

Q1 $z^4 - z^2 - 6 = 0$, $(z^2)^2 - z^2 - 6 = 0$, $(z^2 - 3)(z^2 + 2) = 0$,
 $(z - \sqrt{3})(z + \sqrt{3})(z - i\sqrt{2})(z + i\sqrt{2}) = 0$.
 $\therefore z = \pm\sqrt{3}, \pm i\sqrt{2}$.

Q2 Take upward as the positive direction. Use $\sum \vec{F} = m\vec{a}$.

Q2a $\vec{R} + 50 \times^- 9.8 = 50 \times^- 2$, $\vec{R} = +390$ newtons.

Q2b $\vec{R} + 50 \times^- 9.8 = 50 \times^+ 2$, $\vec{R} = +590$ newtons.

Q3 Unit vector parallel to $-2\tilde{i} - 2\tilde{j} + \tilde{k}$ is

$$\frac{-2\tilde{i} - 2\tilde{j} + \tilde{k}}{\sqrt{(-2)^2 + (-2)^2 + 1^2}} = \frac{1}{3}(-2\tilde{i} - 2\tilde{j} + \tilde{k}).$$

Scalar resolute of $5\tilde{i} + \tilde{j} + 3\tilde{k}$ parallel to $-2\tilde{i} - 2\tilde{j} + \tilde{k}$ is

$$(5\tilde{i} + \tilde{j} + 3\tilde{k}) \cdot \frac{1}{3}(-2\tilde{i} - 2\tilde{j} + \tilde{k}) = -3.$$

$\therefore$ vector resolute of $5\tilde{i} + \tilde{j} + 3\tilde{k}$ parallel to $-2\tilde{i} - 2\tilde{j} + \tilde{k}$ is

$$-3 \times \frac{1}{3}(-2\tilde{i} - 2\tilde{j} + \tilde{k}) = 2\tilde{i} + 2\tilde{j} - \tilde{k}, \text{ and}$$

vector resolute of $5\tilde{i} + \tilde{j} + 3\tilde{k}$ perpendicular to $-2\tilde{i} - 2\tilde{j} + \tilde{k}$ is

$$(5\tilde{i} + \tilde{j} + 3\tilde{k}) - (2\tilde{i} + 2\tilde{j} - \tilde{k}) = 3\tilde{i} - \tilde{j} + 4\tilde{k}.$$

Q4 $\cos(2\theta) = \frac{3}{4}$, $2\cos^2(\theta) - 1 = \frac{3}{4}$, $\cos^2(\theta) = \frac{7}{8}$,

$$\sin^2(\theta) = 1 - \cos^2(\theta) = \frac{1}{8}.$$

Since $\theta \in \left(\frac{3\pi}{4}, \pi\right)$,

$$\therefore \cos(\theta) = -\frac{\sqrt{7}}{2\sqrt{2}} = -\frac{\sqrt{14}}{4} \text{ and } \sin(\theta) = \frac{\sqrt{2}}{4}.$$

$$\therefore \text{cis}(\theta) = \cos(\theta) + i\sin(\theta) = -\frac{\sqrt{14}}{4} + i\frac{\sqrt{2}}{4}.$$

Q5a $3(0)^3 - 4^2 + k(0) + 5(4) - 2(0)(4) = 4$

$\therefore (0, 4)$ satisfies the equation, hence every curve in the family passes through it.

Let $x = 0$ to find the y-intercepts, $y^2 - 5y + 4 = 0$,

$$(y - 4)(y - 1) = 0.$$

$\therefore (0, 1)$ is the other y-intercept.

Q5b Implicit differentiation:

$$9x^2 - 2y \frac{dy}{dx} + k + 5 \frac{dy}{dx} - 2y - 2x \frac{dy}{dx} = 0,$$

$$\therefore (2x + 2y - 5) \frac{dy}{dx} = 9x^2 + k - 2y, \therefore \frac{dy}{dx} = \frac{9x^2 + k - 2y}{2x + 2y - 5}.$$

Q5c If $(1, 1)$ is a point on the curve,

$$3(1)^3 - 1^2 + k(1) + 5(1) - 2(1)(1) = 4, \quad k = -1.$$

At $(1, 1)$, $\frac{dy}{dx} = \frac{9(1)^2 - 1 - 2(1)}{2(1) + 2(1) - 5} = -6.$

Q6 $y = e^{mx}$, $\frac{dy}{dx} = me^{mx}$, $\frac{d^2y}{dx^2} = m^2e^{mx}.$

$$\therefore \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} - 10y = m^2e^{mx} - 3me^{mx} - 10e^{mx} = 0.$$

$$\therefore (m^2 - 3m - 10)e^{mx} = (m - 5)(m + 2)e^{mx} = 0.$$

Since $e^{mx} \neq 0$, $\therefore m = -2$ or 5 .

