



INSIGHT

Trial Exam Paper

2009

SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes

Writing time: 2 hours

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	5	5	58
Total			80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, once bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring sheets of paper or white out liquid/tape into the examination.

Materials provided

- The question and answer book of 29 pages with a separate sheet of miscellaneous formulas.
- Answer sheet for multiple-choice questions.

Instructions

- Write your **name** in the box provided and on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the exam

- Place the multiple-choice answer sheet inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the multiple-choice answer sheet provided.

Choose the response that is **correct** for the question.

One mark will be awarded for a correct answer; no marks will be awarded for an incorrect answer.

Marks **are not** deducted for incorrect answers.

No marks will be awarded if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

Question 1

The parametric equations $x = 2 \sec(t + 4) - 2$ and $y = 3 \tan(t + 4) + 1$ define a relation given by

A. $\frac{(x+2)^2}{4} - \frac{(y+1)^2}{9} = 1$

B. $\frac{(x+2)^2}{4} + \frac{(y-1)^2}{9} = 1$

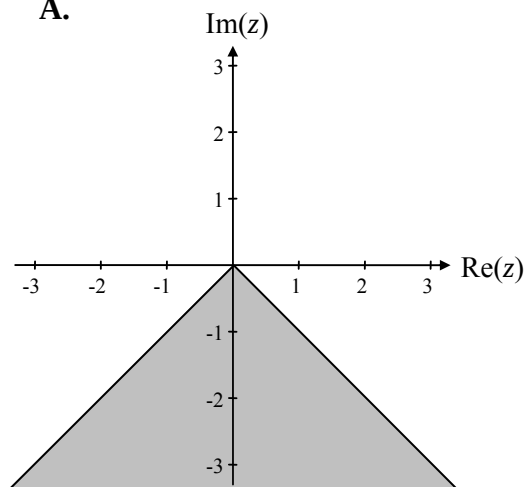
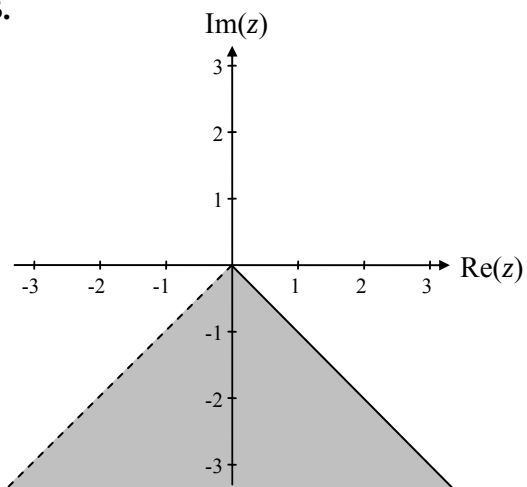
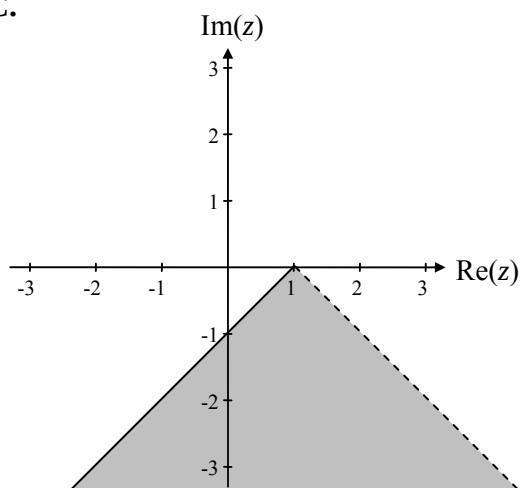
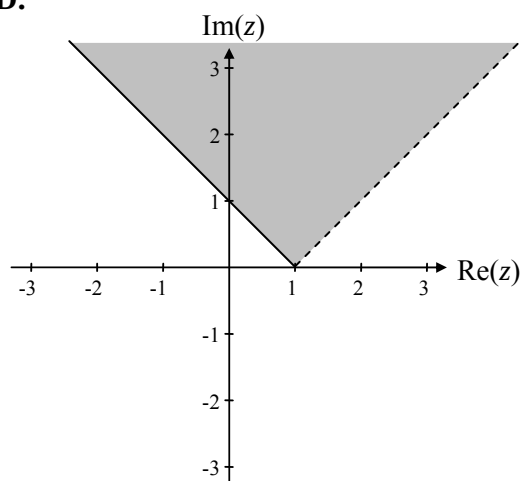
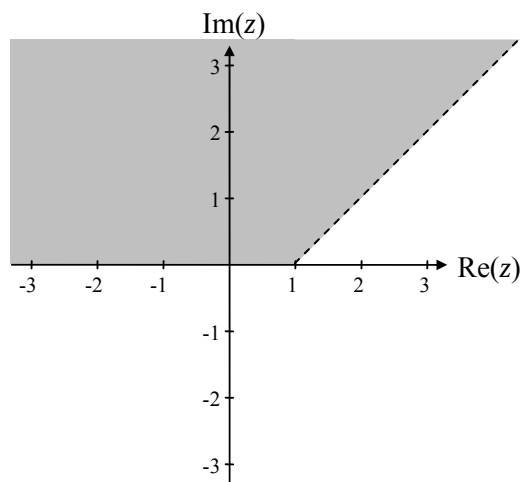
C. $\frac{(x+2)^2}{3} - \frac{(y-1)^2}{2} = 1$

