2009 Specialist Maths Trial Exam 2 Solutions

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Section 1

1	2	3	4	5	6	7	8	9	10	11
D	C	E	D	B	E	B	C	A	C	A

12	13	14	15	16	17	18	19	20	21	22
C	B	A	C	C	D	A	C	A	A	D

Q1 Graph D has $y = 0.5$ as asymptote. Its equation is in the form $y = \dfrac{1}{ax^2 + bx + c} + 0.5$. D

Q2 Expand to obtain

$\left(\sec^2(x+y) - \tan^2(x+y)\right)\left(\operatorname{cosec}^2(x+y) - \cot^2(x+y)\right)$

$= 1 \times 1 = 1$. C

Q3 Equation of inverse: $x = \dfrac{k\pi}{2} - \tan^{-1} y$, $\tan^{-1} y = \dfrac{k\pi}{2} - x$,

$y = \tan\left(\dfrac{k\pi}{2} - x\right)$, $\therefore f^{-1}(x) = \tan\left(\dfrac{k\pi}{2} - x\right)$.

Domain: $-\dfrac{\pi}{2} < \dfrac{k\pi}{2} - x < \dfrac{\pi}{2}$, $-\dfrac{\pi}{2} - \dfrac{k\pi}{2} < -x < \dfrac{\pi}{2} - \dfrac{k\pi}{2}$,

$-\dfrac{\pi}{2} - \dfrac{k\pi}{2} < -x < \dfrac{\pi}{2} - \dfrac{k\pi}{2}$, $\dfrac{\pi}{2} + \dfrac{k\pi}{2} > x > -\dfrac{\pi}{2} + \dfrac{k\pi}{2}$,

$\dfrac{(k-1)\pi}{2} < x < \dfrac{(k+1)\pi}{2}$. E

Q4 For $\cos^{-1}\left(\tan\left(x + \dfrac{\pi}{4}\right)\right)$ to be defined, $-1 \le \tan\left(x + \dfrac{\pi}{4}\right) \le 1$,

$\dfrac{\pi}{2} \pm n\pi \le x \le \pi \pm n\pi$, $\dfrac{(1 \pm 2n)\pi}{2} \le x \le (1 \pm n)\pi$, where $n = 0, 1, 2, \ldots$. D

Q5 B

$$
\begin{array}{r|l}
 & \tfrac{1}{2}x + \tfrac{1}{3} \\
\hline
6x^2 - 12x + 6 & 3x^3 - 4x^2 - x - 4 \\
 & -\left(3x^3 - 6x^2 + 3x\right) \\
\hline
 & \quad 2x^2 - 4x - 4 \\
 & \quad -\left(2x^2 - 4x + 2\right) \\
\hline
 & \qquad\qquad -6
\end{array}
$$

Q6 $z = i\left(\cos\left(\dfrac{\pi}{2} - \theta\right) + i\sin\left(\dfrac{\pi}{2} + \theta\right)\right) = i(\sin\theta + i\cos\theta)$

$= -\cos\theta + i\sin\theta$.

$\dfrac{1}{z} = \dfrac{1}{-\cos\theta + i\sin\theta} \times \dfrac{\cos\theta + i\sin\theta}{\cos\theta + i\sin\theta} = -(\cos\theta + i\sin\theta)$.

$\therefore Arg\left(\dfrac{1}{z}\right) = \theta - \pi$. E

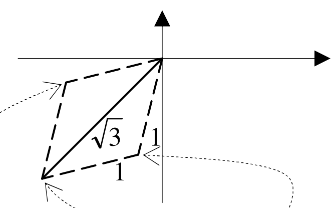
Q7 $z^3 - (1-2i)z^2 + 3z - 3 - 6i = 0$,

$z^2\left(z - (1-2i)\right) + 3\left(z - (1-2i)\right) = 0$,

$\left(z - (1-2i)\right)\left(z^2 + 3\right) = 0$,

$\left(z - (1-2i)\right)\left(z + i\sqrt{3}\right)\left(z - i\sqrt{3}\right) = 0$. B

Q8 $cis\left(-\dfrac{7\pi}{12}\right) - cis\left(\dfrac{\pi}{12}\right) = cis\left(-\dfrac{7\pi}{12}\right) + (-1)cis\left(\dfrac{\pi}{12}\right)$.



