Year 2009

VCE

Specialist Mathematics Solutions Trial Examination 2

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SECTION 1

ANSWERS

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SECTION 1

Question 1 **Answer E**

$$\frac{2+ki}{k+3i} \times \frac{k-3i}{k-3i} = \frac{2k-3ki^2+k^2i-6i}{k^2+9} = \frac{5k+\left(k^2-6\right)i}{k^2+9}$$

if the imaginary part is zero, $\Rightarrow \quad k^2-6=0 \qquad \Rightarrow k=\pm\sqrt{6}$

Question 2 **Answer D**

For perpendicular $\underset{\sim}{a}.\underset{\sim}{b}=0$

$\underset{\sim}{a}=m\underset{\sim}{i}-\sqrt{m}\,\underset{\sim}{j}-3\underset{\sim}{k}$ and $\underset{\sim}{b}=m\underset{\sim}{i}+\sqrt{m}\,\underset{\sim}{j}+2\underset{\sim}{k}$

$\underset{\sim}{a}.\underset{\sim}{b}=m^2-m-6=\left(m-3\right)\left(m+2\right)=0$

$m=3$ and $m=-2$ but $m\ge 0$

$m=3$ is the only answer

Question 3 **Answer D**

$9x^2+6xa+by^2+9=0$

$$9\left(x^2+\frac{2xa}{3}\qquad\right)+by^2=-9$$

$$9\left(x^2+\frac{2xa}{3}+\frac{a^2}{9}\right)+by^2=a^2-9$$

$$9\left(x+\frac{a}{3}\right)^2+by^2=a^2-9$$

if $\left|a\right|>3$ and $b>9$ this represents an ellipse.

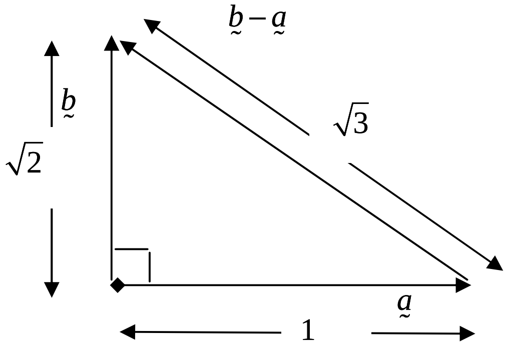
Question 4 **Answer A**

If $\underset{\sim}{a}.\underset{\sim}{b}=0$ then $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}$

and if $\underset{\sim}{a}.\underset{\sim}{a}=1$ then $\underset{\sim}{a}$ is a unit vector

If $\underset{\sim}{b}.\underset{\sim}{b}=2$ then $\left|\underset{\sim}{b}\right|=\sqrt{2}$

It follows that from Pythagoras that $\left|\underset{\sim}{b}-\underset{\sim}{a}\right|=\sqrt{3}$



Question 5 **Answer B**

The domain and range of $y=\cos^{-1}(x)$ are $[-1,1]$ and $[0,\pi]$ respectively.

The domain of $y=4\cos^{-1}\left(\frac{x-3}{2}\right)+1$ is $\left|\frac{x-3}{2}\right|\le 1$ $\Rightarrow$ $-1\le\frac{x-3}{2}\le 1$

$-2\le x-3\le 2$ $\Rightarrow$ $x\in[1,5]$

and the range is $[1,4\pi+1]$, correct answer is **B.**

none of the other alternatives have the correct domain and range.

Question 6 **Answer B**

It follows that

$$z=\left(\sqrt{2}\operatorname{cis}\left(\frac{2\pi}{5}\right)\right)^3=2\sqrt{2}\operatorname{cis}\left(\frac{6\pi}{5}\right)=2\sqrt{2}\operatorname{cis}\left(\frac{6\pi}{5}-2\pi\right)=2\sqrt{2}\operatorname{cis}\left(-\frac{4\pi}{5}\right)$$

Now $\frac{1}{\bar{z}}=\frac{1}{2\sqrt{2}\operatorname{cis}\left(\frac{4\pi}{5}\right)}=\frac{1}{2\sqrt{2}}\operatorname{cis}\left(-\frac{4\pi}{5}\right)=\frac{\sqrt{2}}{4}\operatorname{cis}\left(-\frac{4\pi}{5}\right)$

Question 7 **Answer E**

$$uv=6\operatorname{cis}(\theta)\times r\operatorname{cis}\left(\frac{3\pi}{4}\right)=6r\operatorname{cis}\left(\frac{3\pi}{4}+\theta\right)=12\operatorname{cis}\left(-\frac{7\pi}{12}\right)$$

it follows that $6r=12$, so that $r=2$ and

$\theta+\frac{3\pi}{4}=-\frac{7\pi}{12}$ $\Rightarrow\theta=-\frac{7\pi}{12}-\frac{3\pi}{4}=-\frac{4\pi}{3}$ but to make $-\pi<\theta\le\pi$

$\theta=-\frac{4\pi}{3}+2\pi=\frac{2\pi}{3}$

Question 8 **Answer B**

The line $y=ax$ is an oblique asymptote, with $a<0$.

The graph of $y=\frac{ax^2+b}{x}=ax+\frac{b}{x}$ $\Rightarrow$ $\frac{dy}{dx}=a-\frac{b}{x^2}=0$ has solutions of $x^2=\frac{b}{a}$

Since the graph has turning points, we require $x=\pm\sqrt{\frac{b}{a}}$ to have solutions, since $a<0$, we also require $b<0$.