D. $\frac{(x+2)^2}{4} - \frac{(y-1)^2}{9} = 1$

E. $\frac{(x+2)^2}{2} - \frac{(y-1)^2}{3} = 1$

Question 2

The region of the complex plane defined by $\left\{ z : -\frac{\pi}{4} \leq \text{Arg}(i(z-1)) < \frac{\pi}{4} \right\}$ is

A.**B.****C.****D.****E.**

Question 3

The maximal domain and range of the function $f(x) = 3 \arctan(2x - \pi)$ are given by

- A. $d_f = (\pi, 3\pi)$ and $r_f = R$
- B. $d_f = R$ and $r_f = [-\frac{\pi}{2}, \frac{\pi}{2}]$
- C. $d_f = R$ and $r_f = (-\frac{3\pi}{2}, \frac{3\pi}{2})$
- D. $d_f = R$ and $r_f = (\frac{\pi}{4}, \frac{3\pi}{4})$
- E. $d_f = (-\frac{\pi}{2}, \frac{\pi}{2})$ and $r_f = R$

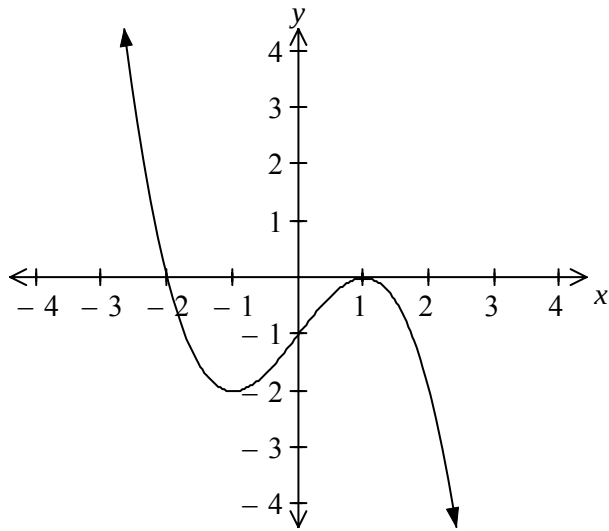
Question 4

If $z^2 - z - 2$ is a factor of $P(z) = z^4 - 3z^3 + 10z^2 - 6z - 20$, $z \in C$, then all of the factors must be

- A. $z - 2, z + 1, z - 1 + 3i$ and $z - 1 - 3i$
- B. $z - 2, z + 1, z - 1 + 3i$ and $z + 1 - 3i$
- C. $z + 2, z - 1, z - 3 + i$ and $z - 3 - i$
- D. $z - 2, z + 1, z + 3 + i$ and $z + 3 - i$
- E. $z - 2, z + 1, z + 2$ and $z + 5$

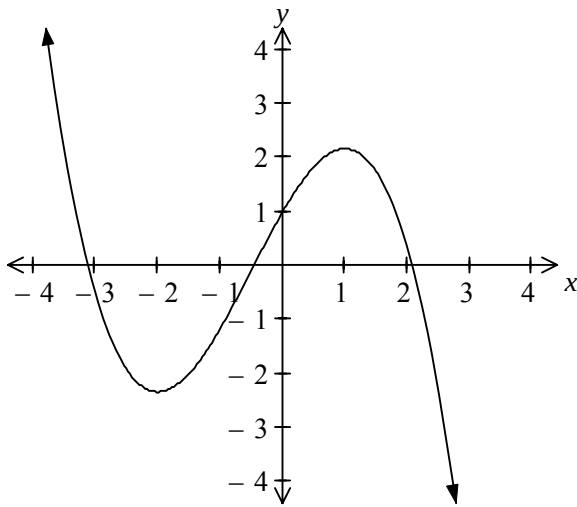
Question 5

The graph of $y = f'(x)$ is shown below.

