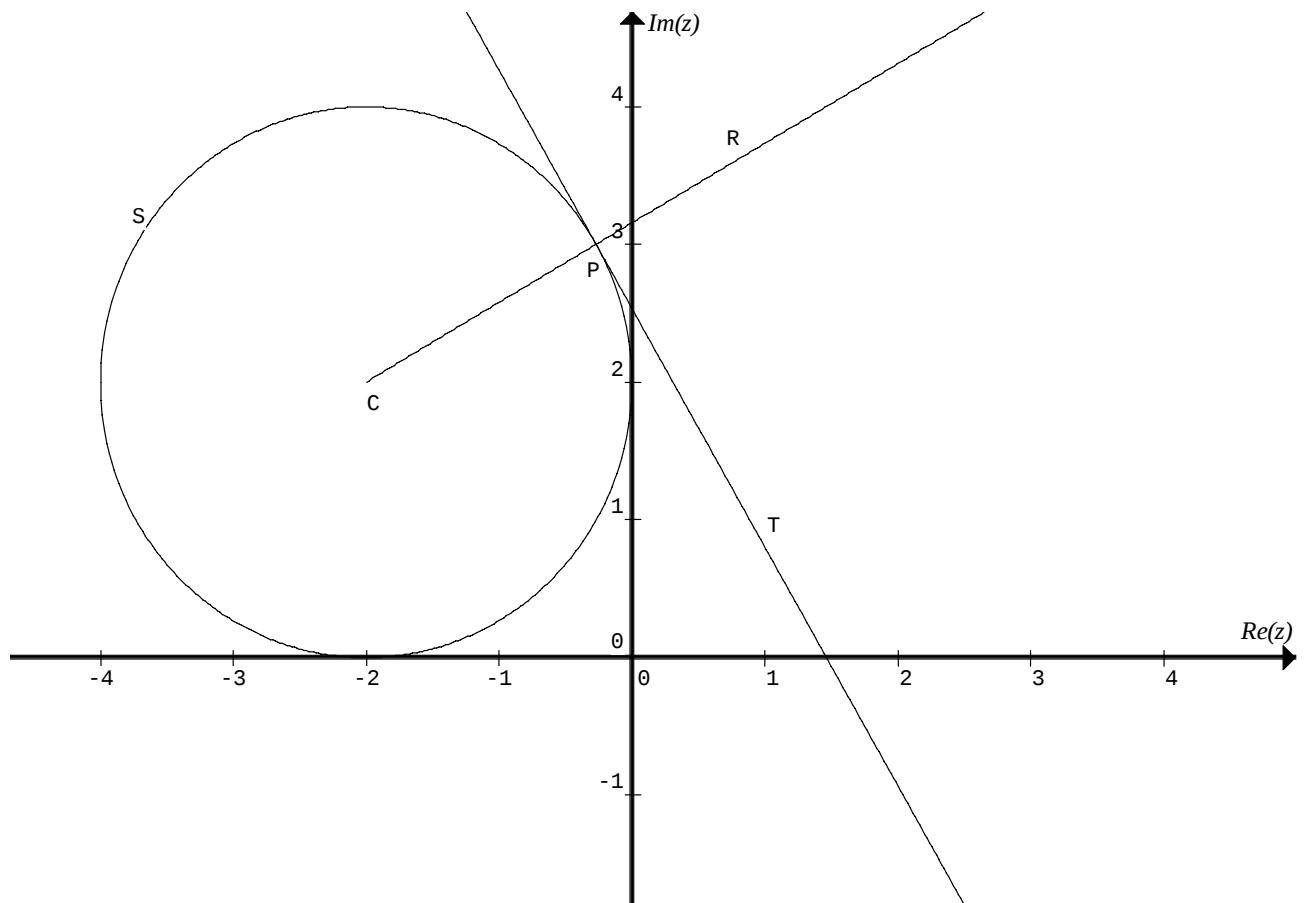
$$\text{so } b = 3 \quad \text{A1}$$

$$a + 2 = \sqrt{3}$$

$$a = \sqrt{3} - 2 \quad \text{A1}$$

$$p = \sqrt{3} - 2 + 3i$$

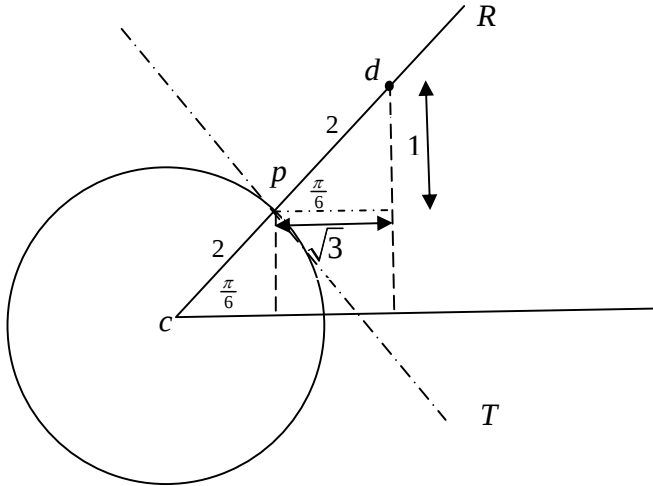
d.



