



Online & home tutors Registered business name: itute ABN: 96 297 924 083

Specialist Mathematics

2010

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 When the quadratic equation $z^2 - z + 1 = 0$ is solved over C ,

- A. it has no solutions.
- B. the solutions are $z = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.
- C. the solutions are $z = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.
- D. the solutions are $z = -\frac{1}{2} \pm \frac{\sqrt{5}}{2}i$.
- E. the solutions are $z = \frac{1}{2} \pm \frac{\sqrt{5}}{2}i$.

Question 2 Given $z = -cis(1)$,

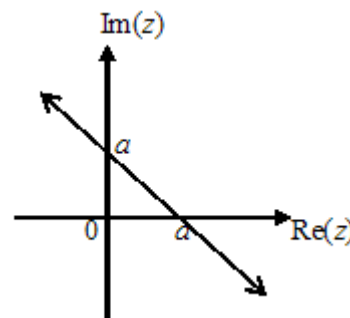
- A. $|z| = -1$ and $Arg(z) = 1$.
- B. $|z| = 1$ and $Arg(z) = -1$.
- C. $|z| = 1$ and $Arg(z) = 1 - \pi$.
- D. $|z| = -1$ and $Arg(z) = \pi$.
- E. $|z| = 1$ and $Arg(z) = \pi - 1$.

Question 3 The polynomial $P(z) = z^3 + 2iz^2 + 2z + 4i$ has

- A. no real solutions.
- B. a pair of conjugate roots.
- C. three linear factors over C .
- D. three solutions over C .
- E. two real solutions and a complex solution.

Question 4 The straight line on the complex plane shown on the left can be defined by

- A. $\left\{ z : \text{Arg}(z) = \frac{3\pi}{4} \right\}$.
- B. $\left\{ z : |z - a - ai| - |z| = 0 \right\}$.
- C. $\left\{ z : |z - a + ai| = |z| \right\}$.
- D. $\left\{ z : |z + a - ai| - |z| = 0 \right\}$.
- E. $\left\{ z : |z + a + ai| = |z| \right\}$.



Question 5 The graph of $y = x + \frac{b}{x}$, where $b \in \mathbb{R} \setminus \{0\}$,

- A. always has two stationary points.
- B. always has two asymptotes.
- C. has $\mathbb{R}$ as its domain.
- D. always has $\mathbb{R}$ as its range.
- E. has a y-intercept.

Question 6 The graph of $y = \frac{2}{4x^2 + px + q^2}$, where $q > 0$, has a stationary point when

- A. $p < -4q$.
- B. $p < 0$.
- C. $p \geq 0$.
- D. $p > q$.
- E. $p < q$.

Question 7 $\sin\left(a + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) =$

- A. $\frac{1}{2}\cos a - \frac{\sqrt{3}}{2}\sin a$.
- B. $-\frac{1}{2}\cos a + \frac{\sqrt{3}}{2}\sin a$.
- C. $\frac{1}{2}\sin a - \frac{\sqrt{3}}{2}\cos a$.
- D. $-\frac{1}{2}\sin a + \frac{\sqrt{3}}{2}\cos a$.
- E. $\sin a + \frac{1}{2}$.

Question 8 The domain of the function $f(x) = \sin^{-1}\left(\frac{x}{a} + b\right) + c$, where $a, b, c \in (-\infty, 0)$, is

- A. $[-a(1+b), a(1-b)]$.
- B. $[-a-b, a-b]$.
- C. $[-a(1+b)-c, a(1-b)-c]$.
- D. $[a-b, -a-b]$.
- E. $[a(1-b), -a(1+b)]$.

Question 9 Let $\tan^{-1} a = 0.3$ and $\tan^{-1} b = 0.2$. In terms of a and/or b , $\tan(0.1) =$

- A. $\frac{a-b}{ab-1}$.
- B. $\frac{a-b}{1-ab}$.
- C. $\frac{\sqrt{1+b^2}-1}{b}$.
- D. $\frac{\sqrt{1+b^2}+1}{b}$.
- E. $a-b$.

Question 10 When $\pi < \theta < \frac{3\pi}{2}$, $\cos^{-1}(\cos \theta) =$

- A. θ .
- B. $\pi - \theta$.
- C. $2\pi - \theta$.
- D. $\theta - 2\pi$.
- E. $\theta - \pi$.