Question 9 **Answer D**

The volume required is $V_y = \pi \int_a^b \left(x_2^2 - x_1^2 \right) dy$ where x_1 and x_2 are the inner and outer radii respectively. Now $b = 3$ and $a = 0$, since $y = -3\cos(2x) \Rightarrow -\frac{y}{3} = \cos(2x)$

$x_1 = \frac{1}{2}\cos^{-1}\left(-\frac{y}{3}\right)$ and $x_2 = \frac{\pi}{2}$, the volume is

$$V = \pi \int_0^3 \left(\frac{\pi}{4}^2 - \left(\frac{1}{2}\cos^{-1}\left(-\frac{y}{3}\right)\right)^2 \right) dy = \frac{\pi}{4} \int_0^3 \left(\pi^2 - \left(\cos^{-1}\left(-\frac{y}{3}\right)\right)^2 \right) dy$$

Question 10 **Answer A**

The graph of $y = \frac{1}{bx - b - x^2}$ has a denominator of $bx - b - x^2$, now the discriminant of this quadratic is $\Delta = b^2 - 4b = b(b-4)$, so if $b > 4$ or $b < 0$, then $\Delta > 0$, so the graph will have two vertical asymptotes.

Question 11 **Answer C**

Resolving horizontally (1) $F_1 \cos\left(30^0\right) - F_2 \sin\left(30^0\right) - F_3 \sin\left(30^0\right) = 0$

Resolving vertically (2) $F_1 \sin\left(30^0\right) + F_2 \cos\left(30^0\right) - F_3 \cos\left(30^0\right) = 0$

$(1) \Rightarrow F_1 = \tan\left(30^0\right)\left(F_2 + F_3\right) \quad \Rightarrow F_1 = \frac{1}{\sqrt{3}}\left(F_2 + F_3\right)$ so that $\sqrt{3}F_1 = F_2 + F_3$

$(2) \Rightarrow \tan\left(30^0\right)F_1 = F_3 - F_2 \quad \Rightarrow \frac{\sqrt{3}}{3}F_1 = F_3 - F_2$ so that $\sqrt{3}F_1 = 3\left(F_3 - F_2\right)$

$3F_3 - 3F_2 = F_2 + F_3 \quad \Rightarrow F_3 = 2F_2$ and $F_1 = \sqrt{3}F_2$

Question 12 **Answer C**

The symbol is made up from Graphs **I** , **III** and **V.**

Graph I $x^2 + 4y^2 = 16$ or $\frac{x^2}{16} + \frac{y^2}{4} = 1$, an ellipse, centre at the origin, semi-major axes 4, semi-minor axes 2, the outer ellipse.

Graph III $4x^2 + y^2 = 4$ or $x^2 + \frac{y^2}{4} = 1$, an ellipse, centre at the origin, semi-minor axes 1, semi-major axes 2, parallel to the *y*-axis, the inner ellipse.

Graph V $x^2 + 9(y-1)^2 = 9$ or $\frac{x^2}{9} + (y-1)^2 = 1$, an ellipse, centre at $(0,1)$, semi-major axes 3, semi-minor axes 1, parallel to the *y*-axis.

Question 13 **Answer C**

Let $v = 3d\,\text{cis}(\theta)$, where $\theta = \text{Arg}(v)$ is the angle between v and the real axis.

Since u is a rotation of 90^0 anti-clockwise, it follows that $u = 2d\,\text{cis}\left(\theta + \frac{\pi}{2}\right)$, then,

$$\frac{u}{v} = \frac{2d\,\text{cis}\left(\theta + \frac{\pi}{2}\right)}{3d\,\text{cis}(\theta)} = \frac{2}{3}\,\text{cis}\left(\frac{\pi}{2}\right) = \frac{2i}{3} \quad \text{or} \quad 3u = 2iv$$

Question 14 **Answer E**

Using $s = ut + \frac{1}{2}at^2$,

For boy 1, $u = 2$, $a = 2$, $x_1 = 2t + t^2$

For boy 2, $u = 4$, $a = 1$ $x_2 = 4t + \frac{1}{2}t^2$, since they are equal $x_1 = x_2$

$$2t + t^2 = 4t + \frac{1}{2}t^2 \quad \text{or} \quad \frac{1}{2}t^2 - 2t = \frac{1}{2}t(t-4) = 0 \;\Rightarrow t = 0 \text{ and } t = 4$$

$$x(4) = 8 + 16 = 16 + 8 = 24\,\text{m}$$

Question 15 **Answer E**

$$\frac{dx}{dt} = \cos\left(\frac{1}{\sqrt{t}}\right)$$

$$x = \int_0^t \cos\left(\frac{1}{\sqrt{u}}\right)du + C \quad \text{now to find } C,\; x = 2 \text{ when } t = 0,$$

$$2 = \int_0^0 \cos\left(\frac{1}{\sqrt{u}}\right)du + C \quad \Rightarrow C = 2$$

$$x = \int_0^t \cos\left(\frac{1}{\sqrt{u}}\right)du + 2 \quad \text{now when} \quad t = 1 \quad x = \int_0^1 \cos\left(\frac{1}{\sqrt{u}}\right)du + 2$$

