SECTION 1: Multiple Choice

ANSWERS

1. D	**2. C**	**3. E**	**4. C**	**5. E**	**6. A**
7. A	**8. E**	**9. A**	**10. D**	**11. C**	**12. D**
13. C	**14. B**	**15. D**	**16. E**	**17. E**	**18. B**
19. A	**20. A**	**21. C**	**22. C**		

SOLUTIONS

Question 1 **Answer D**

$2x+3y+3=0 \Leftrightarrow y=-\frac{2}{3}x-1$

Hence the parallel line will have also a gradient of $-\frac{2}{3}$

$3\underset{\sim}{i}-2\underset{\sim}{j}$ has a gradient of $-\frac{2}{3}$

Question 2 **Answer C**

$(\underset{\sim}{a}.\hat{\underset{\sim}{b}})\hat{\underset{\sim}{b}}=\hat{\underset{\sim}{b}}$, hence $(\underset{\sim}{a}.\hat{\underset{\sim}{b}})=1$

$$\underset{\sim}{a}.\frac{\underset{\sim}{b}}{|\underset{\sim}{b}|}=1$$

Hence $\underset{\sim}{a}.\underset{\sim}{b}=|\underset{\sim}{b}|$

$\underset{\sim}{a}=\underset{\sim}{i}+2\underset{\sim}{j}-2\underset{\sim}{k}$

$\underset{\sim}{b}=2\underset{\sim}{i}+m\underset{\sim}{j}+3\underset{\sim}{k}$

$\underset{\sim}{a}.\underset{\sim}{b}=2+2m-6$

$\quad =-4+2m$

Hence $|\underset{\sim}{b}|=2m-4$

Question 3 **Answer E**

$$\frac{4x^2}{b^2}-\frac{(y-3)^2}{a^2}=1$$

$$\frac{x^2}{\left(\frac{b}{2}\right)^2}-\frac{(y-3)^2}{a^2}=1$$

Equations of asymptotes: $y-3=\pm\dfrac{a}{\left(\frac{b}{2}\right)}x$

$$y-3=\pm\frac{2a}{b}x$$

Hence gradients of asymptotes are $\dfrac{2a}{b}$ and $-\dfrac{2a}{b}$

Product of gradients of the asymptotes is $-\dfrac{4a^2}{b^2}$

Question 4 **Answer C**

Consider $t\in\left(\dfrac{\pi}{3},\dfrac{\pi}{2}\right)$

$x\in\left(\tan\left(2\times\dfrac{\pi}{3}\right),\tan\left(2\times\dfrac{\pi}{2}\right)\right)$ and $y\in\left(\sec\left(2\times\dfrac{\pi}{3}\right),\sec\left(2\times\dfrac{\pi}{2}\right)\right)$

$x\in\left(\tan\left(\dfrac{2\pi}{3}\right),\tan(\pi)\right)$ and $y\in\left(\sec\left(\dfrac{2\pi}{3}\right),\sec(\pi)\right)$

$x\in(-\sqrt{3},0)$ and $y\in(-2,-1)$

The x and y coordinates are both negative in the third quadrant

Question 5 **Answer E**

$|z|+|z+i|=2$ is a general form of an ellipse, so answer is E.

Alternatively, let $z=x+yi$

$$|x+yi|+|x+yi+i|=2$$

$$|x+yi|+|x+(y+1)i|=2$$

$$\sqrt{x^2+y^2}+\sqrt{x^2+(y+1)^2}=2$$

$\sqrt{x^2+(y+1)^2}=2-\sqrt{x^2+y^2}$ squaring gives

$$x^2+y^2+2y+1=4-4\sqrt{x^2+y^2}+x^2+y^2$$

$2y-3=-4\sqrt{x^2+y^2}$ squaring gives

$$4y^2-12y+9=16(x^2+y^2)$$

$16x^2 + 12y^2 + 12y = 9$ This form is also an ellipse.

$16x^2 + 12(y^2 + y + \frac{1}{4} - \frac{1}{4}) = 9$

$16x^2 + 12(y + \frac{1}{2})^2 - 3 = 9$

$16x^2 + 12(y + \frac{1}{2})^2 = 12$

$\frac{4x^2}{3} + (y + \frac{1}{2})^2 = 1$ Ellipse

Question 6 **Answer A**

Let $z = x + yi$ where $x, y \in R^+$

$$\begin{aligned}\frac{\bar{z}}{i^3} &= \frac{x - yi}{i^3} \\ &= \frac{x - yi}{-i} \\ &= \frac{x - yi}{-i} \times \frac{i}{i} \\ &= \frac{xi - yi^2}{1} \\ &= y + xi\end{aligned}$$

Hence $\frac{\bar{z}}{i^3}$ is also in the first quadrant.

z_1 is the only complex number shown in the first quadrant.