Question 11 $-2\tilde{i} + 3\tilde{k}$, $\tilde{j} - 2\tilde{k}$ and which one of the following vectors are linearly independent?

- A. $2\tilde{i} - \tilde{j} - \tilde{k}$
- B. $4\tilde{i} - 3\tilde{j}$
- C. $3\tilde{i} - 4\tilde{j}$
- D. $6\tilde{i} - 2\tilde{j} - 5\tilde{k}$
- E. $2\tilde{i} - 2\tilde{j} + \tilde{k}$

Question 12 A, B and P are points on a **unit** circle centred at O . Given $\overrightarrow{AO} \cdot \overrightarrow{BO} = \frac{1}{2}$ and $\overrightarrow{AP} \cdot \overrightarrow{BP} = \sqrt{3}$, the value of $|\overrightarrow{AP}| |\overrightarrow{BP}|$ is

- A. 0.6
- B. 0.8
- C. 1
- D. 2
- E. 3

Question 13 Point Q divides the line segment joining point $P(-1,0,2)$ and point $R(2,1,-2)$ into a ratio of 2 : 3. The distance of point Q from the origin $O(0,0,0)$ is

- A. $\frac{\sqrt{29}}{5}$.
- B. $\sqrt{29}$.
- C. $\frac{\sqrt{29}}{6}$.
- D. $\frac{1}{2}$.
- E. $\frac{3}{5}$.

Question 14 The angles that $\frac{\sqrt{3}}{2}\tilde{i} - \frac{1}{\sqrt{2}}\tilde{j} - \frac{1}{2}\tilde{k}$ makes with the x , y and z axes are respectively

- A. 30° , 45° and 60° .
- B. 30° , 135° and 120° .
- C. 30° , -45° and -60° .
- D. $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$, $-\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ and $-\cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$.
- E. $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$, $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ and $\cos^{-1}\left(-\frac{1}{\sqrt{6}}\right)$.

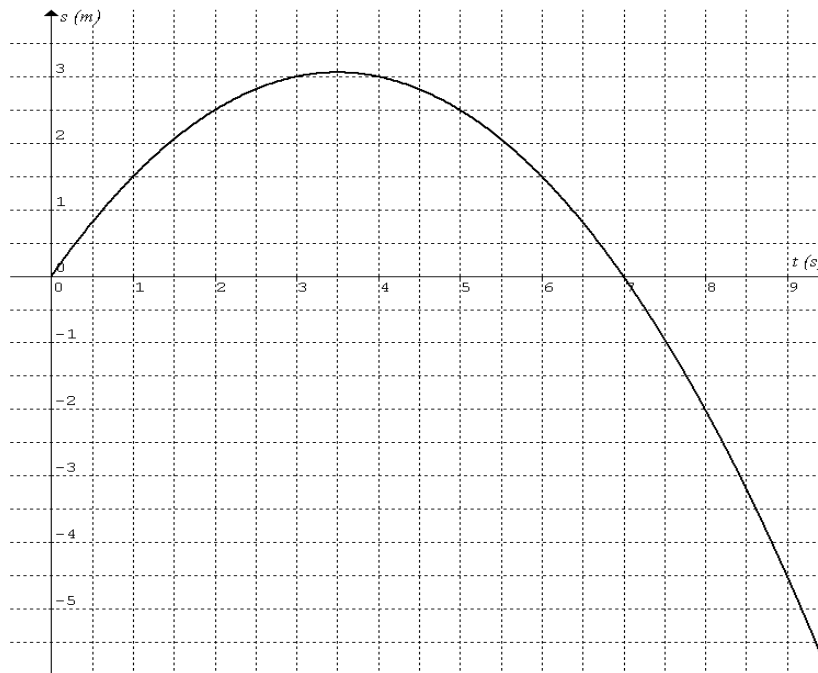
Question 15 Which one of the following statements is **true** about a particle with velocity $\tilde{v} = 2\cos^{-1}(t)\tilde{i} - 3\cos^{-1}(t)\tilde{j} + \cos^{-1}(t)\tilde{k}$, $0 \leq t \leq 1$.

- A. The speed of the particle increases with time t .
- B. The particle moves in a straight line.
- C. The particle moves with constant acceleration.
- D. The distance of the particle from its initial position decreases with time t .
- E. The initial speed of the particle is $\sqrt{14}$.