Question 16 **Answer B**

The solution curves, have the form of hyperbolas, with centre at $(-2, 2)$, the equations are $(x+2)^2 - (y-2)^2 = k$, where k is a positive constant, differentiating implicitly, gives $2(x+2) - 2(y-2)\frac{dy}{dx} = 0$ or $\frac{dy}{dx} = \frac{x+2}{y-2}$

Question 17 **Answer A**

Using Euler's method, with $x_0 = 0$, $y_0 = 1$ $h = \frac{1}{3}$ $\frac{dy}{dx} = f(x) = \log_e(3x+1)$

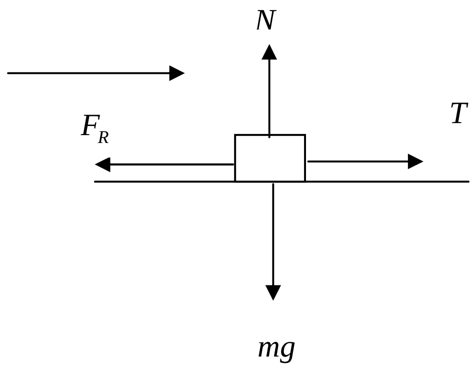
so that $x_1 = \frac{1}{3}$ and $x_2 = \frac{2}{3}$

$y_1 = y_0 + hf(x_0) = 1 + \frac{1}{3}\log_e(1) = 1$

$y_2 = y_1 + hf(x_1) = 1 + \frac{1}{3}\log_e(2)$

$y_3 = y_2 + hf(x_2) = 1 + \frac{1}{3}\log_e(2) + \frac{1}{3}\log_e(3) = 1 + \frac{1}{3}\log_e(6)$

Question 18 **Answer C**



Let T be the horizontal force applied now $T = 10$ N $m = 2$ kg $\mu = 0.25$

resolving parallel to the plane (1) $T - F_R = ma$ and $F_R = \mu N$

resolving perpendicular to the plane (2) $N - mg = 0$

and from (2) $N = mg$ (1) $a = \frac{1}{m}(T - \mu mg)$

$a = \frac{1}{2}(10 - 0.25 \text{x} 2 \text{x} 9.8) = 2.55$, $u = 0$, using $v = u + at$

momentum $mv = 2 \text{ x } 2.55 \text{ x } 2 = 10.2$ kg m/s

Question 19 **Answer C**

$$\int_0^{\frac{1}{4}} \frac{\log_e\left(\cos^{-1}(2x)\right)}{\sqrt{1-4x^2}}\,dx$$

let $u=\cos^{-1}(2x)$ $\quad \dfrac{du}{dx}=\dfrac{-2}{\sqrt{1-4x^2}} \Rightarrow -\dfrac{1}{2}du=\dfrac{1}{\sqrt{1-4x^2}}dx$

terminals when $x=\dfrac{1}{4}$ $\quad u=\cos^{-1}\left(\dfrac{1}{2}\right)=\dfrac{\pi}{3}$ and when $x=0$ $\quad u=\cos^{-1}(0)=\dfrac{\pi}{2}$

the integral becomes $\quad -\dfrac{1}{2}\displaystyle\int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \log_e(u)\,du=\dfrac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \log_e(u)\,du$

Question 20 **Answer E**

The number of rabbits not yet infected is $500-N$ and the initial number infected is 10,

so that $\dfrac{dN}{dt}=kN(500-N)$ $\quad N(0)=10$, the rate $\dfrac{dN}{dt}=49$ when $N=10$

$49=10k(500-10)=4900k \quad \Rightarrow k=\dfrac{1}{100}$, the differential equation is

$$\frac{dN}{dt}=\frac{N(500-N)}{100} \qquad N(0)=10$$

Question 21 **Answer D**

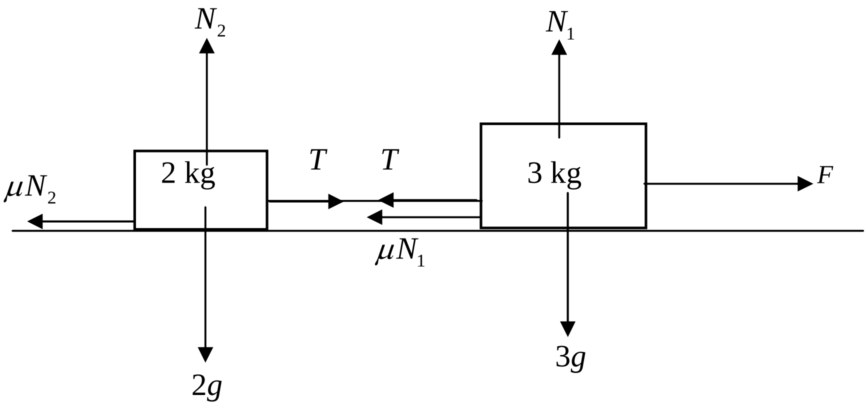
Using constant acceleration formulae $\quad a=-9.8 \quad u=0 \quad s=-150 \quad t=? \quad v=?$

$s=ut+\dfrac{1}{2}at^2 \quad \Rightarrow \quad -150=0-\dfrac{1}{2}\text{x}\,9.8t^2 \quad \Rightarrow t=\sqrt{\dfrac{2\text{x}150}{9.8}}=5.533$

$v=u+at \quad \Rightarrow \quad v=0-9.8\text{x}5.533=-54.22$

The sandbag hits the ground after 5.533 seconds, with a speed of 54.22 m/s.