Question 7 **Answer A**

From the graph it can be seen $1\frac{1}{2}$ cycles occurs over 4 units

1 cycle occurs over $\frac{8}{3}$ units

Period of $\cot(nx)$ is $\frac{\pi}{n}$

$\frac{\pi}{n} = \frac{8}{3}$

$n = \frac{3\pi}{8}$

Therefore $a = \frac{3\pi}{8}$

Question 8 **Answer E**

Graph of $y = \arcsin(x)$ or $y = \arccos(x)$ has been dilated parallel to the y-axis by a factor of 2.
Eliminate A.
It has also been translated horizontally 1 unit left.
Eliminate C and D
$y = 2\arcsin(x+1)$ should be translated vertically by π units to give $y = 2\arcsin(x+1) + \pi$
Eliminate B
$y = 2\arccos(-x-1)$ has domain $[-2, 0]$ and range $[0, 2\pi]$.
This is obtained by reflecting $y = 2\arccos(x)$ in y-axis: $y = 2\arccos(-x)$, then translating 1 unit left: $y = 2\arccos(-(x+1))$. This may be written as $y = 2\arccos(-x-1)$

Question 9 **Answer A**

There is a repeated linear factor: $\dfrac{2x+1}{(x-9)^2} = \dfrac{A}{x-9} + \dfrac{B}{(x-9)^2}$

Partial fractions may be found in the following way (not required):

$$\frac{2x+1}{(x-9)^2} = \frac{A(x-9)}{(x-9)^2} + \frac{B}{(x-9)^2}$$
$$2x+1 = Ax - 9A + B$$

Equating coefficients of x: $A = 2$ and

$$-9A + B = 1$$
$$-9 \times 2 + B = 1$$
$$B = 19$$

$$\therefore \quad \frac{2x+1}{(x-9)^2} = \frac{2}{x-9} + \frac{19}{(x-9)^2}$$

Question 10 **Answer D**

$\dfrac{dy}{dx} = f(x)$ at $x = m$, given $y = b$ when $x = a$ is

$y = \int f(x)\,dx$

$y = F(x) + c$

$x = a, y = b,\ \text{b} = F(a) + c$

$c = b - F(a)$

$y = F(x) - F(a) + b$

When $x = m$, $y = F(m) - F(a) + b$

Hence $y = \int_a^m f(x) + b$

Question 11 **Answer C**

Let $u = 2 - x \quad \Rightarrow \quad x = 2 - u$

$\dfrac{du}{dx} = -1 \qquad \Rightarrow \quad dx = -du$

$$\int (x - 4\sqrt{2-x})\,dx$$
$$= \int ((2-u) - 4\sqrt{u})\,(-du)$$

$= -\int (2 - u - 4\sqrt{u})\, du$

$= \int (-2 + u + 4\sqrt{u})\, du$

$= \int (u - 2 + 4\sqrt{u})\, du$

Question 12 **Answer D**

$\frac{dy}{dx} = \sin^{-1}(x - 2y)$

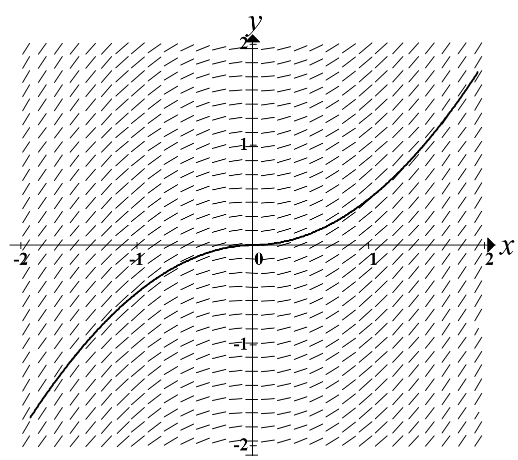
$x_{n+1} = x_n + h,\ h = 0.1$	$y_{n+1} = y_n + hf(x_n, y_n)$, where $f(x_n, y_n) = \frac{dy}{dx}$
2	1
2.1	$1 + 0.1(\sin^{-1}(2 - 2(1)) = 1$
2.2	$1 + 0.1(\sin^{-1}(2.1 - 2(1)) = 1 + 0.1\sin^{-1}(0.1)$

Question 13 **Answer C**

If $\frac{dy}{dx} = \tan(x)$, then $y = -\log_e(\cos(x)) + c$ Eliminate A

If $\frac{dy}{dx} = \sec^2(x)$, then $y = \tan(x) + c$. This has asymptotes at $x = \pm\frac{\pi}{2} = \pm 1.57$ Eliminate B

If $\frac{dy}{dx} = |x|$, then $y = \begin{cases} \frac{1}{2}x^2 + c, & x \ge 0 \\ -\frac{1}{2}x^2 + c, & x < 0 \end{cases}$ This hybrid function satisfies the field diagram as shown