Question 16 If $\frac{dy}{dh} = \sqrt{y(2-y)} \frac{dx}{dh}$, y may be expressed in terms of x as

- A. $y = \sin(x+2)+1$
- B. $y = \sin(x+3)-1$
- C. $y = \sin(x-1)$
- D. $y = \sin(2x-1)+1$
- E. $y = \sin(3x-1)-1$

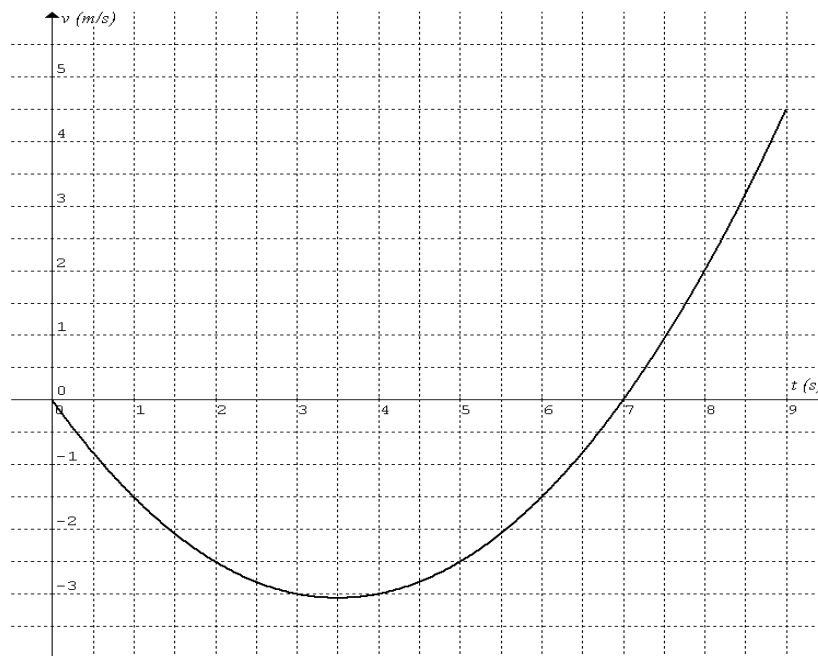
Question 17 The *displacement*-time graph of a particle travelling along the x -axis is shown below.



Initially the particle is at $x = 2$. The particle is at $x = 0$ when

- A. $t = 0$. B. $t = 1.4$. C. $t = 5.6$. D. $t = 7$. E. $t = 8$.

Question 18 The velocity-time graph ($0 \leq t \leq 9$) of a particle is shown below.



The particle is furthest from its initial position when

- A. $t = 0$. B. $t = 3.5$. C. $t = 7$. D. $t = 9$. E. its acceleration is zero.

Question 19 Using a suitable substitution, $\int_0^{\frac{\pi}{2}} \cos^3 x \sqrt{1 - \sin x} dx$ may be expressed completely in terms of u as

- A. $\int_0^1 \left(2u^{\frac{3}{2}} - u^{\frac{5}{2}} \right) du$
- B. $\int_0^1 \left(-2u^{\frac{3}{2}} + u^{\frac{5}{2}} \right) du$
- C. $\int_0^{\sin^{-1}(1)} \left(2u^{\frac{3}{2}} - u^{\frac{5}{2}} \right) du$
- D. $\int_0^{\sin^{-1}(1)} \left(-2u^{\frac{3}{2}} + u^{\frac{5}{2}} \right) du$
- E. $\int_{\cos^{-1}(0)}^1 \left(-2u^{\frac{3}{2}} + u^{\frac{5}{2}} \right) du$

Question 20 An object slides down an inclined plane ($\mu = 0.2$). The object moves at constant speed when the inclined plane is at an angle of θ° with the horizontal. The value of θ

- A. depends on the mass of the object.
- B. depends on the speed of the object.
- C. on earth is different from the value of θ on the moon.
- D. is approximately 11.
- E. is indeterminable without further information.

Question 21 A particle moves along the x -axis. Its velocity is given by $v = -\sqrt{100 - x}$. Which one of the following statements is **NOT** true?

- A. The particle has a constant acceleration.
- B. The particle has a negative acceleration.
- C. The particle speeds up.
- D. The particle slows down.
- E. The particle continues to move in the same direction.

Question 22 The floor of a lift is an inclined plane (10° to the horizontal). A 10-kg object is at rest on the floor whilst the lift is stationary. When the lift moves downwards with speed increasing at 1 m/s in a second, the reaction force of the floor on the object is

- A. 86.7 N perpendicular to the floor.
- B. 88 N vertically upward.
- C. 88 N perpendicular to the floor.
- D. 96.5 N perpendicular to the floor.
- E. 98 N vertically upward.

