Student Name: _____

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 1

2009 Trial Examination

Reading Time: 15 minutes
Writing Time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, and rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 10 pages.
- Working space is provided throughout the book.

Instructions
- Print your name in the space provided on the top of this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working must be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

a. Vectors $\mathbf{u} = \mathbf{a} + k\,\mathbf{b}$ and $\mathbf{v} = \mathbf{a} - \mathbf{b}$ are perpendicular and $|\mathbf{b}| = 2|\mathbf{a}|$. Find the value of k for which the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$ is 120^o.

3 marks

b. Vectors a, b and c are **unit vectors** such that $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$. Find the value of $\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$.

2 marks

TURN OVER

Question 2

Let $z = x + iy$ and $w = \dfrac{1+i}{1-i}$.

a. Show that $w = i$.

_____

_____

_____

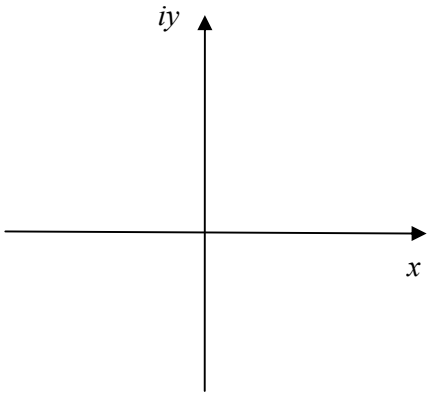
1 mark

b. Find the Cartesian equation of the subset of the complex plane defined by $T = \{ z : z \in |\bar{z} - wz| = 2\sqrt{2} \}$ and sketch its graph.

_____

_____

_____



3 marks

c. Write down all complex numbers $z_1 \in T$ such that $z_1 w \in T$.

_____

_____

_____

2 marks

Question 3

a. Evaluate $\sin\left(\tan^{-1}\frac{1}{\sqrt{3}}\right)-\cos\left(\tan^{-1}\sqrt{3}\right)$.

1 mark

b. If $x=\tan\alpha$, where $\alpha\in\left(0,\frac{\pi}{2}\right)$, find the value of $\sin\left(\tan^{-1}x\right)-\cos\left(\tan^{-1}\frac{1}{x}\right)$.

2 marks

Question 4

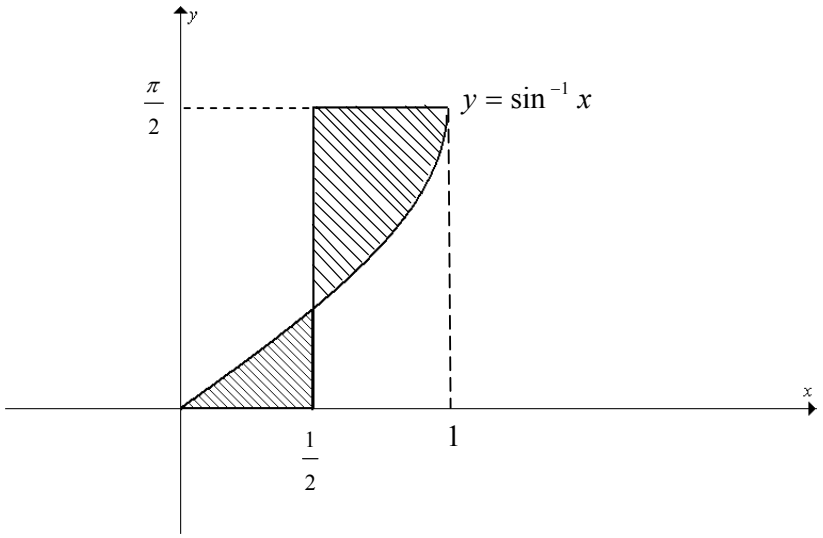
Find the equations of the normals to the curve $y^2-2x^2+2xy-6=0$ at the points where the curve intersects the line $x=1$.

4 marks

TURN OVER

Question 5

Find the exact area of the shaded region.



4 marks

Question 6

Show that $\int_{e^{-3}}^{e^{-2}} \frac{1+\ln x}{x\ln x(1-\ln x)}\,dx = \ln\frac{32}{27}$.

5 marks

TURN OVER

Question 7

A particle is moving so that its position at time t seconds is given by $\mathbf{r}(t) = 2\cos\frac{\pi}{10}t\,\mathbf{i} + 3t\,\mathbf{j}$.

a. Find the Cartesian equation of the path of the particle. State the domain and the range of the path.

2 marks

b. Find the minimum speed of the particle and the time(s) when it occurs during the first 10 seconds of motion.

2 marks

Question 8

A small object is dropped vertically down from the top of a building which is h metres high. It takes 0.4 seconds to travel the last 8 metres before it hits the ground. If the air resistance is negligible, show that the height of the building is: $h = \dfrac{(100+g)^2}{50g}$.

3 marks

TURN OVER

Question 9

An object of mass *M* kg is **pushed** along a rough horizontal surface by a force F_1 inclined α degrees to the horizontal, where $0 < \alpha < \frac{\pi}{2}$. Another object of the same mass *M* kg is **pulled** along the same surface by a force F_2 with the same inclination of α degrees to the horizontal. Both objects are moving with constant velocity.

a. Show that the force F_1 has a greater magnitude than the force F_2.

4 marks

b. Show that the ratio between the two forces is $\frac{F_1}{F_2} = \frac{1 + \mu \tan \alpha}{1 - \mu \tan \alpha}$, where μ is the coefficient of friction.

2 marks

END OF QUESTION AND ANSWER BOOK