

**THE
HEFFERNAN
GROUP**

P.O. Box 1180
Surrey Hills North VIC 3127
Phone 03 9836 5021
Fax 03 9836 5025

info@theheffernangroup.com.au
www.theheffernangroup.com.au

Students Name:.....

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 1

2010

Reading Time: 15 minutes

Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.

All questions should be answered.

There is a total of 40 marks available.

The marks allocated to each of the ten questions are indicated throughout.

Students may not bring any notes or calculators into the exam.

Where more than one mark is allocated to a question, appropriate working must be shown.

Where an exact answer is required to a question, a decimal approximation will not be accepted.

Unless otherwise indicated, diagrams in this exam are not drawn to scale.

The acceleration due to gravity should be taken to have magnitude $g \text{ m/s}^2$ where $g = 9.8$

Formula sheets can be found on pages 12-14 of this exam.

This paper has been prepared independently of the Victorian Curriculum and Assessment Authority to provide additional exam preparation for students. Although references have been reproduced with permission of the Victorian Curriculum and Assessment Authority the publication is in no way connected with or endorsed by the Victorian Curriculum and Assessment Authority.

© THE HEFFERNAN GROUP 2010

This Trial Exam is licensed on a non transferable basis to the purchasing school. It may be copied by the school which has purchased it. This license does not permit distribution or copying of this Trial Exam by any other party.

Question 1

Given that $z = 1 - 2i$ is a solution to the equation $z^3 + 2z^2 - 3z + 20 = 0$, find the other solutions.

2 marks

Question 2

Three points A , B and O are given by $A(2, 1, \sqrt{15})$, $B(2, -4, 0)$ and $O(0, 0, 0)$.

- a. Find the vector $\vec{AB}$ expressed in the form $x\hat{i} + y\hat{j} + z\hat{k}$.

1 mark

- b. Prove that triangle ABO is a right-angled, isosceles triangle.

2 marks

Question 3

Consider the relation $3y^2 + 4x - 2x^2y = 5$.

- a.** Find an expression for $\frac{dy}{dx}$.

2 marks

- b.** Find the gradient of the curve at the point where $x = 1$ in the first quadrant.

2 marks

Question 4

A crate of mass 10kg is sitting on a rough, horizontal floor. A pulling force of 100 newtons, acting at an angle of 30° to the horizontal, is applied to the crate and it accelerates at 8ms^{-2} across the floor.

- a. Show all the forces acting on the crate on the diagram below.



1 mark

- b. Find the value of μ , the coefficient of friction between the crate and the floor.

Given that $g = 9.8\text{ms}^{-2}$, express your answer in the form $\frac{a\sqrt{3}-b}{c}$ where a , b and c are all positive integers.

2 marks

(Question 4 is continued over the page.)

Question 4 (cont'd)

- c. The crate is moved to a different horizontal surface where the coefficient of friction between the crate and the surface is $\frac{1}{10}$. A pulling force of 8 newtons acting at an angle of 30° to the horizontal is applied to the crate.

Explain whether or not the crate is about to move.

2 marks

Question 5

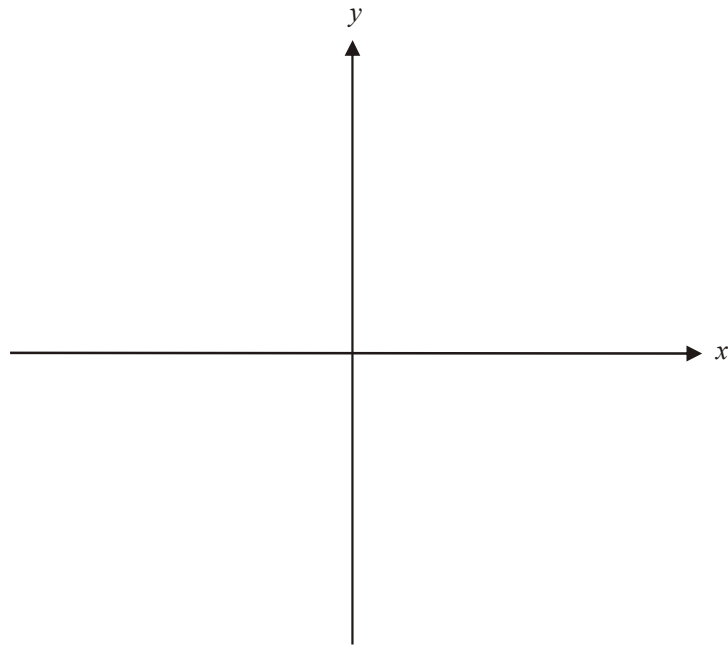
Solve the differential equation $\frac{dy}{dx} = 3\sqrt{4-y^2}$ to find y in terms of x given that $y(0) = 2$.