Question 22 **Answer A**



Resolving horizontally around the 3 kg mass, (1) $F - T - \mu N_1 = 3a$

Resolving vertically around the 3 kg mass, (2) $N_1 - 3g = 0 \quad \Rightarrow \quad N_1 = 3g$

Resolving horizontally around the 2 kg mass, (3) $T - \mu N_2 = 2a$

Resolving vertically around the 2 kg mass, (4) $N_2 - 2g = 0 \quad \Rightarrow \quad N_2 = 2g$

(1) becomes $F - T - 3\mu g = 3a$

(3) becomes $T - 2\mu g = 2a$ adding to eliminate the tension T

$F - 5\mu g = 5a$ but $\mu = \dfrac{1}{7}$ $g = 9.8$ so that $F - 7 = 5a$

If $F > 7$ newtons then $a > 0$, the boxes move with constant acceleration.
All other options are false.

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2

Question 1

a. $\int_{50}^{150} (bt+c)\,dt = 1800$ M1

$\left[\frac{bt^2}{2}+ct\right]_{50}^{150} = 1800$

$\left(\frac{150^2 b}{2}+150c\right)-\left(\frac{50^2 b}{2}+50c\right)=1800$

$b\left(\frac{150^2}{2}-\frac{50^2}{2}\right)+c(150-50)=1800$ A1

$10000b+100c=1800$

$100b+c=18$

b. Since the velocity-time graph is a continuous function

$v(50)=\frac{32}{\pi}\sin^{-1}(1)=16$ and $v(50)=50b+c$ so that

$(1)\quad 50b+c=16$ M1

$(2)\quad 100b+c=18$ from **a.**

subtracting gives $50b=2$

$b=\frac{1}{25}$ and $c=14$

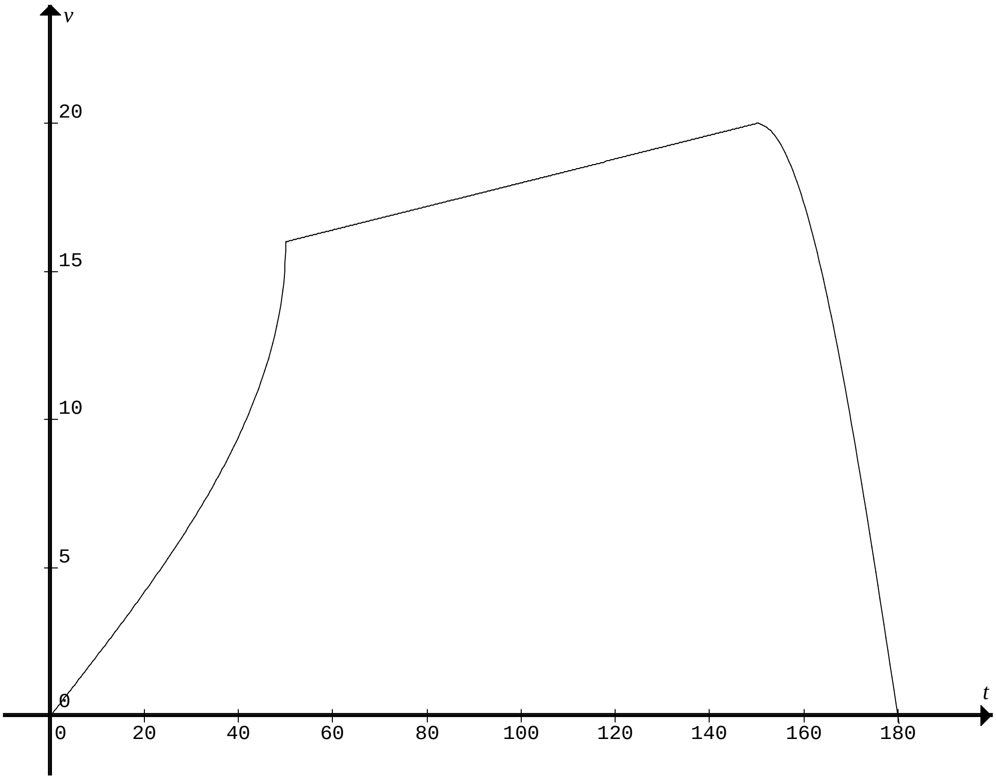
$v(150)=a\cos(0)=a$ $\qquad v(150)=150b+c=6+14=20$

$a=20$ A1

c. $v_{\max}=20\text{m/s}=\frac{20 \times 60 \times 60}{1000}$

$v_{\max}=72$ km/hr A1

d. G1



e. $d = \int_0^{50} \frac{32}{\pi}\sin^{-1}\left(\frac{t}{50}\right)dt + 1800 + \int_{150}^{180} 20\cos\left(\frac{\pi(t-150)}{60}\right)dt$ A1

f. $d = 290.704 + 1800 + 381.972$

$d = 2473\,\text{m}$ A1

g. the retardation is $a = -\frac{20\pi}{60}\sin\left(\frac{\pi(t-150)}{60}\right)$ for $150 < t < 180$

the maximum value is $-\frac{\pi}{3} = -1.472\,\text{m/s}^2$

yes there is cause to be alarmed, the breaking exceeds $1.0\,\text{m/s}^2$ A1

Question 2

a. $\ddot{\underset{\sim}{r}}(t) = -2\underset{\sim}{j} - 9.8\underset{\sim}{k}$ integrating with respect to t A1