SECTION 2 Extended-answer questions

Instructions for Section 2

Answer **all** questions.

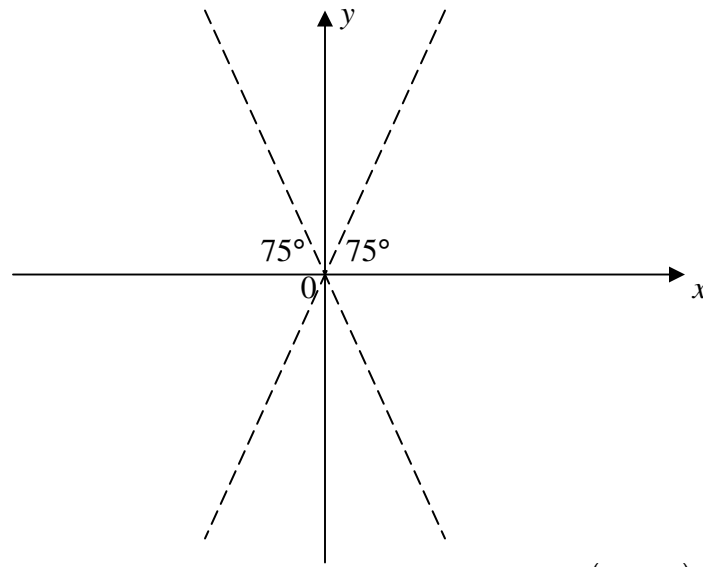
A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1



- a. Show that the equations of the two dotted intersecting lines are $y = \pm(2 + \sqrt{3})x$.

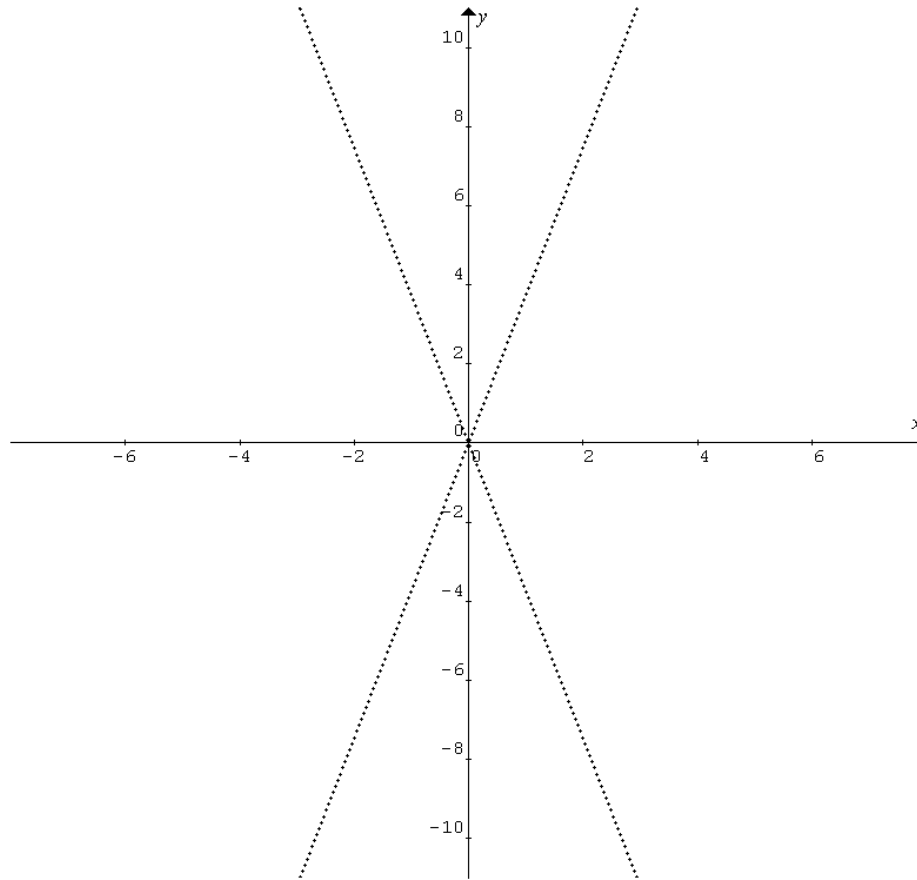
2 marks

- b. A hyperbola has the dotted lines as its asymptotes and $y = \pm(\sqrt{3} + 1)$ as its y-intercepts. Determine the equation of the hyperbola.