$(-1)cis\left(\dfrac{\pi}{12}\right)$ $\qquad -\sqrt{3}cis\left(\dfrac{\pi}{4}\right)$ $\qquad cis\left(-\dfrac{7\pi}{12}\right)$ $\qquad$ C

Q9 A

Q10 $y = 2\cos^{-1}(2x)$, $x = \dfrac{1}{2}\cos\dfrac{y}{2}$.

Area $= 2 \times \displaystyle\int_0^{\pi} x\,dy = 2 \times \int_0^{\pi} \left(\dfrac{1}{2}\cos\dfrac{y}{2}\right)dy = \left[2\sin\dfrac{y}{2}\right]_0^{\pi} = 2$. C

Q11 $\displaystyle\int \left(\dfrac{e^x - e^{-x}}{e^x + e^{-x}} - \dfrac{e^x + e^{-x}}{e^x - e^{-x}}\right)dx$

$= \displaystyle\int \left(\dfrac{1}{u}\right)du - \int \left(\dfrac{1}{v}\right)dv$

$= \log_e u - \log_e v$

$= \log_e\left(\dfrac{u}{v}\right) = \log_e\left(\dfrac{e^x + e^{-x}}{e^x - e^{-x}}\right)$

$= \log_e\left(\dfrac{e^{2x} + 1}{e^{2x} - 1}\right)$. A

Let $u = e^x + e^{-x}$, $\dfrac{du}{dx} = e^x - e^{-x}$.
Let $v = e^x - e^{-x}$, $\dfrac{dv}{dx} = e^x + e^{-x}$.

Q12 $\dfrac{x^2}{2} + y^2 = 1$ and $\dfrac{x^2}{2} + y = c$.

$\therefore c = -y^2 + y + 1$. Let $\dfrac{dc}{dy} = -2y + 1 = 0$. $y = \dfrac{1}{2}$.

Hence maximum $c = -\left(\dfrac{1}{2}\right)^2 + \dfrac{1}{2} + 1 = \dfrac{5}{4}$. C

Q13 $\displaystyle\int \left(\dfrac{1}{2}\sin(2x)\sqrt{1 - \cos x}\right)dx = \int \left(\sin x \cos x \sqrt{1 - \cos x}\right)dx$

$= \displaystyle\int (1-u)u^{\frac{1}{2}}du = \int \left(u^{0.5} - u^{1.5}\right)du$

$= \dfrac{u^{1.5}}{1.5} - \dfrac{u^{2.5}}{2.5} + c$. B

Let $u = 1 - \cos x$, $\dfrac{du}{dx} = \sin x$, $\cos x = 1 - u$.

Q14 $f(x)=\tan^{-1}(x)$, $f'(x)=\dfrac{1}{1+x^2}$, $f''(x)=-\dfrac{2x}{\left(1+x^2\right)^2}$.

$\dfrac{1}{1+x^2}=-\dfrac{2x}{\left(1+x^2\right)^2}$, $1+x^2=-2x$, $x^2+2x+1=0$,

$(x+1)^2=0$, $x=-1$. A

Q15 $y=\int_{-1}^{-2}\left(\tan^{-1}\left(x^2\right)\right)dx+c=-\int_{-2}^{-1}\left(\tan^{-1}\left(x^2\right)\right)dx+c$.

Use calculator to evaluate $\int_{-2}^{-1}\left(\tan^{-1}\left(x^2\right)\right)dx=1.12$.

