INSIGHT

Trial Exam Paper

2010

SPECIALIST MATHEMATICS

Written examination 1

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes
Writing time: 1 hour

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring sheets of paper, notes of any kind or white out liquid/tape into the examination.
- Calculators are not permitted in this examination.

Materials provided

- The question and answer book of 15 pages with a separate sheet of miscellaneous formulas.
- Working space is provided throughout this book.

Instructions

- Write your **name** in the box provided.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

Question 1

The position of a particle is given by $\underset{\sim}{r} = \sec \frac{t}{2} \underset{\sim}{i} + 2 \tan \frac{t}{2} \underset{\sim}{j}$, where $t \in [0, \pi)$.

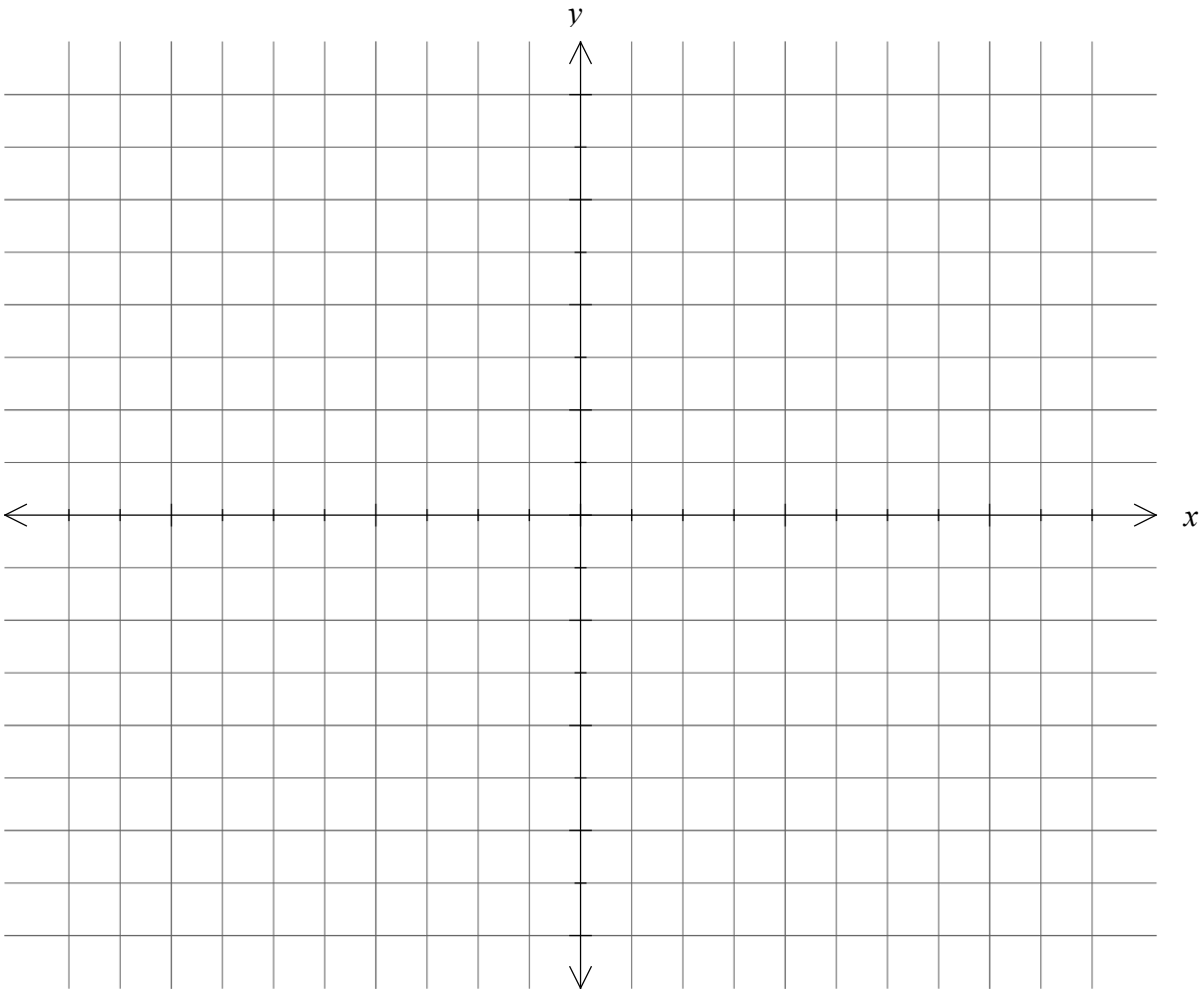
a. Show that the Cartesian equation of the particle is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Find the value of a and b and state the domain and range.

