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Specialist Mathematics

2010

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1

Find all solutions to the equation $4z^3 - i2\sqrt{3}z^2 - 3z = 0$, $z \in \mathbb{C}$.

3 marks

Question 2

Find $\{z : |z + 4| + |z - 3\sqrt{3}| = 11\} \cap \{z : \text{Re}(z) = 0\}$, $z \in \mathbb{C}$.

3 marks

Question 3

The position of a particle in circular motion is given by $\tilde{r} = -3\tilde{i} + 3\cos\frac{t}{10}\tilde{j} - 3\sin\frac{t}{10}\tilde{k}$, where time t is measured in seconds and distance is in metres.

a. What is the exact time taken to complete one turn?

1 mark

b. Find the exact value of the particle's speed.

2 marks

Question 4

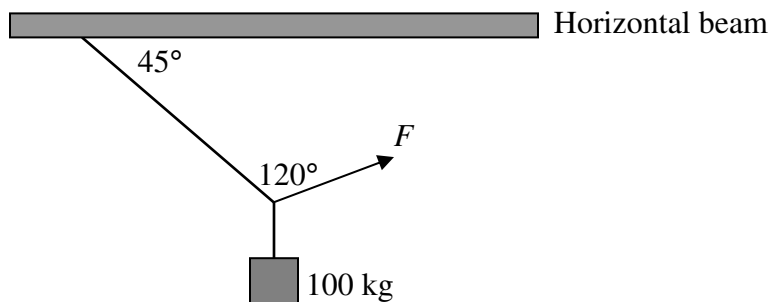
A particle travels in a straight line with a constant acceleration of -3 ms^{-2} . It starts at $x = -2 \text{ m}$ with a velocity of $+8.5 \text{ ms}^{-1}$. Find the time when the particle passes $x = +3 \text{ m}$ in the direction opposite to its initial velocity.

3 marks

Question 5

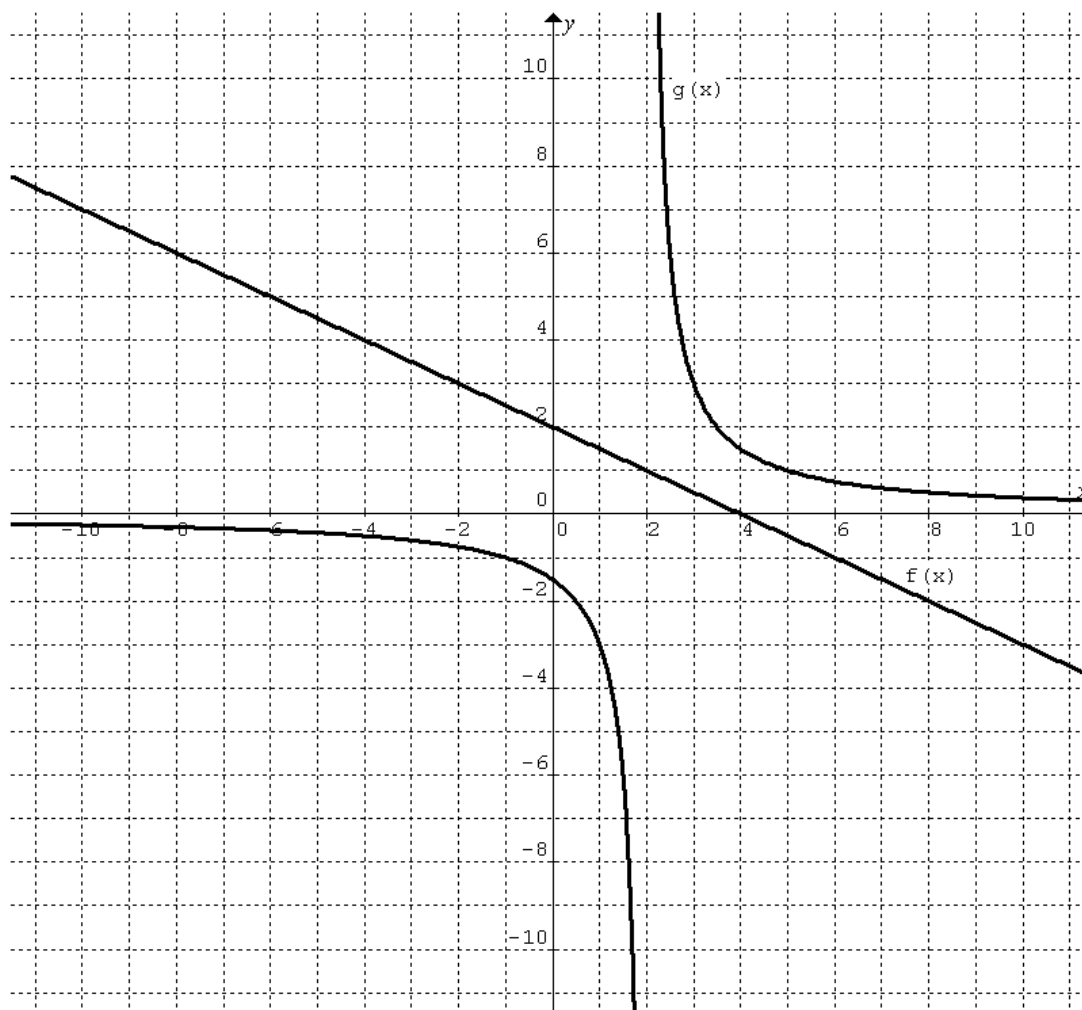
Find the magnitude of force F (newtons) required to keep the 100-kg mass in equilibrium.