$\therefore y=-1.12+c$. C

Q16 $25x+25=4(y-2)\dfrac{dy}{dx}$, $25(x+1)=4(y-2)\dfrac{dy}{dx}$

$\int(x+1)dx=\int\dfrac{4}{25}(y-2)\dfrac{dy}{dx}dx$, $\int(x+1)dx=\int\dfrac{4}{25}(y-2)dy$,

$\therefore\dfrac{4}{25}(y-2)^2=(x+1)^2+c$.

Hence $y=2\pm\dfrac{5}{2}\sqrt{(x+1)^2+c}=2\pm\dfrac{5}{2}\sqrt{x^2+2x+1+c}$. C

Q17 $\left(\tilde{a}+\tilde{b}\right).\left(\tilde{c}+\tilde{d}\right)=0$, $\therefore\tilde{a}.\tilde{c}+\tilde{a}.\tilde{d}+\tilde{b}.\tilde{c}+\tilde{b}.\tilde{d}=0$ ……(1)

$\left(\tilde{b}+\tilde{c}\right).\left(\tilde{d}+\tilde{a}\right)=0$, $\therefore\tilde{b}.\tilde{d}+\tilde{b}.\tilde{a}+\tilde{c}.\tilde{d}+\tilde{c}.\tilde{a}=0$ ……(2)

(1) − (2), $\tilde{b}.\tilde{c}-\tilde{c}.\tilde{d}-\tilde{b}.\tilde{a}+\tilde{a}.\tilde{d}=0$,

$\tilde{c}.\left(\tilde{b}-\tilde{d}\right)-\tilde{a}.\left(\tilde{b}-\tilde{d}\right)=0$, $\therefore\left(\tilde{c}-\tilde{a}\right).\left(\tilde{b}-\tilde{d}\right)=0$.

Since $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ and $\tilde{d}$ are independent of each other,

$\tilde{c}-\tilde{a}\neq\tilde{0}$ and $\tilde{b}-\tilde{d}\neq\tilde{0}$.

$\therefore\tilde{c}-\tilde{a}$ and $\tilde{b}-\tilde{d}$ are perpendicular. D

Q18 $\dfrac{d\left(\frac{1}{2}v^2\right)}{dx}=-2(x-3)^3$,

$\therefore\dfrac{1}{2}v^2=\int\left(-2(x-3)^3\right)dx=-\dfrac{(x-3)^4}{2}+c$.

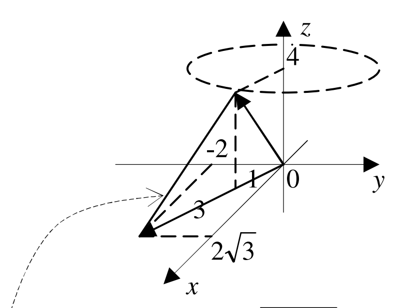
At $x=3+\sqrt{2}$, $v=0$. $\therefore c=2$.

$\therefore\dfrac{1}{2}v^2=2-\dfrac{(x-3)^4}{2}$.

Minimum displacement from O when $v=0$, $\therefore x_{\min}=3-\sqrt{2}$.

Maximum speed occurs when $\dfrac{(x-3)^4}{2}=0$, $\therefore v_{\max}=2$. A

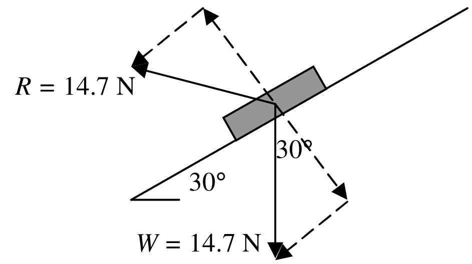
Q19



Closest distance $=\sqrt{3^2+4^2}=5$. C

Q20 $\tilde{i}-2\tilde{j}+2\tilde{k}$ cannot be expressed in terms of $\tilde{i}-2\tilde{j}$ and $-\tilde{j}+2\tilde{k}$. A

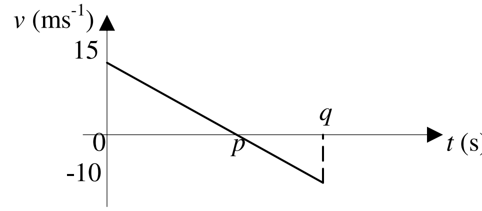
Q21



Resultant force $=2\times14.7\sin30°=14.7$ N down the slope.

