



Victorian Certificate of Education 2009

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

STUDENT NUMBER

Figures

Words

Letter

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SPECIALIST MATHEMATICS

Written examination 1

Friday 30 October 2009

Reading time: 3.00 pm to 3.15 pm (15 minutes)

Writing time: 3.15 pm to 4.15 pm (1 hour)

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are not permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 9 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Working space is provided throughout the book.

Instructions

- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

Find all solutions to the equation $z^4 - z^2 - 6 = 0$, $z \in \mathbb{C}$.

3 marks

Question 2

A 50 kg student stands in a lift which accelerates downwards at a rate of 2 ms^{-2} .

- a.** Find the reaction of the lift floor on the student correct to the nearest newton.

2 marks

A few minutes later the lift accelerates upwards at a rate of 2 ms^{-2} .

- b.** Find the reaction of the lift floor on the student, correct to the nearest newton, during this second stage of the motion.

2 marks

TURN OVER

Question 3

Resolve the vector $5\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ into two vector components, one which is parallel to the vector $-2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and one which is perpendicular to it.

3 marks

Question 4

Given that $\cos(2\theta) = \frac{3}{4}$ where $\theta \in \left(\frac{3\pi}{4}, \pi\right)$, find $\text{cis}(\theta)$ in cartesian form.

4 marks

Question 5

Consider the family of curves defined by the relation $3x^3 - y^2 + kx + 5y - 2xy = 4$ where $k \in \mathbb{R}$.

- a. Verify that every curve in the family passes through the point $(0, 4)$, and find the other point of intersection with the y -axis.

2 marks

- b. Find an expression for $\frac{dy}{dx}$ in terms of x , y and k .

2 marks

- c. **Hence** evaluate the gradient of the curve at the point $(1, 1)$.

2 marks

TURN OVER

Question 6

Find all real values of m such that $y = e^{mx}$ is a solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 10y = 0$.

2 marks

Question 7

A mass has acceleration $a \text{ ms}^{-2}$ given by $a = v^2 - 3$, where $v \text{ ms}^{-1}$ is the velocity of the mass when it has a displacement of x metres from the origin.

Find v in terms of x given that $v = -2$ where $x = 1$.

4 marks

a. Show that $f(x) = \frac{2+x^2}{4-x^2}$ can be written in the form $f(x) = -1 + \frac{6}{4-x^2}$.

b. Find the exact area enclosed by the graph of $f(x) = \frac{2+x^2}{4-x^2}$, the x -axis, and the lines $x = -1$ and $x = 1$.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

TURN OVER

Let $f(x) = \frac{2}{\pi} \arcsin\left(\frac{1}{2}x + 1\right) - 3$.

b. Find $f'(x)$ giving your answer in the form $f'(x) = \frac{a}{\pi\sqrt{bx(x+c)}}$ where a , b and c are integers.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc \sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$

$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$

$\sin(2x) = 2 \sin(x) \cos(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x-y) = \sin(x) \cos(y) - \cos(x) \sin(y)$

$\cos(x-y) = \cos(x) \cos(y) + \sin(x) \sin(y)$

$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$

$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \quad (\text{de Moivre's theorem})$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), \quad x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } \quad v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

TURN OVER

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$