If $\frac{dy}{dx} = \frac{1}{2}x^3$, then $y = \frac{1}{8}x^4 + c$ Eliminate D

If $\frac{dy}{dx} = 2x^4$, then $y = \frac{2}{5}x^5 + c$ This curve passes through $(2, 12.8)$ Eliminate E

Question 14 **Answer B**

$V=\pi\int_a^b \left([f(x)]^2-[g(x)]^2 \right) dx$ where $f(x)=2\cos(2x)$, $g(x)=1$ and $a=0$

Solve $2\cos(2x)=1$ to find b.

$$\cos(2x)=\frac{1}{2}$$
$$2x=\frac{\pi}{3}$$
$$x=\frac{\pi}{6}$$

$$V=\pi\int_0^{\frac{\pi}{6}} \left([2\cos(2x)]^2-1^2 \right) dx$$

$$V=\pi\int_0^{\frac{\pi}{6}} \left(4\cos^2(2x)-1\right) dx$$

$$V=\pi\int_0^{\frac{\pi}{6}} \left([4\cos^2(2x)-2]+1 \right) dx$$

$$V=\pi\int_0^{\frac{\pi}{6}} \left(2\cos(4x)+1\right) dx$$

Question 15 **Answer D**

Options A, B and C all mean $\underset{\sim}{a}$ is a multiple of $\underset{\sim}{b}$, hence indicating $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel.

If two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are parallel then the angle between the vectors is zero degrees.

Hence $\underset{\sim}{a}\,.\,\underset{\sim}{b}=|\underset{\sim}{a}||\underset{\sim}{b}|\cos 0$, since cos 0 = 1, then $\underset{\sim}{a}\,.\,\underset{\sim}{b}=|\underset{\sim}{a}||\underset{\sim}{b}|$ which does not necessarily equal 1.

If $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are unit vectors, then $|\underset{\sim}{a}||\underset{\sim}{b}|$ does equal 1.

Question 16 **Answer E**

$\underset{\sim}{r}(t)=10t\,\underset{\sim}{i}+(2+7t-4t^2)\,\underset{\sim}{j}$

$\underset{\sim}{v}(t)=10\,\underset{\sim}{i}+(7-8t)\,\underset{\sim}{j}$

When the tennis ball strikes the ground the vertical component of its position is zero.

$2+7t-4t^2=0$ Solving this quadratic equation gives $t=2$

$\underset{\sim}{v}(2)=10\,\underset{\sim}{i}-9\,\underset{\sim}{j}$

Question 17 **Answer E**

$$\frac{dy}{dx} = 1 \times e^{\sin(x)} + x \times \cos(x)\, e^{\sin(x)}$$

At $x = 2\pi$, $\quad \frac{dy}{dx} = 1 \times e^{\sin(2\pi)} + 2\pi \times \cos(2\pi)\, e^{\sin(2\pi)}$

$$\frac{dy}{dx} = 1 + 2\pi$$

Applying chain rule for a related rate

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{dy}{dt} = (2\pi + 1) \times 2$$

$$\frac{dy}{dt} = 4\pi + 2 \text{ cm/s}$$

Question 18 **Answer B**

Cyclist is travelling with constant acceleration

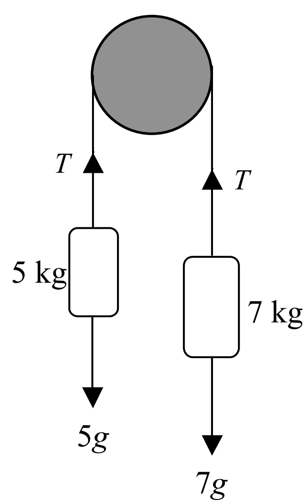
$u = 1, \quad a = 2, \; s = 56$

$v^2 = u^2 + 2as$
$v^2 = 1^2 + 2 \times 2 \times 56$
$v^2 = 225$
$v = 15$ m/s

Change in velocity = 15 – 1 = 14 m/s

Change in momentum
= mass × change in velocity
= 60 × 14
= 840 kg m/s

Question 19 **Answer A**



The 7 kg mass will accelerate downwards.
Equation of motion for 7 kg mass:
$7g - T = 7a$
$T = 7g - 7a \ldots$ (1)

The 5 kg mass will accelerate upwards.
Equation of motion for 5 kg mass:
$T - 5g = 5a$
$T = 5a + 5g$ ... (2)
Equate (1) and (2)
$5a + 5g = 7g - 7a$

