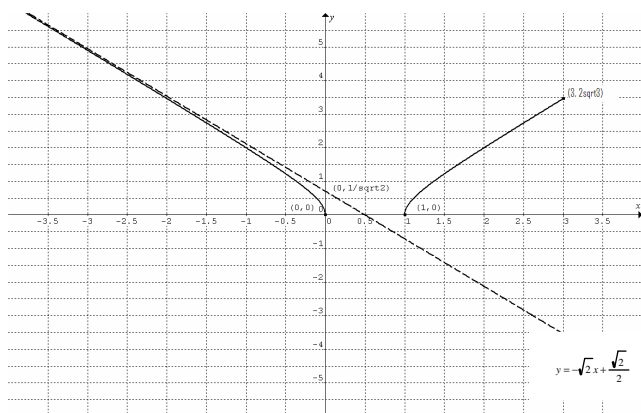


Q1a $y = \sqrt{2x(x-1)}$, $y^2 = 2x(x-1)$, $\frac{y^2}{2} = x^2 - x$,

$$\frac{y^2}{2} = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4}, \left(x - \frac{1}{2}\right)^2 - \frac{y^2}{2} = \frac{1}{4}, \frac{\left(x - \frac{1}{2}\right)^2}{\left(\frac{1}{2}\right)^2} - \frac{\frac{y^2}{2}}{\left(\frac{1}{2}\right)^2} = 1.$$

Asymptote: $y = -\frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}}\left(x - \frac{1}{2}\right) = -\sqrt{2}x + \frac{\sqrt{2}}{2}$.



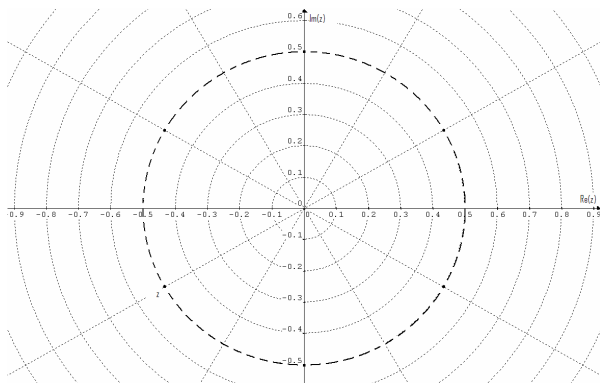
Q2a $\frac{(1+i\sqrt{3})^5}{8(1-i)^6} = \frac{(2\text{cis}\frac{\pi}{3})^5}{8(\sqrt{2}\text{cis}(-\frac{\pi}{4}))^6} = \frac{1}{2}\text{cis}\left(\frac{19\pi}{6}\right) = \frac{1}{2}\text{cis}\left(-\frac{5\pi}{6}\right)$

$$= \frac{1}{2}\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = -\frac{\sqrt{3}}{4} - \frac{1}{4}i.$$

$\therefore a = -\frac{\sqrt{3}}{4}$ and $b = -\frac{1}{4}$.

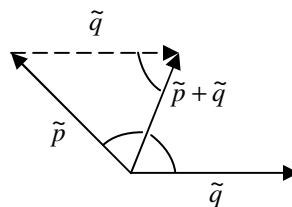
Q2b Since $z = \frac{1}{2}\text{cis}\left(-\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{4} - \frac{1}{4}i$ is a solution to

$64z^6 + 1 = 0$, this solution and the other 5 solutions space out equally on the circle of radius $\frac{1}{2}$ and centred at O in the complex plane.



Hence, the other 5 solutions are $-\frac{\sqrt{3}}{4} + \frac{1}{4}i$, $\frac{\sqrt{3}}{4} - \frac{1}{4}i$, $\frac{\sqrt{3}}{4} + \frac{1}{4}i$, $\frac{1}{2}i$, $-\frac{1}{2}i$.

Q3a $\tilde{p} = 3\tilde{i} + 4\tilde{j}$, $\tilde{q} = -3\tilde{j} + 4\tilde{k}$, $\therefore |\tilde{p}| = |\tilde{q}| = 5$.



$\tilde{p} + \tilde{q}$ bisects the angle between $\tilde{p}$ and $\tilde{q}$.

Let $\tilde{r} = \tilde{p} + \tilde{q} = 3\tilde{i} + \tilde{j} + 4\tilde{k}$, $|\tilde{r}| = \sqrt{3^2 + 1^2 + 4^2} = \sqrt{26}$.