3 marks



Question 6

The graphs of $f(x)$ and $g(x)$ are shown below.



- a. On the same set of axes accurately sketch the graph of $h(x) = f(x) - g(x)$. 3 marks
- b. Determine the equation of each asymptote of $h(x)$. 2 marks

Question 7

- a. Prove that $\frac{1}{1 - \sin x} = \sec x(\sec x + \tan x)$. 2 marks

- b. *Hence* evaluate the exact value of $\int_0^{\frac{\pi}{3}} \frac{1}{1 - \sin x} dx$. 3 marks

Question 8

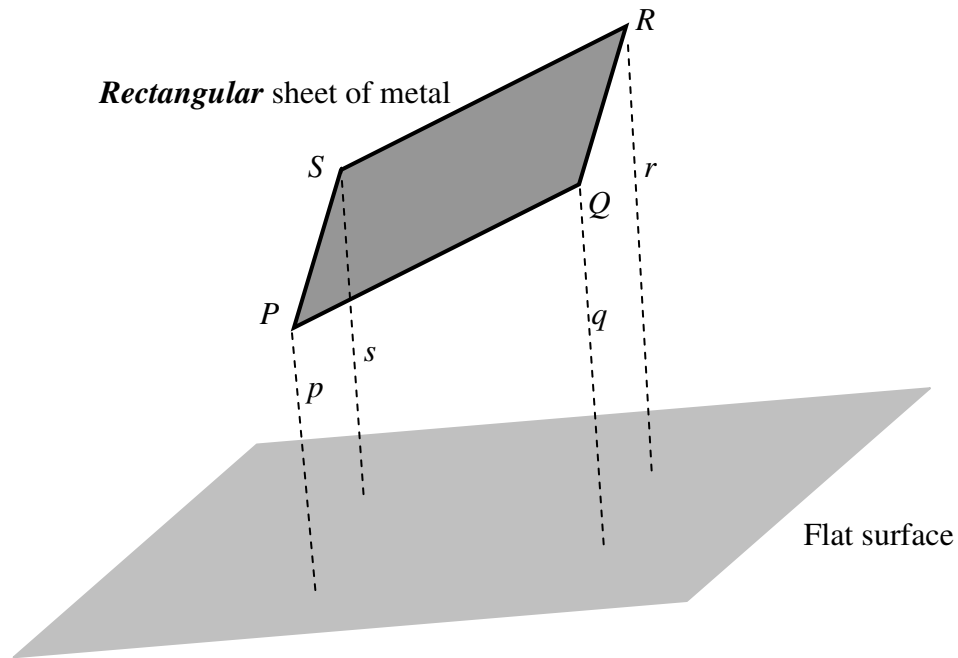
- a. Find the exact area bounded by the graph of $y = \sin^{-1} x$ and the y -axis. 3 marks

- b. Hence or otherwise, find the exact area bounded by the graph of $y = 2\sin^{-1}(2x) + \pi$ and the y -axis. 2 marks

Question 9

$PQRS$ is a rigid rectangular sheet of metal of negligible thickness.

p , q , r and s are respectively the *perpendicular* distances of P , Q , R and S from a flat surface.



- a. Given that the diagonals of the rectangular metal sheet bisect each other, show that $\overrightarrow{OP} + \overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{OS}$, where O is a point on the flat surface.

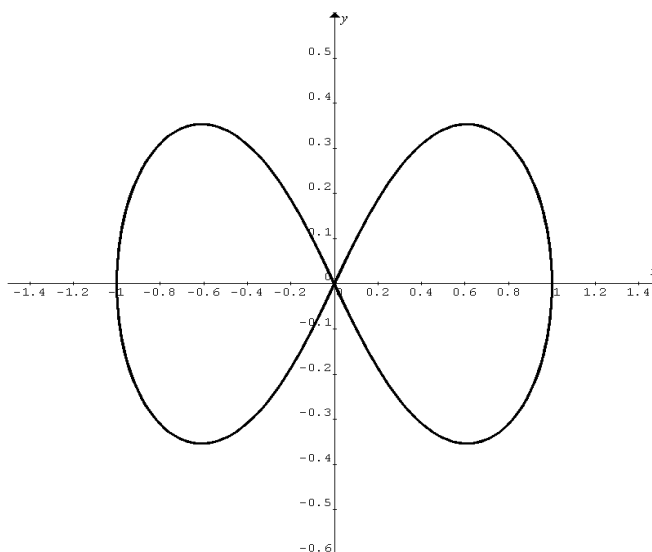
2 marks

- b. **Hence** show that $p + r = q + s$.

3 marks

Question 10

The graph of $(x^2 + y^2)^2 = x^2 - y^2$ is shown below.



- a. Find $\frac{dy}{dx}$ in terms of x and y .

3 marks

- b. Find the exact coordinates of the points on the graph at which the tangents are horizontal.

2 marks

End of Exam 1