$\dot{\underset{\sim}{r}}(t) = -2t\,\underset{\sim}{j} - 9.8t\,\underset{\sim}{k} + \underset{\sim}{C}_1$ but $\dot{\underset{\sim}{r}}(0) = 22\underset{\sim}{i} + 6\underset{\sim}{j} + 9.3\underset{\sim}{k} = \underset{\sim}{C}_1$

$\dot{\underset{\sim}{r}}(t) = 22\underset{\sim}{i} + (6-2t)\underset{\sim}{j} + (9.3-9.8t)\underset{\sim}{k}$ integrating again with respect to t A1

$\underset{\sim}{r}(t) = 22t\underset{\sim}{i} + \left(6t - t^2\right)\underset{\sim}{j} + \left(9.3t - 4.9t^2\right)\underset{\sim}{k} + \underset{\sim}{C}_2$ but $\underline{r}(0) = 1\underset{\sim}{k} = \underset{\sim}{C}_2$

$\underset{\sim}{r}(t) = 22t\underset{\sim}{i} + \left(6t - t^2\right)\underset{\sim}{j} + \left(1 + 9.3t - 4.9t^2\right)\underset{\sim}{k}$ A1

b. strikes the ground when $\underset{\sim}{r}.\underset{\sim}{k} = 0$ or when $1 + 9.3t - 4.9t^2 = 0$ solving gives

$t = -0.1$ and $t = 2$ but $t \geq 0$ so $t = 2\,\text{sec}$ A1

$\underset{\sim}{r}(2) = 44\underset{\sim}{i} + 8\underset{\sim}{j}$ A1

the football hits the ground after 2 seconds, 44 m forward and 8 m to the left.

c. at maximum height $\dot{\underset{\sim}{r}}.\underset{\sim}{k} = 0$ or when $9.3 - 9.8t = 0$ solving gives

$t = 0.95\,\text{sec}$ A1

$\underset{\sim}{r}(0.95) = 20.88\underset{\sim}{i} + 4.79\underset{\sim}{j} + 5.41\underset{\sim}{k}$

the maximum height is 5.41 m above the ground. A1

d. the speed of the football at time t is given by

$\left|\dot{\underset{\sim}{r}}(t)\right| = \sqrt{22^2 + (6-2t)^2 + (9.3-9.8t)^2}$ A1

the maximum value of the speed occurs when

$$\frac{d\left(\left|\dot{\underset{\sim}{r}}(t)\right|\right)}{dt} = 0 = \frac{-4(6-2t) - 19.6(9.3-9.8t)}{\sqrt{22^2 + (6-2t)^2 + (9.3-9.8t)^2}}$$ A1

when $t = 1.03$ and $\dot{\underset{\sim}{r}}(1.03) = 22.68\underset{\sim}{i} + 3.94\underset{\sim}{j} - 0.80\underset{\sim}{k}$

$\left|\dot{\underset{\sim}{r}}(t)\right|_{\min} = \sqrt{22.68^2 + 3.94^2 + (-0.80)^2} = 22.36\,\text{m/s}$ A1

this can also be found by graphing $y = \sqrt{22^2 + (6-2x)^2 + (9.3-9.8x)^2}$

and finding the minimum value of $(1.03, 22.36)$

Question 3

a.i. $\overrightarrow{OA} = -2\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\overrightarrow{OB} = u\underset{\sim}{i} + v\underset{\sim}{j}$ A1

ii. $\left|\overrightarrow{OA}\right| = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$ and $\left|\overrightarrow{OB}\right| = \sqrt{u^2 + v^2}$ A1

since $\left|\overrightarrow{OA}\right| = \left|\overrightarrow{OB}\right| \Rightarrow \sqrt{u^2 + v^2} = \sqrt{8}$ or $u^2 + v^2 = 8$ A1

since OAB is an equilateral triangle angle AOB is 60^0

$$\cos(\theta) = \frac{\overrightarrow{OA}.\overrightarrow{OB}}{\left|\overrightarrow{OA}\right|\left|\overrightarrow{OB}\right|}$$

$$\cos\left(60^0\right) = \frac{1}{2} = \frac{-2u + 2v}{8} \qquad \Rightarrow v - u = 2$$ A1

iii. (1) $v = u + 2$ into (2) $u^2 + v^2 = 8$

$u^2 + (u+2)^2 = 2u^2 + 4u + 4 = 8$

$u^2 + 2u + 1 = 3$ M1

$(u+1)^2 = 3$

$u = -1 \pm \sqrt{3}$ but $u > 0$ so $u = \sqrt{3} - 1$

$v = u + 2$ $\qquad v = 1 + \sqrt{3}$ A1

iv. $\overrightarrow{OC} = \frac{2}{3}\overrightarrow{OD} = \frac{2}{3}\left(\overrightarrow{OA} + \overrightarrow{AD}\right) = \frac{2}{3}\left(\overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}\right)$

$\overrightarrow{OC} = \frac{2}{3}\left(\overrightarrow{OA} + \frac{1}{2}\left(\overrightarrow{OB} - \overrightarrow{OA}\right)\right) = \frac{2}{3}\left(\frac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{OB}\right)\right)$ M1