The point d , is on the ray R , and the distance $d(cp) = d(pd) = 2$

$$|z + 2 - 2i| = |z - d|, \text{ since } p = \sqrt{3} - 2 + 3i$$

M1



$\text{Im}(d) = 4$ (one up from p), and so $\text{Re}(d) = 2\sqrt{3} - 2$ ($\sqrt{3}$ across from p)

$$d = 2\sqrt{3} - 2 + 4i$$

A1

e. $|z + 2 - 2i| = |z - 2\sqrt{3} + 2 - 4i|$ let $z = x + yi$

$$|x + 2 + (y - 2)i| = |x - 2\sqrt{3} + 2 + (y - 4)i|$$

$$\sqrt{(x + 2)^2 + (y - 2)^2} = \sqrt{(x + 2 - 2\sqrt{3})^2 + (y - 4)^2}$$

$$(x + 2)^2 + y^2 - 4y + 4 = (x + 2)^2 - 4\sqrt{3}(x + 2) + 12 + y^2 - 8y + 16$$

M1

$$4y = -4\sqrt{3}x - 8\sqrt{3} + 24$$

$$y = -\sqrt{3}x + 6 - 2\sqrt{3} \quad \text{Im}(z) = y \text{ and } \text{Re}(z) = x$$

$$\text{Im}(z) = m \text{Re}(z) + k$$

$$m = -\sqrt{3} \quad k = 6 - 2\sqrt{3}$$

A1

Alternatively using geometry, $P(\sqrt{3} - 2, 3)$ $C(-2, 2)$

$$\text{the gradient of } m(PC) = \frac{y_p - y_c}{x_p - x_c} = \frac{3 - 2}{\sqrt{3} - 2 + 2} = \frac{1}{\sqrt{3}}$$

so the gradient of the line perpendicular is $m = -\sqrt{3}$

and the line through P , is $y - 3 = -\sqrt{3}(x - (\sqrt{3} - 2))$

$$y = -\sqrt{3}x + 6 - 2\sqrt{3} \Rightarrow k = 6 - 2\sqrt{3}$$

Question 5

- a. let $y = 14 \log_e (14 - \sqrt{x}) + \sqrt{x}$ for $0 < x < b$

$$\frac{dy}{dx} = \frac{-14}{2\sqrt{x}} \left(\frac{1}{14 - \sqrt{x}} \right) + \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{-7}{\sqrt{x}(14 - \sqrt{x})} + \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{-14 + (14 - \sqrt{x})}{2\sqrt{x}(14 - \sqrt{x})}$$

M1

$$\frac{dy}{dx} = \frac{-\sqrt{x}}{2\sqrt{x}(14 - \sqrt{x})}$$

$$\frac{dy}{dx} = \frac{-1}{2(14 - \sqrt{x})} = \frac{1}{2(\sqrt{x} - 14)}$$

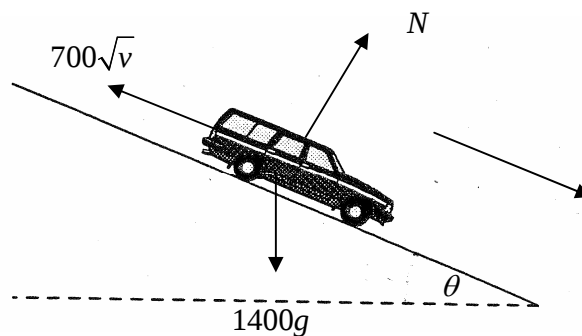
$$\text{so that } \frac{d}{dx} [14 \log_e (14 - \sqrt{x}) + \sqrt{x}] = \frac{1}{2(\sqrt{x} - 14)}$$

since $\sqrt{x} > 0$ and $14 - \sqrt{x} > 0$ it follows that

$$0 < x < 196 \Rightarrow b = 196$$

A1

- b.i.



$$ma = mg \sin(\theta) - R \quad \text{and correct forces}$$

A1

$$m = 1400 \text{ kg} \quad R = 700\sqrt{v} \quad \theta = \sin^{-1}\left(\frac{5}{7}\right) \Rightarrow \sin(\theta) = \frac{5}{7}$$

$$1400a = 1400 \times 9.8 \times \frac{5}{7} - 700\sqrt{v}$$

$$a = 7 - \frac{\sqrt{v}}{2}$$

A1

ii. use $a = \ddot{x} = \frac{dv}{dt} = 7 - \frac{\sqrt{v}}{2} = \frac{14 - \sqrt{v}}{2}$ inverting gives

$$\frac{dt}{dv} = \frac{2}{14 - \sqrt{v}}$$

$$t = \int_0^{16} \left(\frac{2}{14 - \sqrt{v}} \right) dv \quad \text{from a.}$$

$$t = -4 \left[14 \log_e (14 - \sqrt{v}) + \sqrt{v} \right]_0^{16} \quad \text{M1}$$

$$t = -4 \left[(14 \log_e (10) + 4) - 14 \log_e (14) \right]$$

$$t = 56 \log_e \left(\frac{7}{5} \right) - 16 \text{ sec} \quad \text{A1}$$

iii. use $\ddot{x} = v \frac{dv}{dx} = \frac{14 - \sqrt{v}}{2}$ inverting gives

$$\frac{dx}{dv} = \frac{2v}{14 - \sqrt{v}} \quad \text{A1}$$

$$x = \int_0^v \left(\frac{2v}{14 - \sqrt{v}} \right) dv \quad \text{now when } x = 20 \text{ } V = ? \quad \text{M1}$$

$$20 = \int_0^v \left(\frac{2v}{14 - \sqrt{v}} \right) dv \quad \text{since } 0 < V < 196$$

solving gives $V = 14.76 \text{ m/s}$ A1

END OF SECTION 2 SUGGESTED ANSWERS