3 marks

Question 6

Consider the function f with the rule $f(x) = 1 + \cos^{-1}\left(\frac{x}{2}\right)$. The function has its maximal domain.

- a.** Sketch the graph of $y = f(x)$ on the set of axes below.



2 marks

- b.** Find

- i.** the domain of f .

- ii.** the range of f .

1 + 1 = 2 marks

- c.** Find the equation of the normal to f at the point where $x = \sqrt{3}$.

2 marks

Question 8

A particle moves in a straight line and at time t seconds it has acceleration $a \text{ ms}^{-2}$, velocity $v \text{ ms}^{-1}$ and position $x \text{ m}$ from a fixed point. Its acceleration is given by $a = v - 5$. Find x in terms of v given that $v = 6$ when $x = 2$.

3 marks

Question 9

- a.** Express the complex number $2 + 2\sqrt{3}i$ in polar form.

1 mark

- b.** Hence find the solutions to the equation $\sqrt{3}z^2 + \sqrt{2}z - \frac{i}{2} = 0$.

3 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\begin{aligned} \cos^2(x) + \sin^2(x) &= 1 \\ 1 + \tan^2(x) &= \sec^2(x) & \cot^2(x) + 1 &= \operatorname{cosec}^2(x) \\ \sin(x+y) &= \sin(x)\cos(y) + \cos(x)\sin(y) & \sin(x-y) &= \sin(x)\cos(y) - \cos(x)\sin(y) \\ \cos(x+y) &= \cos(x)\cos(y) - \sin(x)\sin(y) & \cos(x-y) &= \cos(x)\cos(y) + \sin(x)\sin(y) \\ \tan(x+y) &= \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)} & \tan(x-y) &= \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)} \\ \cos(2x) &= \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x) \\ \sin(2x) &= 2\sin(x)\cos(x) & \tan(2x) &= \frac{2\tan(x)}{1 - \tan^2(x)} \end{aligned}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$\begin{aligned} z &= x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta \\ |z| &= \sqrt{x^2 + y^2} = r & -\pi < \operatorname{Arg} z \leq \pi \\ z_1 z_2 &= r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2) & \frac{z_1}{z_2} &= \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2) \\ z^n &= r^n \operatorname{cis}(n\theta) & & \text{(de Moivre's theorem)} \end{aligned}$$

Reproduced with permission of the Victorian Curriculum and Assessment Authority, Victoria, Australia.

*This formula sheet has been copied in 2010 from the VCAA website www.vcaa.vic.edu.au
The VCAA publish an exam issue supplement to the VCAA bulletin.*

Calculus

$$\begin{aligned} \frac{d}{dx}(x^n) &= nx^{n-1} & \int x^n dx &= \frac{1}{n+1} x^{n+1} + c, n \neq -1 \\ \frac{d}{dx}(e^{ax}) &= ae^{ax} & \int e^{ax} dx &= \frac{1}{a} e^{ax} + c \\ \frac{d}{dx}(\log_e(x)) &= \frac{1}{x} & \int \frac{1}{x} dx &= \log_e|x| + c \\ \frac{d}{dx}(\sin(ax)) &= a \cos(ax) & \int \sin(ax) dx &= -\frac{1}{a} \cos(ax) + c \\ \frac{d}{dx}(\cos(ax)) &= -a \sin(ax) & \int \cos(ax) dx &= \frac{1}{a} \sin(ax) + c \\ \frac{d}{dx}(\tan(ax)) &= a \sec^2(ax) & \int \sec^2(ax) dx &= \frac{1}{a} \tan(ax) + c \\ \frac{d}{dx}(\sin^{-1}(x)) &= \frac{1}{\sqrt{1-x^2}} & \int \frac{1}{\sqrt{a^2-x^2}} dx &= \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0 \\ \frac{d}{dx}(\cos^{-1}(x)) &= \frac{-1}{\sqrt{1-x^2}} & \int \frac{-1}{\sqrt{a^2-x^2}} dx &= \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0 \\ \frac{d}{dx}(\tan^{-1}(x)) &= \frac{1}{1+x^2} & \int \frac{a}{a^2+x^2} dx &= \tan^{-1}\left(\frac{x}{a}\right) + c \end{aligned}$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b,$$

$$\text{then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

Reproduced with permission of the Victorian Curriculum and Assessment Authority, Victoria, Australia.

*This formula sheet has been copied in 2010 from the VCAA website www.vcaa.vic.edu.au
The VCAA publish an exam issue supplement to the VCAA bulletin.*

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Mechanics

momentum: $\vec{p} = m\vec{v}$

equation of motion: $\vec{R} = m\vec{a}$

friction: $F \leq \mu N$

Reproduced with permission of the Victorian Curriculum and Assessment Authority, Victoria, Australia.

*This formula sheet has been copied in 2010 from the VCAA website www.vcaa.vic.edu.au
The VCAA publish an exam issue supplement to the VCAA bulletin.*