3 marks

Question 1 – continued

b. Sketch the graph of its path on the axes below.

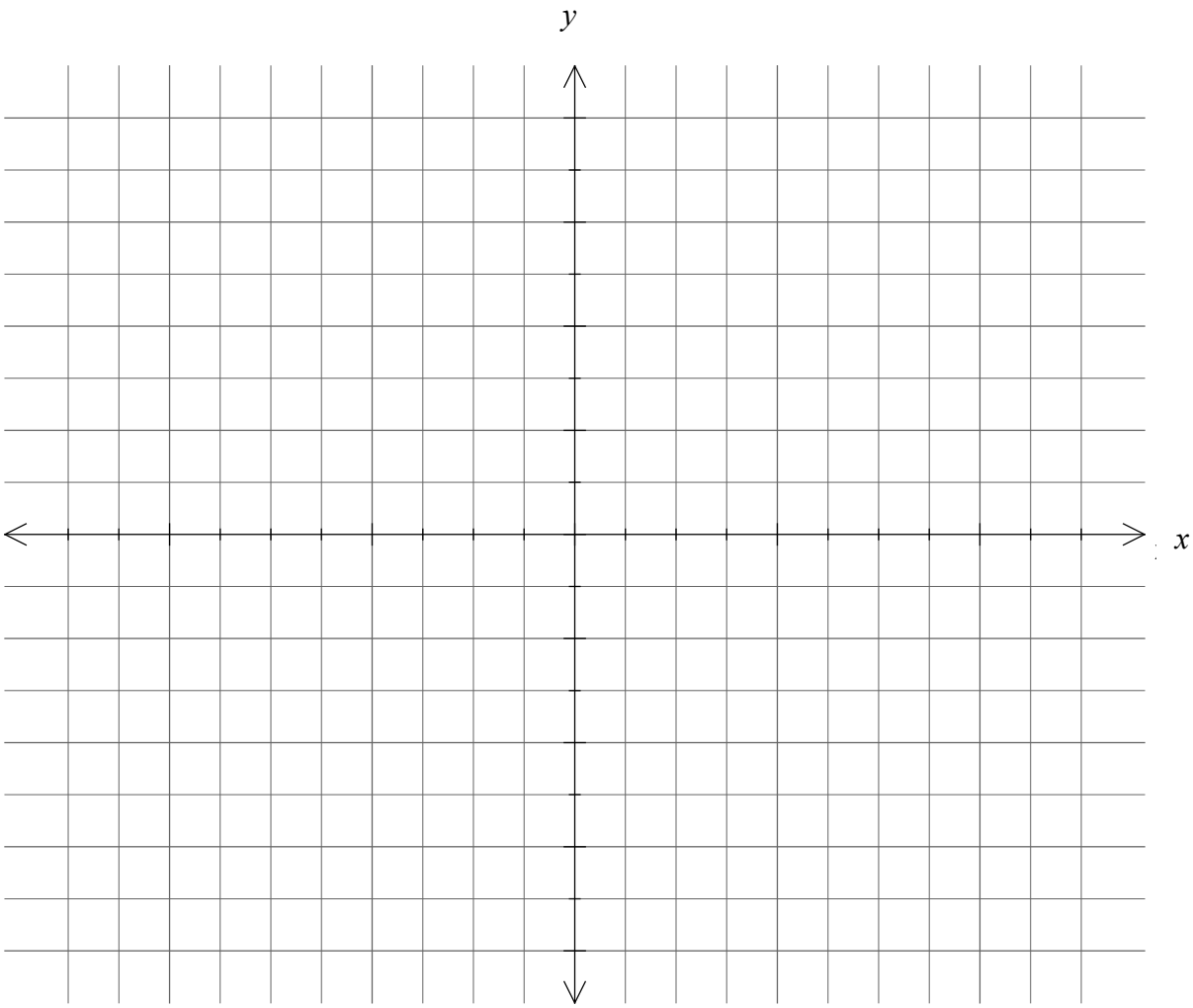


1 mark

End of Question 1
TURN OVER

Question 2

a. On the set of axes below, shade the region enclosed by the graph of $y = \sin^{-1} x$ and the lines $y = 0$ and $x = 1$.