Hence the required vector is $2\tilde{r} = 6\tilde{i} + 2\tilde{j} + 8\tilde{k}$.

Q3b Let S be the point dividing PQ into the ratio 3:1.

$$\overrightarrow{OS} = \frac{1 \times \overrightarrow{OP} + 3 \times \overrightarrow{OQ}}{3+1} = \frac{1(3\tilde{i} + 4\tilde{j}) + 3(-3\tilde{j} + 4\tilde{k})}{4}$$

$$= \frac{3}{4}\tilde{i} - \frac{5}{4}\tilde{j} + 3\tilde{k}.$$

Hence S is $\left(\frac{3}{4}, -\frac{5}{4}, 3\right)$.

Q4a Let $y = -2(\cos^{-1}(x+1) - \pi)$.

The equation of $f^{-1}(x)$ is $x = -2(\cos^{-1}(y+1) - \pi)$.

$$\therefore \cos^{-1}(y+1) = -\frac{1}{2}x + \pi, \quad y = \cos\left(-\frac{x}{2} + \pi\right) - 1,$$

$$\therefore y = -\cos\left(\frac{x}{2}\right) - 1. \text{ Hence } f^{-1}(x) = -\cos\left(\frac{x}{2}\right) - 1.$$

Domain is $[0, 2\pi]$, range is $[-2, 0]$.

Q4b $g(x) = \cos x$,

$$g(f(x)) = \cos(f(x)) = \cos(-2(\cos^{-1}(x+1) - \pi))$$

$$= \cos(2(\cos^{-1}(x+1) - \pi)) = \cos(2\cos^{-1}(x+1) - 2\pi)$$

$$= \cos(2\cos^{-1}(x+1)) = 2(\cos(\cos^{-1}(x+1)))^2 - 1$$

$$= 2(x+1)^2 - 1, \text{ for } x \in [-2, 0].$$

Q5 $3y^2\sqrt{x+1} = x + y$ and $\frac{dy}{dx} > 0$.

At $x=0$, $3y^2 - y = 0$, $\therefore y=0$ or $y=\frac{1}{3}$.

Implicit differentiation:

$$3y^2 \frac{d(\sqrt{x+1})}{dx} + \frac{d(3y^2)}{dx} \sqrt{x+1} = 1 + \frac{dy}{dx},$$

$$3y^2 \frac{1}{2\sqrt{x+1}} + 6y \frac{dy}{dx} \sqrt{x+1} = 1 + \frac{dy}{dx},$$

$$6y \frac{dy}{dx} \sqrt{x+1} - \frac{dy}{dx} = 1 - \frac{3y^2}{2\sqrt{x+1}},$$

$$(6y\sqrt{x+1} - 1) \frac{dy}{dx} = 1 - \frac{3y^2}{2\sqrt{x+1}}, \therefore \frac{dy}{dx} = \frac{1 - \frac{3y^2}{2\sqrt{x+1}}}{6y\sqrt{x+1} - 1}.$$

$$\frac{dy}{dx} > 0 \text{ at } \left(0, \frac{1}{3}\right) \text{ only.}$$

Gradient of tangent at $\left(0, \frac{1}{3}\right)$ is $\frac{dy}{dx} = \frac{1 - \frac{1}{6}}{2 - 1} = \frac{5}{6}$.

Q6a $f(x) = \tan^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{x}\right) + \sin^{-1}\left(\frac{1}{x}\right) = \tan^{-1}\left(\frac{1}{x}\right) + \frac{\pi}{2}$.

$$f(\sqrt{3}) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \frac{\pi}{2} = \frac{\pi}{6} + \frac{\pi}{2} = \frac{2\pi}{3}.$$

Q6b $f(x) = \tan^{-1}\left(\frac{1}{x}\right) + \frac{\pi}{2}$, $f'(x) = \frac{1}{1 + \left(\frac{1}{x}\right)^2} \times \frac{-1}{x^2} = -\frac{1}{x^2 + 1}$.

$$\therefore f'(\sqrt{3}) = -\frac{1}{3+1} = -\frac{1}{4}.$$

Q7a $\int_0^2 f(x) dx = \int_0^2 \cos^3(x-1) dx = \int_0^2 \cos^2(x-1) \cos(x-1) dx$

$$= \int_0^2 (1 - \sin^2(x-1)) \cos(x-1) dx$$

$$= \int_{-p}^p (1 - u^2) du$$

$$= \left[u - \frac{u^3}{3} \right]_{-p}^p = 2 \left(p - \frac{p^3}{3} \right).$$

Let $u = \sin(x-1)$,

$$\frac{du}{dx} = \cos(x-1).$$

When $x=0$,

$$u = \sin(-1) = -\sin(1) = -p.$$

When $x=2$, $u = p$.