$|a|=\dfrac{F}{m}=\dfrac{F}{\frac{W}{g}}=\dfrac{14.7}{\frac{14.7}{9.8}}=9.8\text{ ms}^{-2}$. A

Q22



Distance $=\dfrac{1}{2}\times15p+\dfrac{1}{2}\times10(q-p)=65.0$ ……(1)

$a=gradient=\dfrac{-15}{p}=\dfrac{-10}{q-p}$, $\therefore q-p=\dfrac{2p}{3}$ ……(2)

Substitute (2) into (1), $\dfrac{1}{2}\times15p+\dfrac{1}{2}\times10\times\dfrac{2p}{3}=65.0$.

$\therefore\dfrac{65p}{6}=65.0$, $p=6$.

$\therefore a=\dfrac{-15}{6}=-2.50$. D

Section 2

Q1ai. $f(x)=\dfrac{1}{4}\log_e\left(\dfrac{x^2+x+1}{x^2-x+1}\right)$,

$f'(x)=\dfrac{1}{4}\left(\dfrac{x^2-x+1}{x^2+x+1}\right)\dfrac{\left(x^2-x+1\right)(2x+1)-\left(x^2+x+1\right)(2x-1)}{\left(x^2-x+1\right)^2}$

$=\dfrac{1-x^2}{2\left(x^2+x+1\right)\left(x^2-x+1\right)}$.

Q1aii. $g(x)=\tan^{-1}\left(\dfrac{2x+1}{\sqrt{3}}\right)+\tan^{-1}\left(\dfrac{2x-1}{\sqrt{3}}\right)$,

$g'(x)=\dfrac{2}{\sqrt{3}}\dfrac{1}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2}+\dfrac{2}{\sqrt{3}}\dfrac{1}{1+\left(\frac{2x-1}{\sqrt{3}}\right)^2}$

$=2\sqrt{3}\left(\dfrac{1}{3+(2x+1)^2}+\dfrac{1}{3+(2x-1)^2}\right)$

$=2\sqrt{3}\left(\dfrac{1}{4x^2+4x+4}+\dfrac{1}{4x^2-4x+4}\right)=\dfrac{\sqrt{3}\left(1+x^2\right)}{\left(x^2+x+1\right)\left(x^2-x+1\right)}$.

Q1aiii. $y = f(x) + \frac{1}{2\sqrt{3}} g(x)$, $\frac{dy}{dx} = f'(x) + \frac{1}{2\sqrt{3}} g'(x)$

$= \frac{1-x^2}{2(x^2+x+1)(x^2-x+1)} + \frac{1}{2\sqrt{3}} \frac{\sqrt{3}(1+x^2)}{(x^2+x+1)(x^2-x+1)}$

$= \frac{1}{(x^2+x+1)(x^2-x+1)} = \frac{1}{x^4+x^2+1}$.

Q1b. Area $= \int_0^1 (h(x))dx = \left[f(x) + \frac{1}{2\sqrt{3}} g(x) \right]_0^1$

$= \left[\log_e \sqrt[4]{\frac{x^2+x+1}{x^2-x+1}} + \frac{1}{2\sqrt{3}} \left(\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) \right) \right]_0^1$

$= \log_e \sqrt[4]{3} + \frac{1}{2\sqrt{3}} \left(\frac{\pi}{3} + \frac{\pi}{6} \right) = \frac{1}{4} \log_e 3 + \frac{\pi}{4\sqrt{3}}$

$= \frac{1}{4} \left(\log_e 3 + \frac{\pi}{\sqrt{3}} \right)$.

Q1c. $h'(x) = -\frac{4x^3+2x}{(x^4+x^2+1)^2}$. Use calculator to draw the graph of $h'(x)$ and find the coordinates of the stationary points, $(-0.6426, 0.6315)$ and $(0.6426, 0.6315)$.