$\overrightarrow{OC} = \frac{1}{3}\left(\overrightarrow{OA} + \overrightarrow{OB}\right) = \frac{1}{3}\left(\left(-2\underset{\sim}{i} + 2\underset{\sim}{j}\right) + \left(\left(\sqrt{3} - 1\right)\underset{\sim}{i} + \left(1 + \sqrt{3}\right)\underset{\sim}{j}\right)\right)$

$\overrightarrow{OC} = \frac{1}{3}\left(\left(\sqrt{3} - 3\right)\underset{\sim}{i} + \left(3 + \sqrt{3}\right)\underset{\sim}{j}\right)$ A1

b. i. $c=\frac{1}{3}\left(\sqrt{3}-3\right)+\frac{1}{3}\left(3+\sqrt{3}\right)i$

$|c|=\sqrt{\frac{1}{9}\left(\left(\sqrt{3}-3\right)^{2}+\left(3+\sqrt{3}\right)^{2}\right)}$

$|c|=\sqrt{\frac{1}{9}\left(3-6\sqrt{3}+9+9+6\sqrt{3}+3\right)}=\sqrt{\frac{24}{9}}$

$|c|=\frac{2\sqrt{6}}{3}$ A1

ii. $a=-2+2i$, $b=\sqrt{3}-1+\left(1+\sqrt{3}\right)i$ and $S=\{z:|z-c|\le|c|\}$

and $T=\{z:\frac{5\pi}{12}\le \operatorname{Arg}(z)\le\frac{3\pi}{4}\}$.

Now $\operatorname{Arg}(a)=\frac{3\pi}{4}$ so $a\in T$ A1

and $a-c=-\frac{1}{3}\left(3+\sqrt{3}\right)+\frac{1}{3}\left(3-\sqrt{3}\right)i$

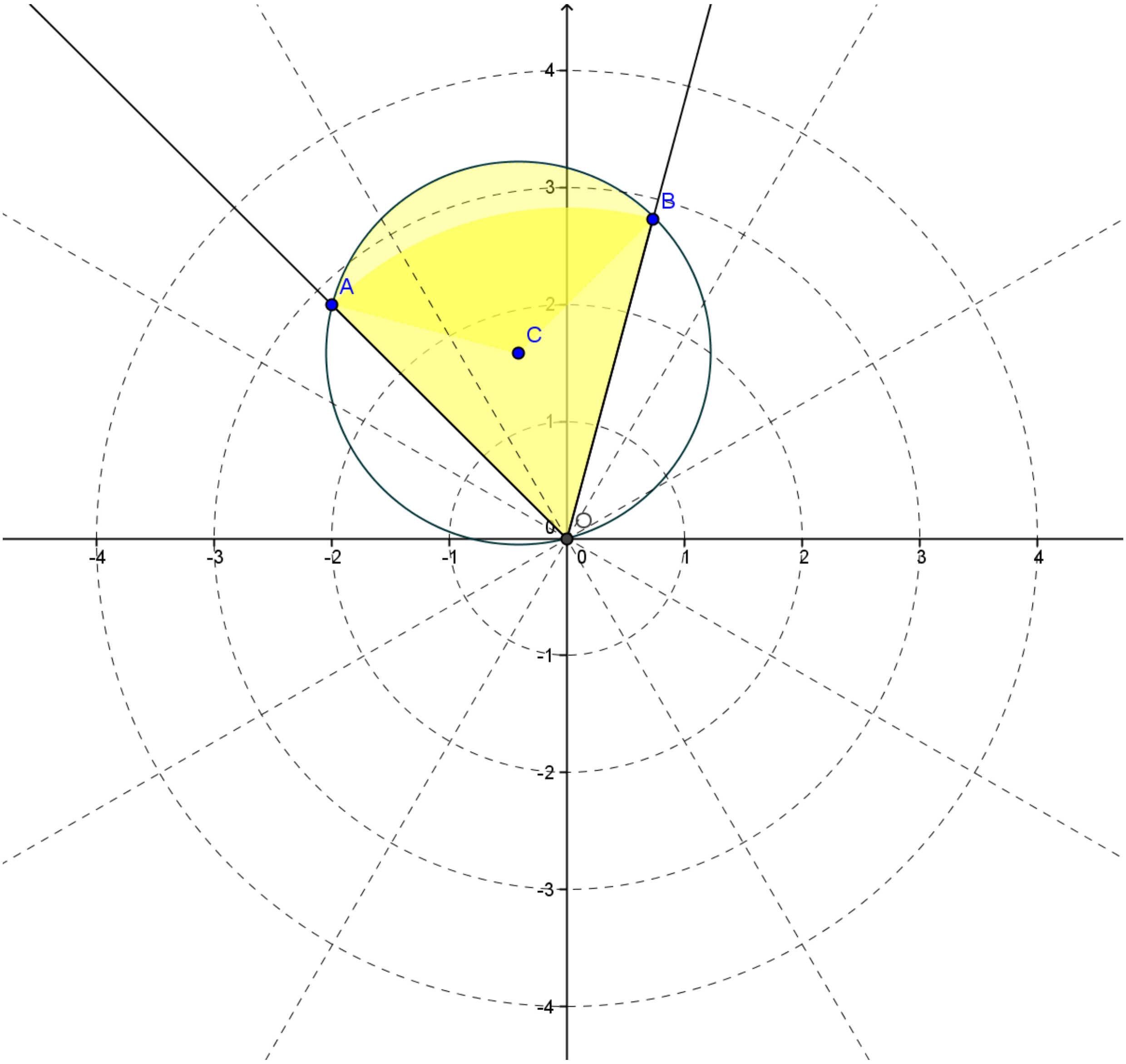
$|a-c|=\left|-\frac{1}{3}\left(3+\sqrt{3}\right)+\frac{1}{3}\left(3-\sqrt{3}\right)i\right|$