$12a = 2g$

$a = \frac{g}{6}$ m/s^2

Question 20 **Answer A**

$\frac{dv}{dt} = -\frac{4}{v}$

$\frac{dt}{dv} = -\frac{v}{4}$

$t = \int -\frac{v}{4}\,dv$

$t = -\frac{v^2}{8} + c$

$t = 0, v = -4 \Rightarrow c = 2$

$t = -\frac{v^2}{8} + 2$

$t - 2 = -\frac{v^2}{8}$

$16 - 8t = v^2$

$v = \pm\sqrt{16 - 8t}$

Therefore $v = -\sqrt{4(4-2t)}$, since when $t = 0, v = -4$

$v = -2\sqrt{(4-2t)}$

Question 21 **Answer D**

$R = ma$, hence $2a = 3 + 2t$

$a = \frac{3}{2} + t$

$\frac{dv}{dt} = \frac{3}{2} + t$

$v = \frac{3}{2}t + \frac{t^2}{2} + c$

$t = 0$, $v = 0$, hence $c = 0$

When $t = 4$, $v = \frac{3}{2}(4) + \frac{16}{2}$

$= 14$ m/s

Question 22 **Answer C**

$N = 5g\cos(30°)$

$5g\sin(30°) - F = 5 \times 2$

$F = 5g\sin(30°) - 10$

$F = \mu N = \mu \times 5g\cos(30°)$

Equating $\mu \times 5g\dfrac{\sqrt{3}}{2} = \dfrac{5g}{2} - 10$ multiply by $\dfrac{2}{5}$

$\mu \times g\sqrt{3} = g - 4$

$\mu = \dfrac{g-4}{g\sqrt{3}}$

$\mu = 0.34$ (correct to 2 decimal places)

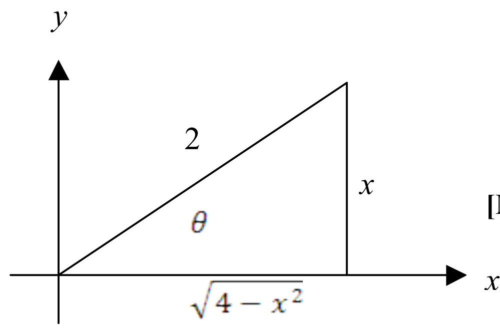
SECTION 2

Question 1

a.

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ $\qquad \sin(\theta) = \dfrac{x}{2}$

$\cos(\theta) = \dfrac{\sqrt{4-x^2}}{2}$



[M1]

$$\sin(2\theta) = 2\left(\frac{x}{2}\right)\frac{\sqrt{4-x^2}}{2}$$

$$\sin(2\theta) = \frac{x\sqrt{4-x^2}}{2}$$ **[A1]**

b.

$\int \sqrt{4-x^2}\,dx$ $\qquad$ Let $x = 2\sin(\theta),\ \dfrac{dx}{d\theta} = 2\cos(\theta)$

$= \int \sqrt{4-4\sin^2(\theta)}\,(2\cos(\theta))\dfrac{d\theta}{dx}\,dx$ **[M1]**

$= \int \sqrt{4(1-\sin^2(\theta)}\,(2\cos(\theta))\,d\theta$

$= \int (2\cos(\theta))(2\cos(\theta))\,d\theta$

$= \int 4\cos^2(\theta)\,d\theta$ **[A1]**

$= \int 4\times\dfrac{1}{2}(1+\cos(\theta))\,d\theta$ **[M1]**

$= 2\int (1+\cos(\theta))\,d\theta$

$= 2\left(\theta + \dfrac{1}{2}\sin(2\theta)\right) + c$ $\qquad$ but $\sin(\theta) = \dfrac{x}{2}$ and from **(a)** $\sin(2\theta) = \dfrac{x\sqrt{4-x^2}}{2}$ **[A1]**

$= 2\sin^{-1}\left(\dfrac{x}{2}\right) + \dfrac{x}{2}\sqrt{4-x^2} + c$

c.

$\int_{-2}^{2} \sqrt{4-x^2}\,dx$ $\qquad$ (by symmetry) **[M1]**

$= 2\int_{0}^{2} \sqrt{4-x^2}\,dx$

$= 2\left[2\sin^{-1}\left(\dfrac{x}{2}\right) + \dfrac{x}{2}\sqrt{4-x^2}\right]_0^2$ **[M1]**

$= 2\left[(2\sin^{-1}(1) + 0) - (2\sin^{-1}(0) + 0)\right]$

$= 4 \times \dfrac{\pi}{2}$

$= 2\pi$ square units **[A1]**

d.

$V = \pi \int x^2 dy$ $\qquad y = \sqrt{4 - x^2}$

$y^2 = 4 - x^2$ **[M1]**

$V = \pi \int_0^2 (4 - y^2) dy$ $\qquad x^2 = 4 - y^2$ **[M1]**

$= \pi \left[4y - \dfrac{y^3}{3}\right]_0^2$

$= \pi(8 - \dfrac{8}{3})$

$= \dfrac{16\pi}{3}$ cubic units **[A1]**

Total 12 marks

Question 2
a.
i.