Q2a. Let $x = \tan\theta$.

$\int \frac{dx}{\sqrt{1+x^2}} = \int \frac{\sec^2\theta d\theta}{\sqrt{1+\tan^2\theta}} = \int \frac{\sec^2\theta d\theta}{\sec\theta} = \int \sec\theta d\theta$.

Q2bi $\int \sec\theta \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} d\theta$

$= \int \frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta} d\theta$

$= \int \frac{du}{u} = \log_e|u| + c$

$= \log_e|\sec\theta + \tan\theta| + c$.

Let $u = \sec\theta + \tan\theta$.
$\frac{du}{d\theta} = \sec^2\theta + \sec\theta\tan\theta$

Q2bii Area $= \int_0^{2\sqrt{2}} \frac{dx}{\sqrt{1+x^2}}$

$= \int_0^{\tan^{-1}(2\sqrt{2})} \sec\theta d\theta$

$= [\log_e|\sec\theta + \tan\theta|]_0^{\tan^{-1}(2\sqrt{2})}$

$= [\log_e(3+2\sqrt{2})] - [\log_e 1]$

$= \log_e(3+2\sqrt{2})$.

$x = \tan\theta$
$\theta = \tan^{-1}(x)$

$\theta = \tan^{-1}(2\sqrt{2})$
$\tan\theta = 2\sqrt{2}$
$\sec\theta = \sqrt{1+\tan^2\theta} = 3$

Q2c. Volume $= \int_0^{2\sqrt{2}} \pi y^2 dx = \int_0^{2\sqrt{2}} \frac{\pi}{1+x^2} dx = [\pi \tan^{-1}(x)]_0^{2\sqrt{2}}$

$= \pi \tan^{-1}(2\sqrt{2})$.

Q2d. $y = \frac{1}{\sqrt{1+x^2}}$. When $x = 0$, $y = 1$. When $x = 2\sqrt{2}$, $y = \frac{1}{3}$.

Also, $y^2 = \frac{1}{1+x^2}$, $\therefore x^2 = \frac{1}{y^2} - 1$.

Volume $= \int_{\frac{1}{3}}^1 \pi x^2 dy = \int_{\frac{1}{3}}^1 \pi\left(\frac{1}{y^2} - 1\right) dy = \left[\pi\left(-\frac{1}{y} - y \right) \right]_{\frac{1}{3}}^1 = \frac{4\pi}{3}$.

Q2e. $y = \frac{1}{\sqrt{1+x^2}} + 1$, $y^2 = \frac{1}{1+x^2} + \frac{2}{\sqrt{1+x^2}} + 1$.

Volume $= \int_0^{2\sqrt{2}} \pi y^2 dx = \int_0^{2\sqrt{2}} \pi \left(\frac{1}{1+x^2} + \frac{2}{\sqrt{1+x^2}} + 1 \right) dx$

$= \int_0^{2\sqrt{2}} \frac{\pi}{1+x^2} dx + 2\pi \int_0^{2\sqrt{2}} \frac{dx}{\sqrt{1+x^2}} + \int_0^{2\sqrt{2}} \pi dx$

$= \pi \tan^{-1}(2\sqrt{2}) + 2\pi \log_e(3+2\sqrt{2}) + 2\sqrt{2}\pi$.

Q2f. The difference is the volume of a cylinder, radius $2\sqrt{2}$ and height 1.

Difference $= \pi r^2 h = \pi(2\sqrt{2})^2(1) = 8\pi$.

Q3a. $\frac{dx}{dt} = v_x = xe^{-t}$ and $x(0) = 1$.

When $t = 0$, $x = 1$, $\frac{dx}{dt} = 1$.

When $t = 0.1$, $x \approx 1 + 0.1 \times 1 = 1.1$, $\frac{dx}{dt} \approx 1.1 \times e^{-0.1} \approx 0.9953$.