2 marks

c. Sketch accurately the graph of the hyperbola showing the important features on the axes below.

1 mark



Let O be the origin $(0,0)$. $\overrightarrow{OA}$ is the position vector of moving point A which lies on the dotted line with positive gradient, and $\overrightarrow{OB}$ is the position vector of moving point B which lies on the dotted line with negative gradient. The unit vectors $\tilde{i}$ and $\tilde{j}$ are in the positive x and y directions respectively.

d i. Given the x -coordinates of A and B are a and b respectively, express $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in terms of a , b , $\tilde{i}$ and $\tilde{j}$.

2 marks

d ii. Hence determine the position vector $\overrightarrow{OM}$ in terms of a , b , $\tilde{i}$ and $\tilde{j}$, where M is the mid-point of $\overline{AB}$.

1 mark

d iii. Given $|\vec{BA}| = 2$, show that $(2 + \sqrt{3})^2(a + b)^2 + (a - b)^2 = 4$.

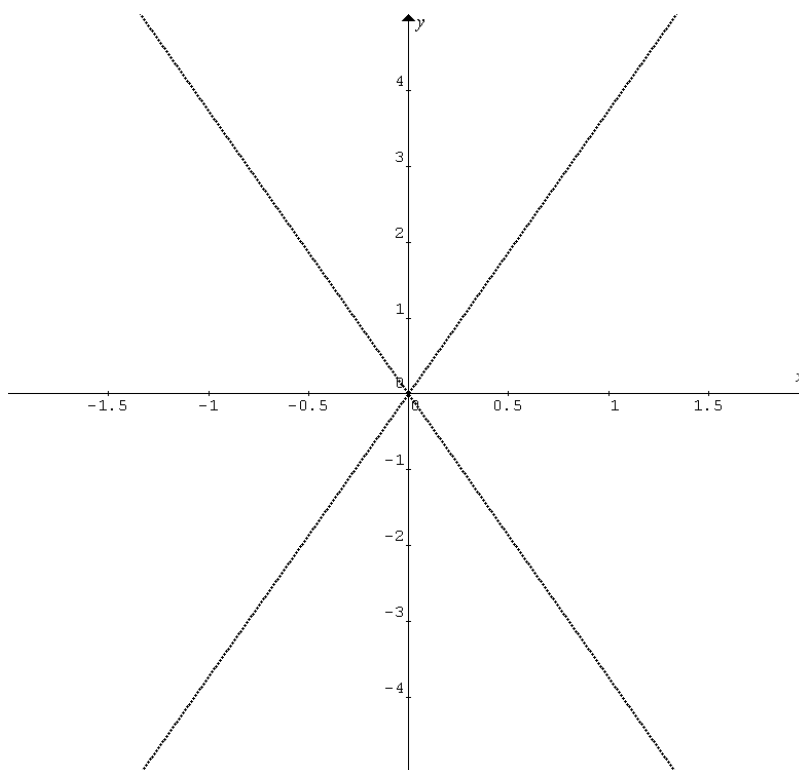
1 mark

d iv. Hence show that the locus of point M is an ellipse when a and b are varied and the condition $|\vec{BA}| = 2$ is satisfied.

2 marks

d v. Sketch accurately the graph of the ellipse showing the important features on the axes below.

1 mark



d vi. Find the values of b (correct to three decimal places) when $a = 0.500$. Hence draw accurately the line segment $\overline{AB}$ on the axes above, with B in the second quadrant. Mark the point M and label the ends of line segment $\overline{AB}$.