$z^6 = 64 + 0i$

$z^6 = 64\,\text{cis}(0)$

$z = \left(64\,\text{cis}(0)\right)^{\frac{1}{6}}$ **[A1]**

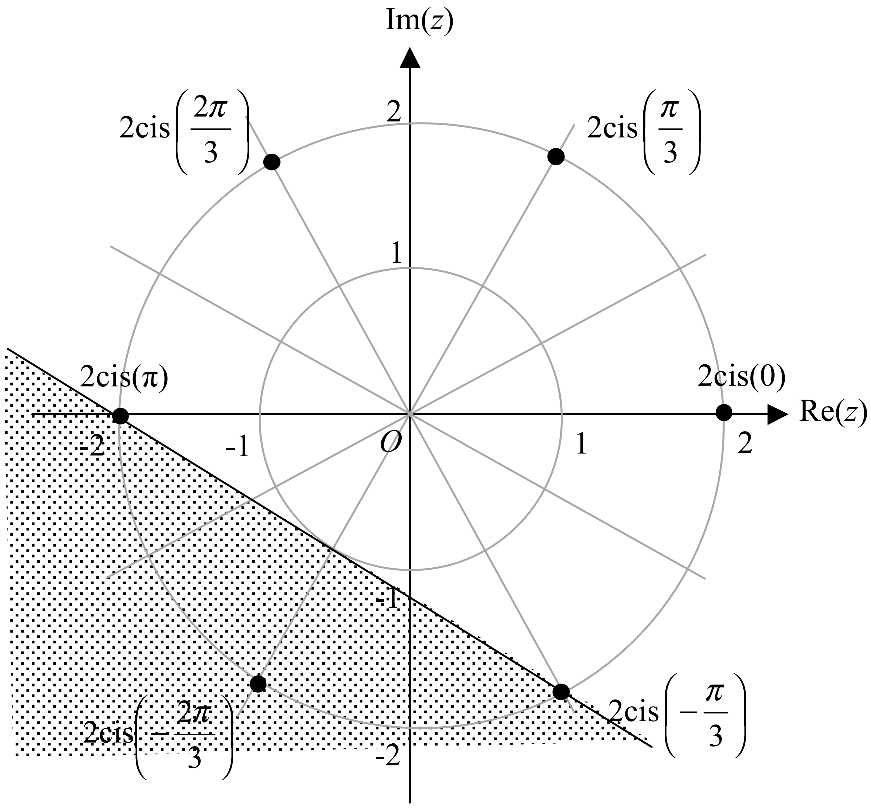
$z = 2\,\text{cis}\left(\frac{1}{6}\left(0 + 2k\pi\right)\right)$ **[M1]**

$z = 2\,\text{cis}\left(\frac{k\pi}{3}\right) \qquad k = 0, \quad \pm 1, \quad \pm 2, \quad 3$

Solutions are: $2\,\text{cis}\left(-\frac{2\pi}{3}\right),\ 2\,\text{cis}\left(-\frac{\pi}{3}\right),\ 2\,\text{cis}(0),\ 2\,\text{cis}\left(\frac{\pi}{3}\right),\ 2\,\text{cis}\left(\frac{2\pi}{3}\right),\ 2\,\text{cis}(\pi)$ **[A1]**

ii.

The solutions to $z^6 - 64 = 0$ will be equally spaced around the circumference of circle $|z| = 2$ as shown.



Solutions plotted **[A1]**

b.

i. $\left| z+1+\sqrt{3}\,i \right| \le | z |$

$\sqrt{(x+1)^2+(y+\sqrt{3})^2} \le \sqrt{x^2+y^2}$ **[M1]**

$x^2+2x+1+y^2+2\sqrt{3}y+3 \le x^2+y^2$

$2\sqrt{3}y \le -2x-4$

$y \le -\frac{1}{\sqrt{3}}x-\frac{2}{\sqrt{3}}$

$y \le -\frac{\sqrt{3}}{3}x-\frac{2\sqrt{3}}{3}$ **[A1]**

ii. Sketch line $y=-\frac{\sqrt{3}}{3}x-\frac{2\sqrt{3}}{3}$

x-intercept, $y=0$ $\qquad$ y-intercept, $x=0$

$0=-\frac{\sqrt{3}}{3}x-\frac{2\sqrt{3}}{3}$ $\qquad$ $y=-\frac{2\sqrt{3}}{3}$

$\frac{\sqrt{3}}{3}x=-\frac{2\sqrt{3}}{3}$ line sketched **[A1]**

$x=-2$

For the region $y \le -\frac{\sqrt{3}}{3}x-\frac{2\sqrt{3}}{3}$,

Test point – select (0, 0) $\qquad 0 \le -\frac{\sqrt{3}}{3}\times 0-\frac{2\sqrt{3}}{3}$ False