When $t = 0.2$, $x \approx 1.1 + 0.1 \times 0.9953 \approx 1.20$.

Q3bi. $x = e^{1-e^{-t}}$.

$y = \int [-(5t-1)]dt = -\frac{5t^2}{2} + t + c$. When $t = 0$, $y = 2$, $\therefore c = 2$

and $y = -\frac{5t^2}{2} + t + 2$.

$\therefore \tilde{r}(t) = e^{1-e^{-t}} \tilde{i} + \left(-\frac{5t^2}{2} + t + 2 \right) \tilde{j} + 3\tilde{k}$.

Q3bii. Distance from the origin

$D(t) = \sqrt{\left(e^{1-e^{-t}}\right)^2 + \left(-\frac{5t^2}{2} + t + 2 \right)^2 + 3^2}$.

Use calculator to sketch the graph of $D(t)$ and find the time $t = 1.05$ when D is a minimum.

Q3c. Speed $= \sqrt{(xe^{-t})^2 + (-(5t-1))^2}$.

When $t = 0.3$, $x = e^{1-e^{-0.3}}$.

Speed $= \sqrt{\left(e^{1-e^{-0.3}} e^{-0.3}\right)^2 + (5(0.3)-1)^2} = 1.08$.

Q3d. $\tilde{v} = \left(xe^{-t}\right)\tilde{i} - (5t-1)\tilde{j}$.

$\tilde{a} = \frac{d\tilde{v}}{dt} = \frac{d\left(xe^{-t}\right)}{dt}\tilde{i} - \frac{d(5t-1)}{dt}\tilde{j}$

$= \left(\frac{dx}{dt}e^{-t} + x\frac{d\left(e^{-t}\right)}{dt}\right)\tilde{i} - 5\tilde{j}$

$= \left(xe^{-t}e^{-t} - xe^{-t}\right)\tilde{i} - 5\tilde{j}$

$= xe^{-t}\left(e^{-t}-1\right)\tilde{i} - 5\tilde{j}$.

When $t = 0.3$, $\tilde{a} \approx -0.25\tilde{i} - 5\tilde{j}$.

Q3e. The first particle moves in the plane defined by $z = 3$.
The z-coordinate of the second particle at time t:

$z = \int \frac{3}{\sqrt{t}}dt = 6\sqrt{t} + c$. When $t = 0$, $z = 0$.

$\therefore c = 0$ and $z = 6\sqrt{t}$.

Let $6\sqrt{t} = 3$, $t = \frac{1}{4}$.

Q4ai. $z^4 + z^2 + 1 = \left(z^2 + h\right)^2 - kz^2 = z^4 + 2hz^2 + h^2 - kz^2$.

$\therefore h^2 = 1$ and $2h - k = 1$.

Since $h, k \in R^+$, $\therefore h = 1$ and $k = 1$.

Q4aii. $z^4 + z^2 + 1 = \left(z^2+1\right)^2 - z^2 = \left(z^2+1-z\right)\left(z^2+1+z\right) = 0$.

$\therefore z^2 - z + 1 = 0$ or $z^2 + z + 1 = 0$.

Hence $z = \frac{1 \pm i\sqrt{3}}{2}$ or $z = \frac{-1 \pm i\sqrt{3}}{2}$.

Q4b.

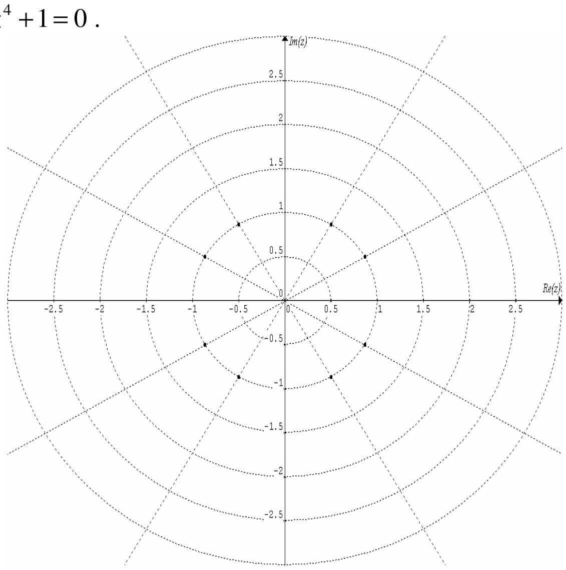
$z^4 - z^2 + 1 = \left(z^2+1\right)^2 - 3z^2 = \left(z^2+1-\sqrt{3}z\right)\left(z^2+1+\sqrt{3}z\right) = 0$.