3 marks

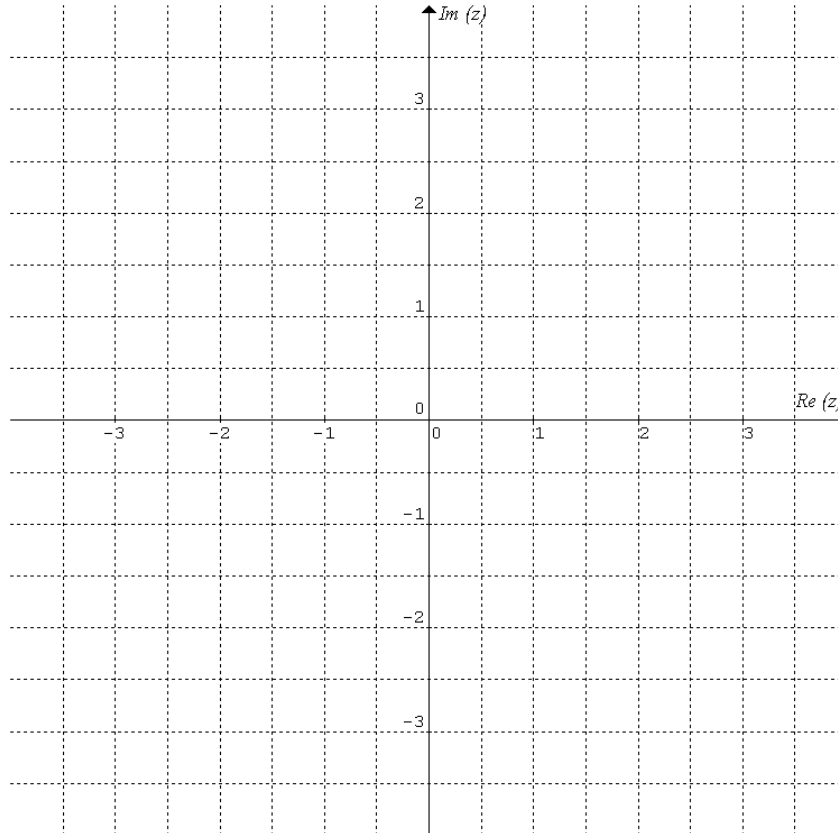
Question 2 Consider $f(z) = z^3 - 6iz^2 - 12z + 7i$.

a i. Express $f(z)$ in the form $(z - a)^3 + b$, where $a, b \in \mathbb{C}$.

2 marks

a ii. Sketch $\{z : |z - 2i| = 1\}$ on the argand diagram below.

1 mark



a iii. Hence sketch the roots of $f(z) = 0$ on the argand diagram above.

2 marks

a iv. Determine the roots of $f(z) = 0$ in exact $x + yi$ form.

2 marks

b. Determine the roots of $-iz^3 - 6z^2 + 12iz + 7 = 0$.

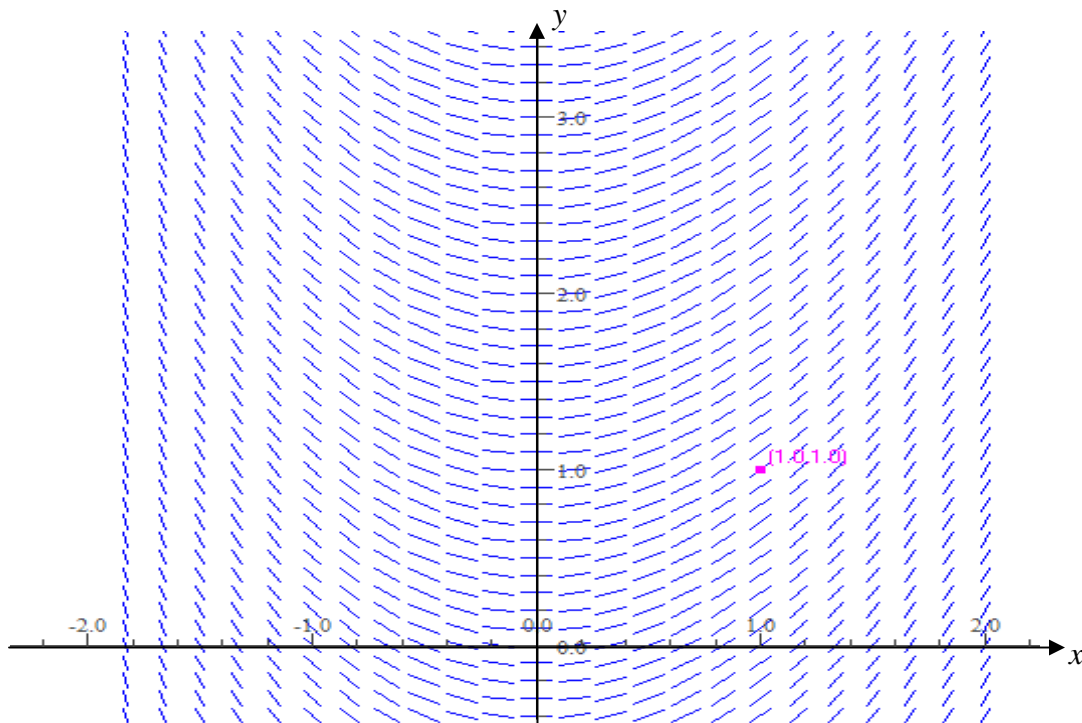
1 mark

c. Let $g(z) = f(z) - 7i$. Sketch the roots of $g(z) = 0$ on the argand diagram above. Describe the locations of the roots of $g(z) = 0$ in relation to the locations of the roots of $f(z) = 0$.