Q7b Let $u = x-1$.

$$\int_0^2 f(x) g(x) dx = \int_0^2 (\cos^3(x-1))(x-1) dx = \int_{-1}^1 (\cos^3 u) u du.$$

Since $\cos^3 u$ is an even function and u an odd function, the product is an odd function.

$$\therefore \int_0^2 f(x) g(x) dx = \int_{-1}^1 (\cos^3 u) u du = 0.$$

Q8a $\vec{r}(0) = \vec{i} + 2\vec{j}$, $\vec{r}(2) = 3\vec{i} - 2\vec{j} - 6\vec{k}$.

Displacement $= \vec{r}(2) - \vec{r}(0) = 2\vec{i} - 4\vec{j} - 6\vec{k}$.

Q8b Velocity $= \frac{d\vec{r}}{dt} = \vec{i} - 2\vec{j} - 3\vec{k}$ is constant.

$\therefore$ the particle moves in a straight line.

Q8c From velocity $= \vec{i} - 2\vec{j} - 3\vec{k}$, $\tan \theta = \frac{-3}{\sqrt{1^2 + (-2)^2}} = -\frac{3}{\sqrt{5}}$.

Hence $\theta = \tan^{-1}\left(-\frac{3}{\sqrt{5}}\right) = -\tan^{-1}\left(\frac{3}{\sqrt{5}}\right)$.

Q9a $\vec{v}(0) = 2\vec{i}$, $\vec{v}(\sqrt{5}) = 2\vec{i} - 5\vec{j} + 2.5\vec{k}$.

Change in momentum $= 4.0(\vec{v}(\sqrt{5}) - \vec{v}(0)) = -20\vec{j} + 10\vec{k} \text{ kg ms}^{-1}$.

Q9b $\vec{a}(t) = \frac{d\vec{v}}{dt} = (-2t)\vec{j} + \frac{\sqrt{5}}{2}\vec{k}$, $\therefore \vec{a}(0) = \frac{\sqrt{5}}{2}\vec{k}$.

Initial resultant force $= m\vec{a}(0) = 2\sqrt{5}\vec{k} \text{ N}$.

Q9c Speed $= \sqrt{2^2 + (-t^2)^2 + \left(\frac{t\sqrt{5}}{2}\right)^2} = \sqrt{4 + t^4 + \frac{5t^2}{4}} = 2.5$.

$$\therefore 4t^4 + 5t^2 - 9 = 0, (4t^2 + 9)(t+1)(t-1) = 0.$$

Since $t \geq 0$, $\therefore t = 1 \text{ s}$.

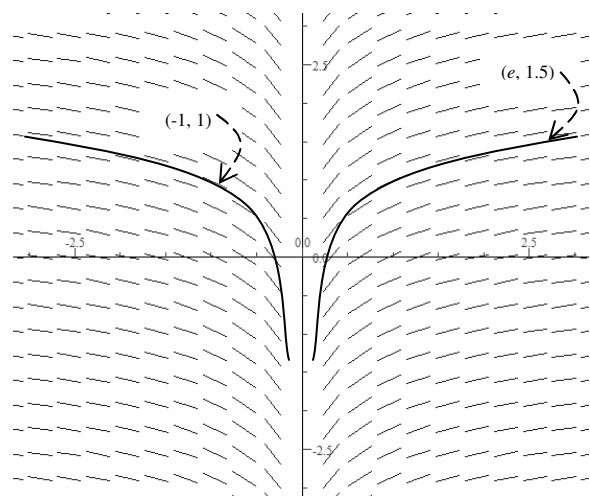
Q10a $y = a \log_e |x| + c$, where a and c are constants. $\therefore \frac{dy}{dx} = \frac{a}{x}$.

Q10b $(-1, 1) \rightarrow 1 = a \log_e |-1| + c$, $\therefore c = 1$.

$(e, 1.5) \rightarrow 1.5 = a \log_e e + 1$, $\therefore a = 0.5$.

The particular solution is $y = 0.5 \log_e |x| + 1$.

Q10c



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