$\therefore z^2 - \sqrt{3}z + 1 = 0$ or $z^2 + \sqrt{3}z + 1 = 0$.

Hence $z = \frac{\sqrt{3} \pm i}{2}$ or $z = \frac{-\sqrt{3} \pm i}{2}$.

Q4c. $z^8 + z^4 + 1 = \left(z^4 + z^2 + 1\right)\left(z^4 - z^2 + 1\right) = 0$.

$\therefore$ the roots of $z^4 + z^2 + 1 = 0$ and $z^4 - z^2 + 1 = 0$ are the roots of $z^8 + z^4 + 1 = 0$.



Q4d. Let $z_1, z_2, z_3, \ldots\ldots, z_8$ be the roots of $z^8 + z^4 + 1 = 0$.

$\therefore z^8 + z^4 + 1 = (z - z_1)(z - z_2)(z - z_3)\ldots\ldots(z - z_8)$

$= z^8 + \ldots\ldots + z_1 z_2 z_3 \ldots z_8$.

$\therefore z_1 z_2 z_3 \ldots z_8 = 1$

Q4ei. $\left|\mathrm{Im}(z - 2i)\right| \le \sqrt{2}\left|z + 2i\right|$,

$\left|\mathrm{Im}(x + yi - 2i)\right| \le \sqrt{2}\left|x + yi + 2i\right|$,

$\left|\mathrm{Im}(x + (y-2)i)\right| \le \sqrt{2}\left|x + (y+2)i\right|$,

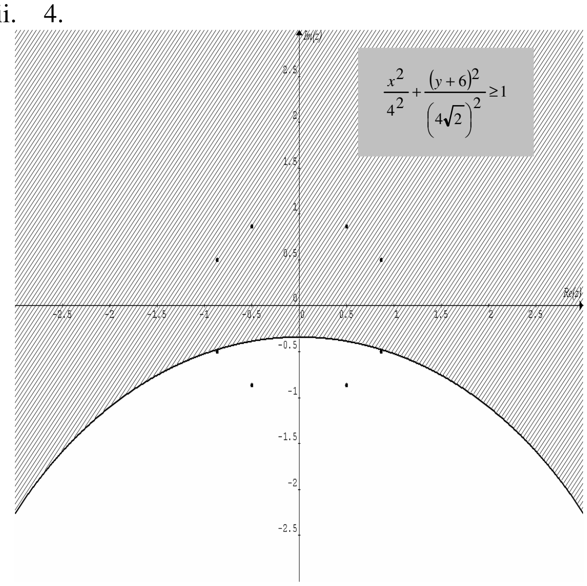
$\left|y - 2\right| \le \sqrt{2}\left|x + (y+2)i\right|$,

$\left|y - 2\right|^2 \le 2\left|x + (y+2)i\right|^2$,

$(y-2)^2 \le 2\left(x^2 + (y+2)^2\right)$, which can be simplified to

$\frac{x^2}{4^2} + \frac{(y+6)^2}{\left(4\sqrt{2}\right)^2} \ge 1$, which is a region in the complex plane on and outside the ellipse $\frac{x^2}{4^2} + \frac{(y+6)^2}{\left(4\sqrt{2}\right)^2} = 1$. The ellipse is centred at $(0,-6)$, and intersects the y-axis at $y = -6 + 4\sqrt{2}$ and $y = -6 - 4\sqrt{2}$. See diagram below.