$|a-c|=\frac{2\sqrt{6}}{3}=|c|$ A1

so $a\in S$

so $a\in S\cap T$

iii. S is the inside of a circle with center c and radius $|c|=\frac{2\sqrt{6}}{3}$, it passes through the points a, b and the origin. T is the set of points between the rays making angles of 75^0 $\left(\frac{5\pi}{12}\right)$ and 135^0 $\left(\frac{3\pi}{4}\right)$ G2



iv. Since OC bisects OB and OA and $\angle BOA = 60^0 \Rightarrow \angle COB = \angle COA = 30^0$

it follows that $\text{Arg}(c) = 75^0 + 30^0$

$\text{Arg}(c) = \dfrac{7\pi}{12}$ or (105^0) A1

Question 4

a. g is the reflection of the graph of f in the x-axis. A1

b. using implicit differentiation on $y^2 = 4x^3 - x^4$

$2y\dfrac{dy}{dx} = 12x^2 - 4x^3 \quad \Rightarrow \quad y\dfrac{dy}{dx} = 2x^2(3-x)$ M1

$\dfrac{dy}{dx} = \dfrac{2x^2(3-x)}{y} = \dfrac{2x^2(3-x)}{x^{\frac{3}{2}}\sqrt{4-x}} = \dfrac{2\sqrt{x}(3-x)}{\sqrt{4-x}}$ since $x \in (0,4)$ A1

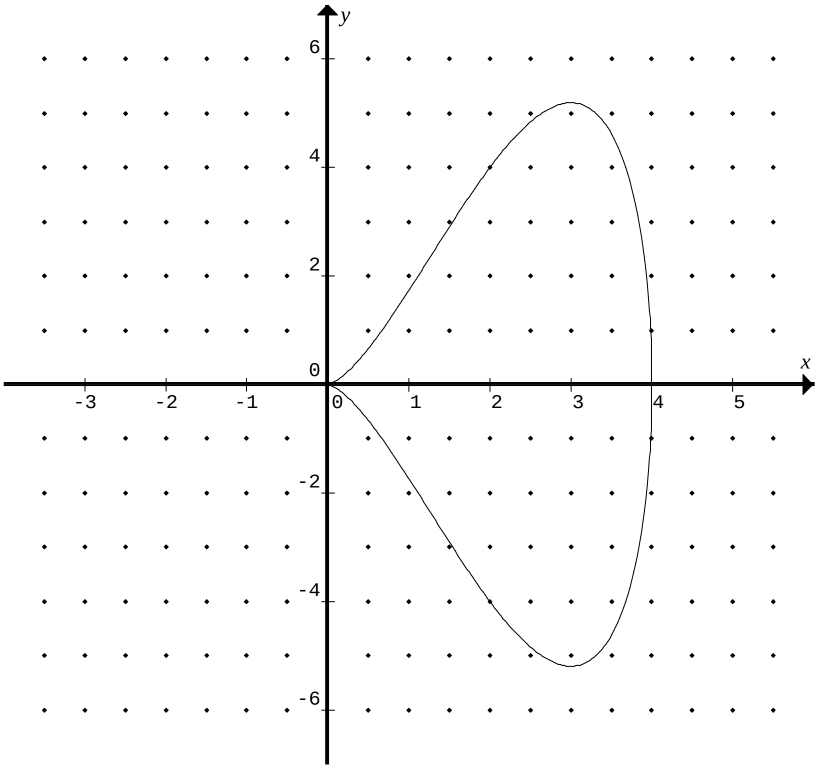
c. for turning points, $\dfrac{dy}{dx} = 0 \Rightarrow x = 3$

$f(3) = 3\sqrt{3}$ and $f''(3) = -2\sqrt{3} < 0$ A1

the point $\left(3, 3\sqrt{3}\right)$ is a maximum turning point. A1

d. as $x \to 4$ the gradients of both functions, becomes infinite. A1

e. correct max $\left(3, 3\sqrt{3}\right)$ min $\left(3, -3\sqrt{3}\right)$ passes through $(0,0)$ and $(4,0)$ G1



f. $\underset{\sim}{r}(t) = 4\sin^2\left(\frac{t}{2}\right)\underset{\sim}{i} + 16\cos\left(\frac{t}{2}\right)\sin^3\left(\frac{t}{2}\right)\underset{\sim}{j}$

$x = 4\sin^2\left(\frac{t}{2}\right) \quad y = 16\cos\left(\frac{t}{2}\right)\sin^3\left(\frac{t}{2}\right)$

$RHS \quad x^3(4-x) = 64\sin^6\left(\frac{t}{2}\right)\left(4 - 4\sin^2\left(\frac{t}{2}\right)\right)$ M1

$= 256\sin^6\left(\frac{t}{2}\right)\left(1 - \sin^2\left(\frac{t}{2}\right)\right)$