The required region is on and below the line.
Shown on diagram for part a. iii. **[A1]**

iii. From graph, the solution to $z^6-64=0$ which clearly lies in region $y < -\frac{\sqrt{3}}{3}x-\frac{2\sqrt{3}}{3}$ is:

$z=2\text{cis}\left(-\frac{2\pi}{3}\right)=2\left(\cos\left(-\frac{2\pi}{3}\right)+\sin\left(-\frac{2\pi}{3}\right)i\right)=2\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=-1-\sqrt{3}\,i$

The solutions on the line $y=-\frac{\sqrt{3}}{3}x-\frac{2\sqrt{3}}{3}$ are:

$z=2\text{cis}\left(-\frac{\pi}{3}\right)=1-\sqrt{3}\,i$ $\qquad$ and $z=2\text{cis}(\pi)=-2+0i$

Hence in Cartesian form $\{\ z : | z+1+\sqrt{3}\,i | \le | z | \} \ \cap\ \{ z : z^6-64=0 \}$ is
$\{ z :\ \ z=-2+0i,\ \ -1-\sqrt{3}\,i,\ \ 1-\sqrt{3}\,i \}$

One correct solution (polar or Cartesian form) **[A1]**
All correct solutions in Cartesian form **[A1]**

Total 10 marks

Question 3

a.

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = \frac{dQ}{dt}_{\mathrm{IN}} - \frac{dQ}{dt}_{\mathrm{OUT}}$$

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = \frac{dV}{dt}\frac{dQ}{dV}_{\mathrm{IN}} - \frac{dV}{dt}\frac{dQ}{dV}_{\mathrm{OUT}}$$ **[M1]**

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = 3 - 5 \times \frac{Q}{180}$$

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = 3 - \frac{Q}{36}$$

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = \frac{108 - Q}{36}$$ **[A1]**

$$\frac{\mathrm{dQ}}{\mathrm{dt}} = -\frac{Q - 108}{36}$$

b.

i.

$$\frac{\mathrm{dt}}{\mathrm{dQ}} = -\frac{36}{Q - 108}$$

$$t = \int -\frac{36}{Q - 108}\, dt$$

$$t = -36\log_e\left|Q - 108\right| + c$$ **[M1]**

$t = 0$, $Q = 120$, hence $0 = -36\log_e\left|120 - 108\right| + c$

$$c = 36\log_e 12$$

$$t = 36\log_e\left|\frac{12}{Q - 108}\right|$$ **[M1]**

$$\frac{t}{36} = \log_e\left|\frac{12}{Q - 108}\right|$$

$$e^{\frac{t}{36}} = \frac{12}{Q - 108}$$

$$Q - 108 = 12e^{-\frac{t}{36}}$$

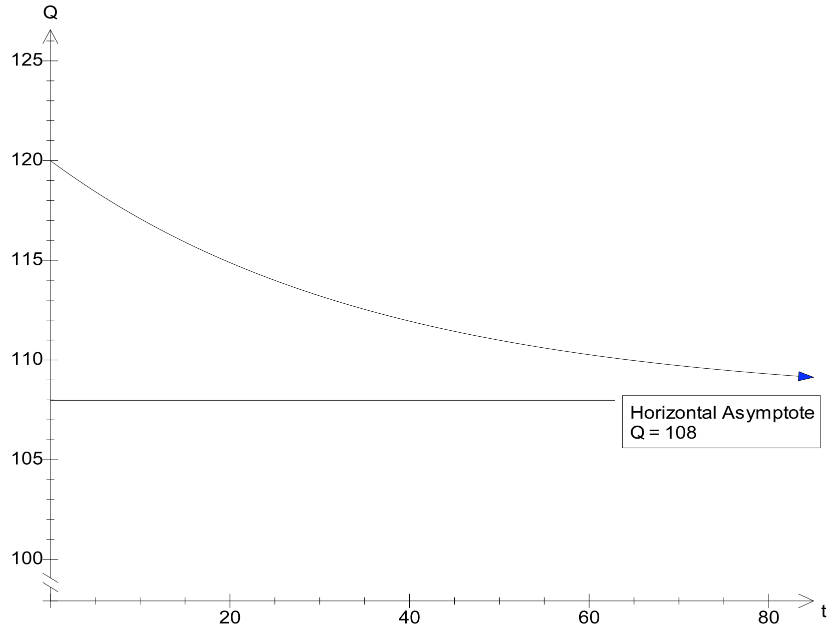
$$Q = 12e^{-\frac{t}{36}} + 108$$ **[A1]**

ii.