Q4eii. 4.



Q5a. Let T newtons be the tension in the rope at the pulley, and a ms^{-2} be the acceleration of the rope.

For the left side, $T - 0.50xg = 0.50xa$ …… (1)

For the right side, $0.50(5-x)g - T = 0.50(5-x)a$ …… (2)

(1) + (2), $0.50(5-x)g - 0.50xg = 2.50a$.

$\therefore a = (1 - 0.4x)g$ ms^{-2}.

Q5bi. $a = \left|\frac{d\left(\frac{1}{2}v^2\right)}{dx}\right| = (1 - 0.4x)g$.

Since v increases as x decreases, $\therefore \frac{d\left(\frac{1}{2}v^2\right)}{dx}$ is a negative value.

$\therefore \frac{d\left(\frac{1}{2}v^2\right)}{dx} = -(1 - 0.4x)g$.

Q5bii. $\frac{1}{2}v^2 = \int\left(-(1-0.4x)g\right)dx = -\left(x-0.2x^2\right)g + c$.

When $x = 2.5$, $v = 0.20$, $\therefore c = 0.02 + 1.25g$.

$\therefore \frac{1}{2}v^2 = -\left(x-0.2x^2\right)g + 0.02 + 1.25g$.

When $x = 0$, $v^2 = 0.04 + 2.5g$, $\therefore v = 4.95$ ms^{-1}.

Q5biii. $v^2 = -2\left(x-0.2x^2\right)g + 0.04 + 2.5g$,

$v = \sqrt{-2\left(x-0.2x^2\right)g + 0.04 + 2.5g}$.

$\frac{dx}{dt} = -\sqrt{-2\left(x-0.2x^2\right)g + 0.04 + 2.5g}$.

Note: Since *x* decreases as *t* increases, $\therefore \frac{dx}{dt}$ is a negative rate.

$\frac{dt}{dx} = -\frac{1}{\sqrt{-2\left(x-0.2x^2\right)g + 0.04 + 2.5g}}$,

$t = \int_{2.5}^{0} -\frac{1}{\sqrt{-2\left(x-0.2x^2\right)g + 0.04 + 2.5g}}dx$

$= \int_{0}^{2.5} \frac{1}{\sqrt{-2\left(x-0.2x^2\right)g + 0.04 + 2.5g}}dx = 1.97$ s (By calculator)

Q5biv.

Initial momentum $= (0.50\times 2.5)(^{+}0.20) + (0.50\times 2.5)(^{-}0.20) = 0$.

Final momentum $= (0.50\times 5.0)(^{-}4.95) = {}^{-}12.38$.

|Change in momentum| $= 12.38$ kg ms^{-1}.

Q5ci. Total mass of box and rope $= 15 + 2.5 = 17.5$ kg.

Force of friction $= 0.90\times 15\times 9.8 = 132.3$ N.

Resultant force $= 150 - 132.3 = 17.7$ N.

$a = \frac{17.7}{17.5} = 1.0114 \approx 1.01$ ms^{-2}.

Q5cii. For constant acceleration, average speed $= \frac{u+v}{2}$.

$\therefore \frac{0+v}{2} = 1.0$, $v = 2.0$ ms^{-1}.

$v^2 = u^2 + 2as$, $2.0^2 = 0 + 2(1.0114)s$, $s = 1.98$ m.

Distance travelled $= 1.98$ m.

Q5d. Tension at front end $= 150$ N.

Tension at rear end: $T - 132.3 = 15\times 1.0114$, $T = 147.47$ N.

Difference $= 150 - 147.47 = 2.53$ N.

Alternatively, difference $= (0.5\times 5)1.0114 = 2.53$ N.

Q5e. Friction = pulling force.

$0.90\times m\times 9.8 = 150$, $m = 17$ kg.

Minimum additional mass $= 17 - 15 = 2$ kg.

Please inform mathline@itute.com re conceptual, mathematical and/or typing errors