2 marks

Question 3

A slope field of the differential equation $\frac{dy}{dx} = \sin^{-1}\left(\frac{x}{2}\right) + \log_e\left(\frac{x+2}{2}\right)$ is shown below. The graph of a particular solution of the differential equation passes through the point (1,1).



a. State the maximal domain of the differential equation $\frac{dy}{dx} = \sin^{-1}\left(\frac{x}{2}\right) + \log_e\left(\frac{x+2}{2}\right)$.

1 mark

b i. On the slope field above sketch the graph of the particular solution of the differential equation.

2 marks

b ii. Hence estimate the value of y (correct to one decimal place) when $x = 1.5$.

1 mark

c. Divide the section from $x = 1$ to $x = 1.5$ into two equal intervals. Given $y = 1$ when $x = 1$, by Euler's method (first-order approximation) find the approximate value of y (correct to two decimal places) when $x = 1.5$.

2 marks

d i. Given $y = 1$ when $x = 1$, a numerical solution of $\frac{dy}{dx} = \sin^{-1}\left(\frac{x}{2}\right) + \log_e\left(\frac{x+2}{2}\right)$ when $x = 1.5$ can be found by evaluation of a definite integral. Write down this definite integral.

1 mark

d ii. Hence find the value of y (correct to two decimal places) when $x = 1.5$.

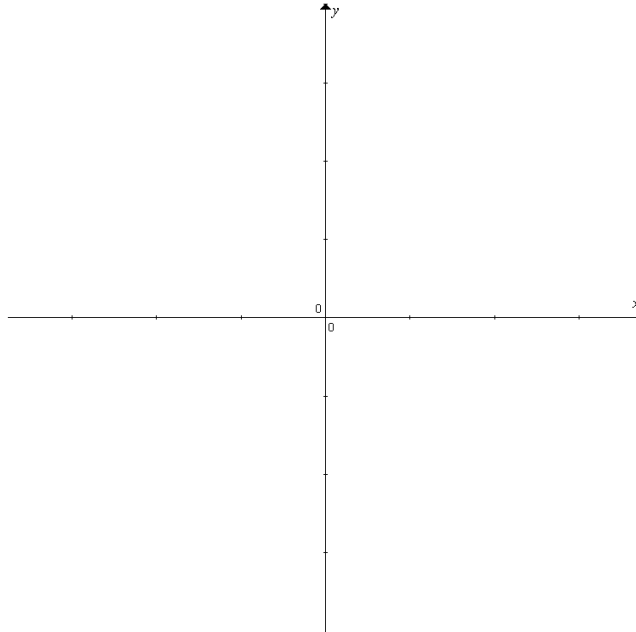
1 mark

Question 4

Consider $f(x) = \frac{x}{\sqrt{p^2 - x^2}} + q$, where $p, q \in \mathbb{R}^+$.

- a.** Sketch the graph of $f(x)$ on the axes below. Show and label the axis-intercept(s) and asymptote(s) in terms of p and q .

2 marks



Now let $p = \sqrt{3}$ and $q = \sqrt{3}$.

- b.** Without using CAS or calculator show that the area of the region bounded by the graph of $f(x)$, the x -axis and $x = \frac{3}{2}$ is $3\sqrt{3}$.