When Q = 115, $t = 36\log_e\left|\frac{12}{115 - 108}\right|$

t = 19.4 minutes **[A1]**

c.
i.



Shape **[A1]**
y-intercept, asymptote **[A1]**

ii. As $t \to \infty, e^{-\frac{t}{36}} \to 0$, hence the quantity of green paint stabilizes to 108 litres. **[M1]**
Hence the quantity of red paint stabilizes to 180 – 108 = 72 litres. **[A1]**

Total 10 marks

Question 4

a.

$\underset{\sim}{r}(t) = (1 + 3\cos(\pi t))\underset{\sim}{i} + 3\sin(\pi t)\,\underset{\sim}{j} + 5\,\underset{\sim}{k}$

$\dot{\underset{\sim}{r}}(t) = -3\pi\sin(\pi t)\,\underset{\sim}{i} + 3\pi\cos(\pi t)\,\underset{\sim}{j}$

Initial position: $\underset{\sim}{r}(0) = 4\,\underset{\sim}{i} + 5\,\underset{\sim}{k}$ **[A1]**

Initial velocity: $\dot{\underset{\sim}{r}}(0) = 3\pi\,\underset{\sim}{j}$

Initial speed = 3π m/s **[A1]**

b.

i.

$x = 1 + 3\cos(\pi t)$, $y = 3\sin(\pi t)$ **[M1]**

$\dfrac{x-1}{3} = \cos(\pi t)$, $\dfrac{y}{3} = \sin(\pi t)$

Since $\sin^2(\pi t) + \cos^2(\pi t) = 1$, $\dfrac{(x-1)^2}{9} + \dfrac{y^2}{9} = 1$ **[A1]**

$(x-1)^2 + y^2 = 9$

Domain: Since $x = 1 + 3\cos(\pi t)$ and $-1 \le \cos(\pi t) \le 1$, then $-2 \le x \le 4$ **[A1]**

ii. The plane moves anticlockwise in a circular path of radius 3 metres, at a height of 5 metres above the ground. The centre of the circle, *O*, is one metre from Angela. **[A2]**

c.

$\dot{\underset{\sim}{r}}(t) = -3\pi\sin(\pi t)\,\underset{\sim}{i} + 3\pi\cos(\pi t)\,\underset{\sim}{j}$

$\ddot{\underset{\sim}{r}}(t) = -3\pi^2\cos(\pi t)\,\underset{\sim}{i} - 3\pi^2\sin(\pi t)\,\underset{\sim}{j}$ **[A1]**

$\dot{\underset{\sim}{r}}(t).\ddot{\underset{\sim}{r}}(t) = 9\pi^3\sin(\pi t)\cos(\pi t) - 9\pi^3\sin(\pi t)\cos(\pi t)$ **[M1]**

$\dot{\underset{\sim}{r}}(t).\ddot{\underset{\sim}{r}}(t) = 0$ hence the acceleration is perpendicular to the velocity

d.

i.

$\dot{\underset{\sim}{r}}(12) = 3\pi\,\underset{\sim}{j}$ **[A1]**

d.

ii.

$\ddot{\underset{\sim}{r}}(t) = -9.8\,\underset{\sim}{k}$

$\dot{\underset{\sim}{r}} = -9.8\,t\,\underset{\sim}{k} + \dot{\underset{\sim}{c}}$ **[A1]**

$\dot{\underset{\sim}{r}}(0) = 3\pi\,\underset{\sim}{j}$, hence $\dot{\underset{\sim}{c}} = 3\pi\,\underset{\sim}{j}$

$\dot{\underset{\sim}{r}} = 3\pi\,\underset{\sim}{j} - 9.8\,t\,\underset{\sim}{k}$ **[A1]**

Total 12 marks

Question 5

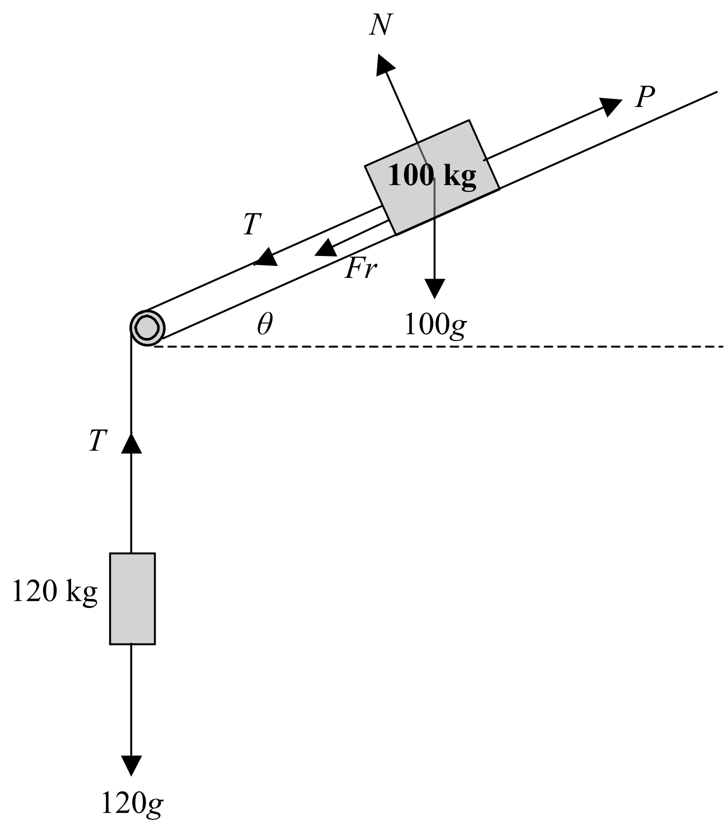
a.