$= 256\sin^6\left(\frac{t}{2}\right)\cos^2\left(\frac{t}{2}\right)$

$LHS \quad y^2 = \left(16\cos\left(\frac{t}{2}\right)\sin^3\left(\frac{t}{2}\right)\right)^2 = 256\sin^6\left(\frac{t}{2}\right)\cos^2\left(\frac{t}{2}\right)$ shown A1

g. $x = 3 \quad 4\sin^2\left(\frac{t}{2}\right) = 3$

$\sin\left(\frac{t}{2}\right) = \pm\frac{\sqrt{3}}{2}$ taking positive only

$\frac{t}{2} = \frac{\pi}{3}$

$t = \frac{2\pi}{3}$ A1

h. $x = 4\sin^2\left(\frac{t}{2}\right) \quad \Rightarrow \frac{dx}{dt} = \dot{x} = 4\sin\left(\frac{t}{2}\right)\cos\left(\frac{t}{2}\right) = 2\sin(t)$

$\left.\frac{dx}{dt}\right|_{t=\frac{2\pi}{3}} = 2\sin\left(\frac{2\pi}{3}\right) = \sqrt{3}$. Since $\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx}$ M1

at the maximum the particle is no longer rising, so $\frac{dy}{dt} = \dot{y} = 0$ at $t = \frac{2\pi}{3}$

or alternatively at the maximum $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 0$, the velocity vector at the

maximum point is $\dot{\underset{\sim}{r}}\left(\frac{2\pi}{3}\right) = \sqrt{3}\,\underset{\sim}{i}$ A1

Question 5

a. $ma = mg - R \qquad g = 9.8 \quad m = 9\,\text{kg} \quad R = 0.01v^2$

$9a = 9 \text{x} 9.8 - 0.01v^2$

$a = 9.8 - \dfrac{0.01v^2}{9} = 9.8 - \dfrac{v^2}{900} = \dfrac{8820 - v^2}{900}$ A1

b. Use $a = v\dfrac{dv}{dx} = \dfrac{8820 - v^2}{900}$

$x = \displaystyle\int \frac{900v}{8820 - v^2}\,dv$, where x is the distance fallen A1

$x = -450\log_e\left(8820 - v^2\right) + C \quad \text{but} \quad x = 0 \ \text{ when } \ v = 0$

$0 = -450\log_e\left(8820\right) + C \quad \text{so} \quad C = 450\log_e\left(8820\right)$ A1

$x = 450\log_e\left(8820\right) - 450\log_e\left(8820 - v^2\right)$

$x = 450\log_e\left(\dfrac{8820}{8820 - v^2}\right)$ A1

c. $v = ? \quad \text{when} \quad x = 150$

$150 = 450\log_e\left(\dfrac{8820}{8820 - v^2}\right)$

$\log_e\left(\dfrac{8820}{8820 - v^2}\right) = \dfrac{1}{3}$

$\dfrac{8820}{8820 - v^2} = e^{\frac{1}{3}}$

$8820 - v^2 = 8820e^{-\frac{1}{3}}$

$v = \sqrt{8820\left(1 - e^{-\frac{1}{3}}\right)}$

$v = 50.002 \text{ m/s}$ A1

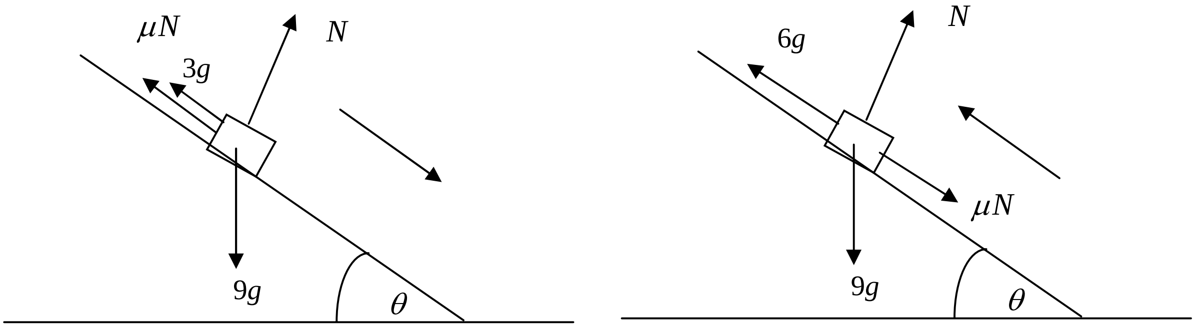
this could easily be obtained graphically.

d. use $a = \frac{dv}{dt} = \frac{8820 - v^2}{900}$

$T = \int_0^{50.002} \frac{900}{8820 - v^2}\, dv$ A1

e. $T = 5.69$ sec A1

f.



in both situations resolving perpendicular to the plane

$N - 9g\cos(\theta) = 0 \quad \Rightarrow \quad N = 9g\cos(\theta)$

when the sandbag is on the point of moving down the plane

$(1)\ 3g + \mu N - 9g\sin(\theta) = 0$ A1

when the sandbag is on the point of moving up the plane

$(2)\ 6g - \mu N - 9g\sin(\theta) = 0$ A1

adding $(1)+(2)$ gives

$9g = 18g\sin(\theta)$

$\sin(\theta) = \frac{1}{2}$ M1

$\theta = 30^0$ A1

and subtracting $(2)-(1)$ gives

$3g = 2\mu N$

$3g = 2\mu \times 9g\cos\left(30^0\right)$ M1

$\mu = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{9}$ A1

END OF SECTION 2 SUGGESTED ANSWERS