2 marks

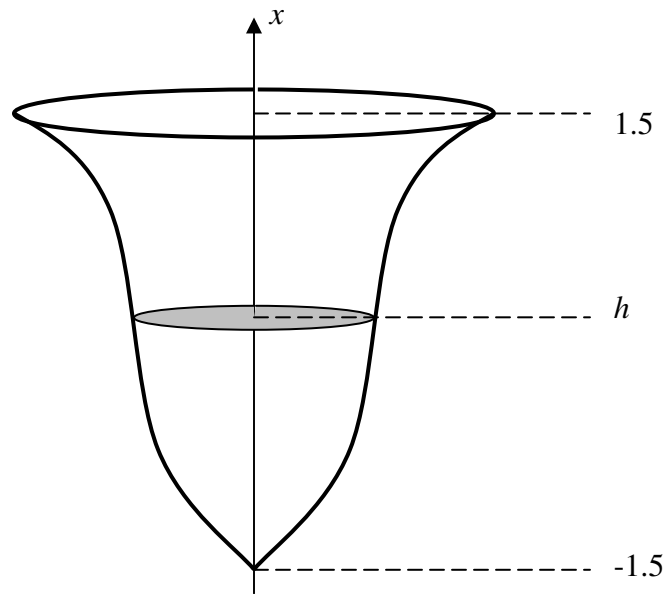
- c i.** The region specified in part **b.** is rotated about the x -axis to form a solid of revolution with volume V . Write a definite integral for finding V .

1 mark

- c ii.** Find the value of V correct to one decimal place.

1 mark

A container (wall of negligible thickness) in the shape of the solid in **c i.** is filled with water at a rate of 0.5 cm^3 per second. Linear measurements are in cm.



- d.** Use calculus to find the volume of water (in terms of h in cm^3) in the container when $x = h$, where $-1.5 \leq h \leq 1.5$.

4 marks

- e.** Find the exact rate of increase (cm per second) in the depth of water when $h = 0$.

2 marks

Question 5

The speed of a cyclist (total mass 85 kg) travelling along a straight road is $v = \frac{1}{2} \tan^{-1}(5t + 2.5) + \frac{\pi}{2}$ m/s, $t \geq 0$.

A car (total mass 1200 kg) travels in the same direction at constant speed of 20 m/s. The car passes the cyclist at $t = 0$. Two seconds later the car slows down and its speed is given by $v = \frac{20}{1 + 0.5(t - 2)^4}$ m/s, $t \geq 2$.

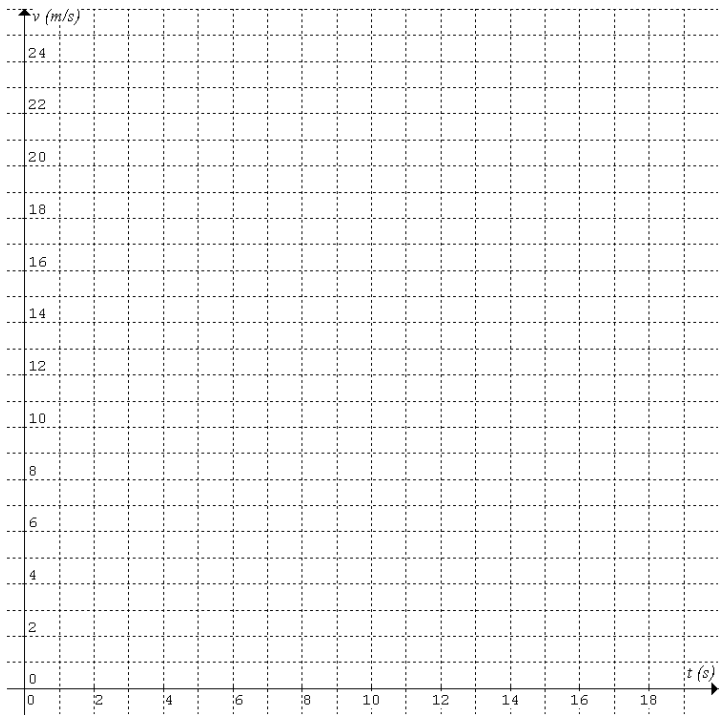
a. Determine the initial speed and acceleration of the cyclist correct to three decimal places. 3 marks

b. Determine the resultant force on the cyclist at $t = 0$. Give your answer correct to the nearest newton. 1 mark

c. Determine the difference in initial momentum of the cyclist and the car. Give your answer correct to the nearest kg m/s. 1 mark

d. Sketch the speed-time graphs for the cyclist and the car on the same axes below. Show and label intercept(s) and asymptote(s).

4 marks



e. Determine whether the cyclist or the car is ahead at $t = 10$ s. Find the distance between the cyclist and the car at $t = 10$ s. Give your answer correct to the nearest metre.

2 marks

f. Determine the time when the cyclist and the car are next to each other. Give your answer correct to the nearest second.

2 marks

End of Exam 2