Q7 $a = v^2 - 3$, $\therefore \frac{1}{2} \frac{d(v^2)}{dx} = v^2 - 3$, $\frac{dx}{d(v^2)} = \frac{1}{2} \times \frac{1}{v^2 - 3},$

$$x = \frac{1}{2} \int \frac{1}{v^2 - 3} d(v^2).$$

$$\therefore 2x = \log_e |v^2 - 3| + c.$$

At $x = 1$, $v = -2$, $\therefore c = 2$ and $2x = \log_e |v^2 - 3| + 2.$

$$\therefore |v^2 - 3| = e^{2(x-1)}, \therefore v^2 - 3 = e^{2(x-1)} \text{ or } v^2 - 3 = -e^{2(x-1)}.$$

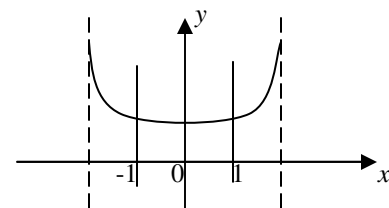
Hence $v^2 = 3 + e^{2(x-1)}$ or $v^2 = 3 - e^{2(x-1)}.$

Only $v^2 = 3 + e^{2(x-1)}$ satisfies the condition that at $x = 1$, $v = -2$.

$$\therefore v = -\sqrt{3 + e^{2(x-1)}}.$$

Q8a $f(x) = \frac{2 + x^2}{4 - x^2} = \frac{-(4 - x^2) + 6}{4 - x^2} = -1 + \frac{6}{4 - x^2}.$

Q8b



$$\text{Area} = \int_{-1}^1 \left(-1 + \frac{6}{4 - x^2}\right) dx = 2 \times \int_0^1 \left(-1 + \frac{6}{4 - x^2}\right) dx$$

$$= 2 \times \int_0^1 \left(-1 + \frac{\frac{3}{2}}{2 - x} + \frac{\frac{3}{2}}{2 + x}\right) dx \quad [\text{Partial fractions}]$$

$$= 2 \left[-x - \frac{3}{2} \log_e |2 - x| + \frac{3}{2} \log_e |2 + x| \right]_0^1 = \log_e 27 - 2.$$

$$\text{Q9a } \frac{dy}{dx} = 4 + (y+2)^2, \quad \frac{dx}{dy} = \frac{1}{4 + (y+2)^2},$$

$$x = \int \frac{1}{4 + (y+2)^2} dy = \frac{1}{2} \int \frac{2}{4 + (y+2)^2} dy = \frac{1}{2} \times \tan^{-1}\left(\frac{y+2}{2}\right) + c.$$

$$\text{Given } y(0) = 0, \therefore c = -\frac{\pi}{8}.$$

$$x = \frac{1}{2} \times \tan^{-1}\left(\frac{y+2}{2}\right) - \frac{\pi}{8}, \quad \tan^{-1}\left(\frac{y+2}{2}\right) = 2x + \frac{\pi}{4},$$

$$\frac{y+2}{2} = \tan\left(2x + \frac{\pi}{4}\right), \therefore y = 2 \tan\left(2x + \frac{\pi}{4}\right) - 2.$$

$$\text{Q9b } x_0 = 0, \quad y_0 = 0, \quad \frac{dy}{dx} = 8.$$

$$x_1 = 0.1, \quad y_1 \approx 0 + 0.1 \times 8 = 0.8.$$

$$\text{Q10a } f(x) = \frac{2}{\pi} \arcsin\left(\frac{1}{2}x + 1\right) - 3 = \frac{2}{\pi} \arcsin\left(\frac{1}{2}(x+2)\right) - 3.$$

Comparing with $\arcsin(x)$, which has a domain of $[-1, 1]$ and a range of $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the domain of $f(x)$ is dilated by a factor of 2 and translated to the left by 2, i.e. $[-4, 0]$, the range of $f(x)$ is dilated by a factor of $\frac{2}{\pi}$ and translated downwards by 3, i.e. $[-4, -2]$.

$$\text{Q10b } f(x) = \frac{2}{\pi} \arcsin\left(\frac{1}{2}x + 1\right) - 3,$$

$$\begin{aligned} f'(x) &= \frac{2}{\pi} \times \frac{1}{\sqrt{1 - \left(\frac{1}{2}x + 1\right)^2}} \times \frac{1}{2} = \frac{1}{\pi \sqrt{1 - \left(\frac{1}{2}x + 1\right)^2}} \\ &= \frac{1}{\pi \sqrt{-\frac{1}{4}x(x+4)}} = \frac{2}{\pi \sqrt{-x(x+4)}}. \end{aligned}$$

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