1 mark

Question 2 – continued

b. Find the exact area enclosed by the graph of $y = \sin^{-1} x$ and the lines $y = 0$ and $x = 1$.

2 marks

c. Find the exact volume generated by rotating this area about the y-axis.

3 mark

End of Question 2

TURN OVER

Question 3

Given that $z_1 = 2$ is a solution to the equation $z^2 + (i\sqrt{3} - 3)z + 2 - 2\sqrt{3}\,i = 0,\ z \in \mathrm{C}$

a. Show that the other solution is $z_2 = 1 - \sqrt{3}\,i$.

1 mark

b. Find the square root of z_2 in the form $a + bi$.

3 marks

End of Question 3

Question 4

Find the equation of the tangent to the function $x \log_e y + 2x^2 = 3$ at the point where $x = 1$.

4 marks

End of Question 4

TURN OVER

Question 5

a. Show that $\frac{2x}{4-x^2} = \frac{1}{2-x} - \frac{1}{2+x}$.

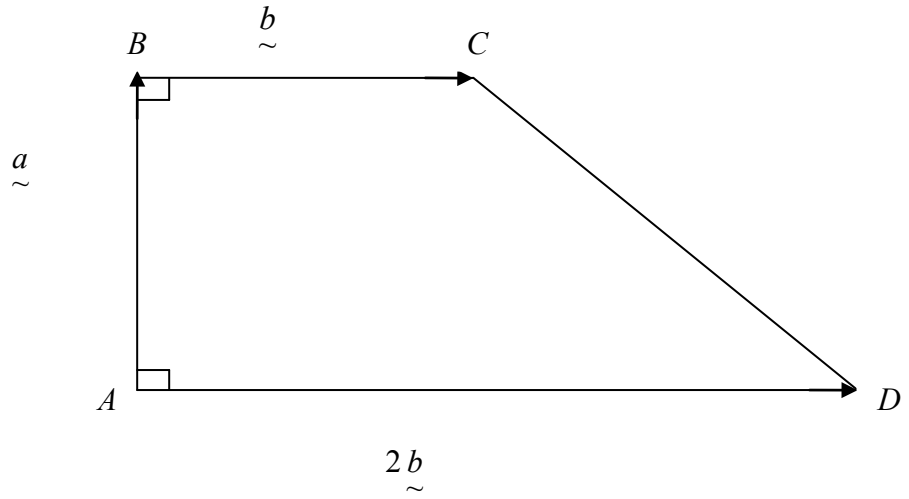
2 marks

b. Hence, or otherwise, find $\int_{-1}^{1} \frac{2x}{4-x^2}.dx$.

2 marks

End of Question 5

Question 6



Given that $|\underset{\sim}{a}| = |\underset{\sim}{b}|$ in the trapezium shown above, use vectors to show that $\triangle ACD$ is right angled.

3 marks

End of Question 6

TURN OVER

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Question 7

A parachutist free-falls from rest and reaches a speed of $2g$ m/s when his parachute is activated.
The acceleration of the parachutist from this point is $(g - 2v)$ m/s^2, where v is the speed of the parachutist t seconds after the parachute is activated.

Five seconds after the parachute is activated, the speed of the parachutist is $\frac{g}{2}(a + be^{-10})$.

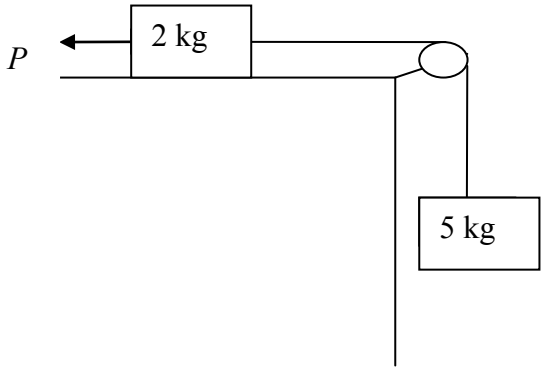
Find the values of a and b.
The acceleration due to gravity is $g = 9.8$ m/s^2.