N is the normal reaction of the plane

T is the tension in the rope

Fr is the frictional force on the inclined plane acting to oppose motion

$100g$ and $120g$ are the weight forces of the 100 kg mass and the 120 kg mass respectively



[A1]

b.

Resolving forces around the 100 kg mass perpendicular to the plane to find N.

$N = mg\cos(\theta)$ $\qquad \cos(\theta) = \sqrt{1-\sin^2(\theta)} = \sqrt{1-(0.6)^2} = 0.8$

$N = 100g \times 0.8$

$N = 80g$ **[A1]**

$Fr = \mu N$

$Fr = 0.25 \times 80g$

$Fr = 20g$ newtons **[A1]**

c.

Resolving forces vertically around the 120 kg mass

Equation of motion:

$T-120g=120a$ …. (1)
$T-120g=0$ Since the mass is on the point of moving upwards $a=0$
$T=120g$ newtons **[A1]**

Resolving forces parallel to the plane around the 100 kg mass.
Equation of motion: $P-100g\sin(\theta)-T-Fr=100a$

$P-100g\times\frac{3}{5}-T-20g=100a$
$P-T-80g=100a$ …. (2) **[M1]**
$P-T-80g=100\times 0$
$P=80g+T$
$P=80g+120g$
$P=120g$ newtons **[A1]**

d.
From (1) $T=120a+120g$
Substituting P and T into (2)
$(55t+200g)-(120a+120g)-80g=100a$ **[M1]**
$55t+200g-200g=220a$
$220a=55t$ **[A1]**
$a=\frac{t}{4}$ m/s^2

e.
i.
Use integration to find velocity and displacement when the force is variable
$v=\int\frac{t}{4}\,dt$
$v=\frac{1}{8}t^2+c$
When $t=0$, $v=0$, $\Rightarrow$ $c=0$
$\therefore$ $v=\frac{1}{8}t^2$
When $t=4$, $v=\frac{1}{8}\times 4^2=2$ m/s **[A1]**
The system is travelling at 2 m/s at 4 seconds.

ii.
Finding the distance travelled over $0\le t\le 4$
$x=\int\frac{1}{8}t^2\,dt$

$x = \frac{1}{24}t^3 + c$,

When $t = 0$, $x = 0$, $\Rightarrow \quad c = 0$

$\therefore \quad x = \frac{1}{24}t^3$

When $t = 4$, $x = \frac{1}{24} \times 4^3 = 2\frac{2}{3}$ m

After 4 seconds the 120 kg mass has risen $2\frac{2}{3}$ m. **[A1]**

The system is moving under constant acceleration for the 2 seconds between $4 < t \le 6$

$s = ut + \frac{1}{2}at^2 \qquad a = 1,\ u = 2,\ t = 2$

$s = 2 \times 2 + \frac{1}{2} \times 1 \times 2^2$

$s = 6$ m

The mass rises 6 m between $4 < t \le 6$ seconds **[A1]**

Finding the velocity of the system after 6 seconds.

$v = u + at$

$v = 2 + 1 \times 2$

$v = 4$ m/s

For $6 < t \le 12$ seconds $a = 4 - \frac{t}{2}$ m/s^2

$v = \int \left(4 - \frac{t}{2}\right) dt$

$v = 4t - \frac{1}{4}t^2 + c$

When $t = 6$, $v = 4 \quad \Rightarrow \quad 4 = 4 \times 6 - \frac{1}{4} \times 6^2 + c, \quad c = -11$

$\therefore \quad v = 4t - \frac{1}{4}t^2 - 11$ **[A1]**

Vertical distance travelled over $6 < t \le 12$

$= \int_6^{12} \left(4t - \frac{1}{4}t^2 - 11\right) dt = 24$ m **[A1]**

Total height the 120 kg mass rises in 12 seconds $= 2\frac{2}{3} + 6 + 24 = 32\frac{2}{3}$ metres **[A1]**

Total 14 marks