4 marks

End of Question 7
TURN OVER

Question 8

A 2 kg mass on a horizontal bench is connected by an inextensible string via a smooth pulley to a 5 kg mass, as shown below. The force of P newtons is acting on the 2 kg mass, as shown. The coefficient of friction between the 2 kg mass and the bench is 0.25.



a. Label all forces on the diagram above.

1 mark

b. Show that the minimum value of the force P required to maintain equilibrium is $4.5g$ newtons.

2 marks

c. If $P = 2.5g$, find the exact acceleration of the system.

2 marks

Question 9

Evaluate $\int_{e}^{e^2} \frac{1}{x \ln x} .dx$.

3 marks

End of Question 9

TURN OVER

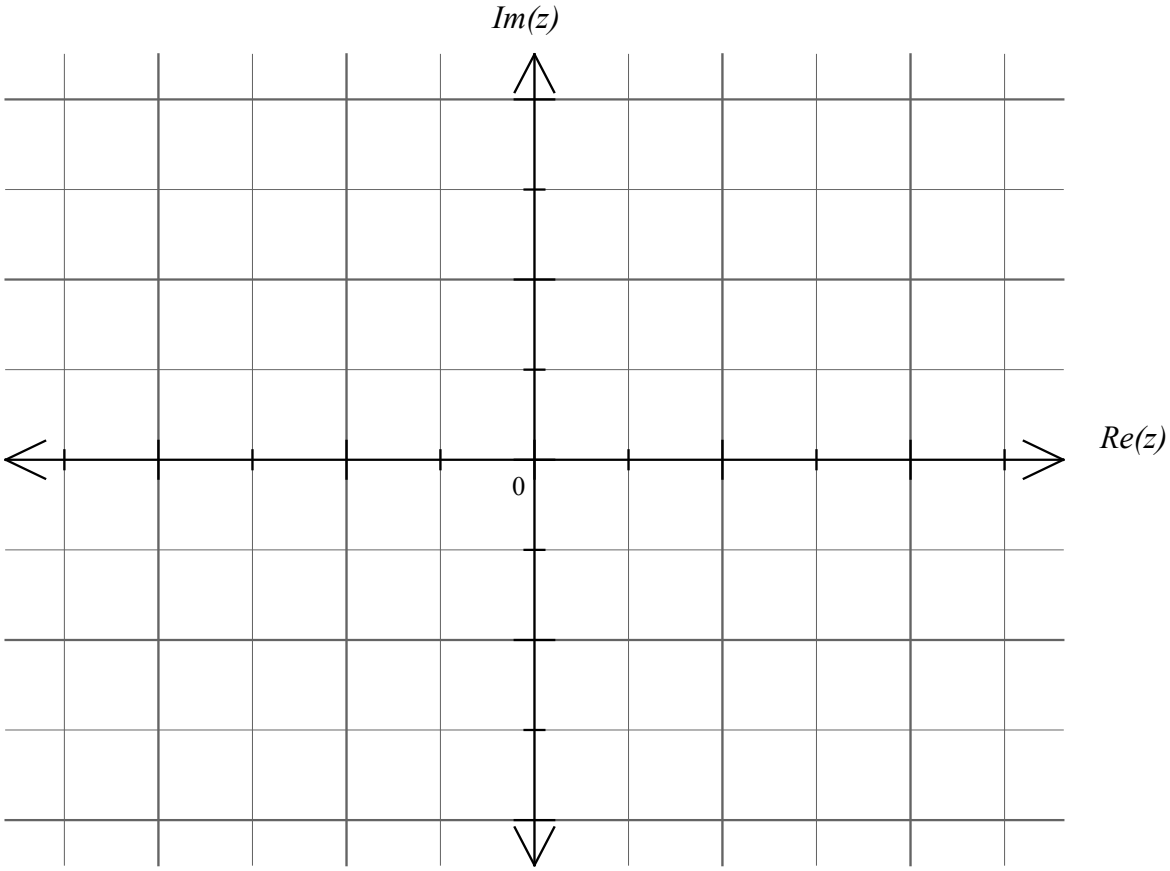
Question 10

$S = \{z: (z + 1)(\bar{z} + 1) = 4,\ z \in \mathrm{C}\}$ and $T = \{z: \mathrm{Arg}\,(z + 1) = \frac{-3\pi}{4},\ z \in \mathrm{C}\}$.

a. Find the Cartesian equations of S and T.

2 marks

b. Show the subsets S and T on the complex plane below.



1 mark

END OF QUESTION AND ANSWER BOOK