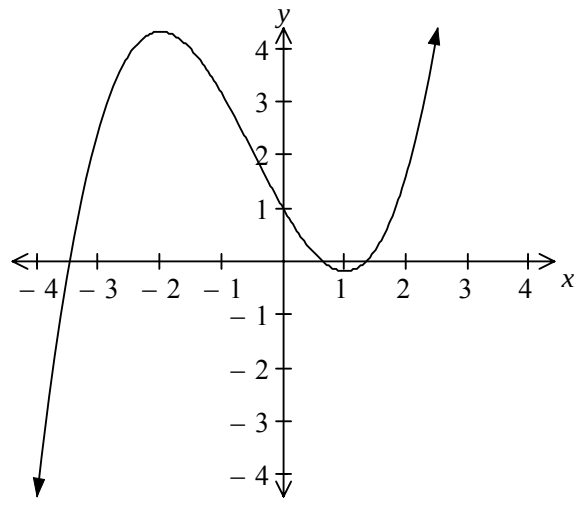


If $f(0) = 1$, then the graph of $y = f(x)$ could be

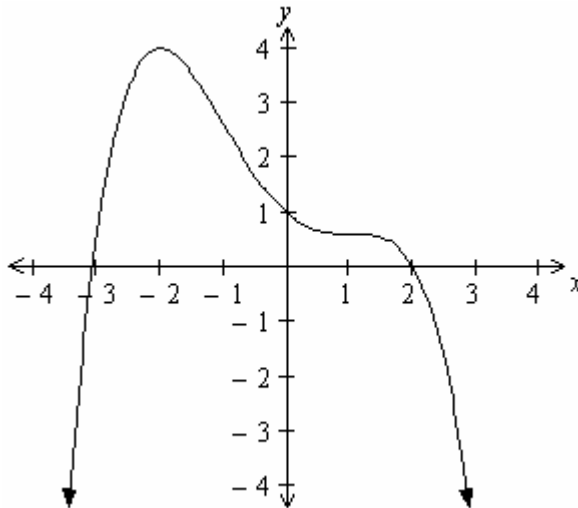
A.



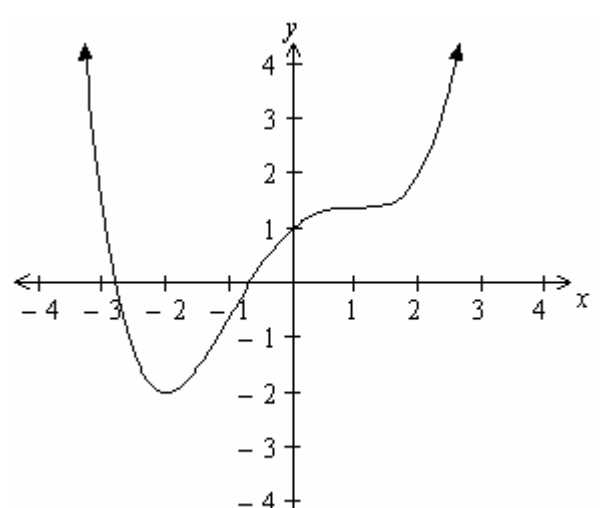
B.



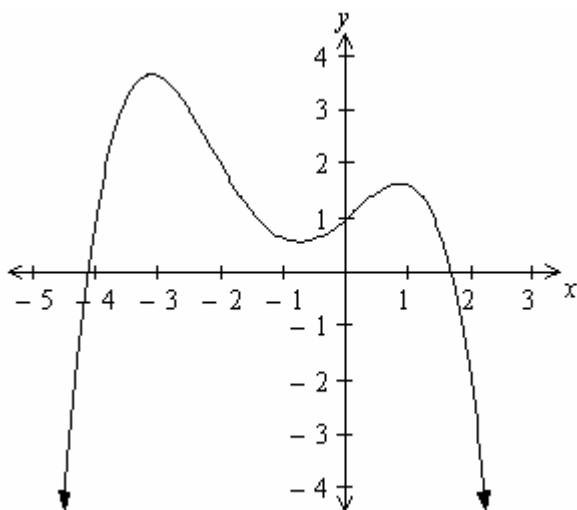
C.



D.

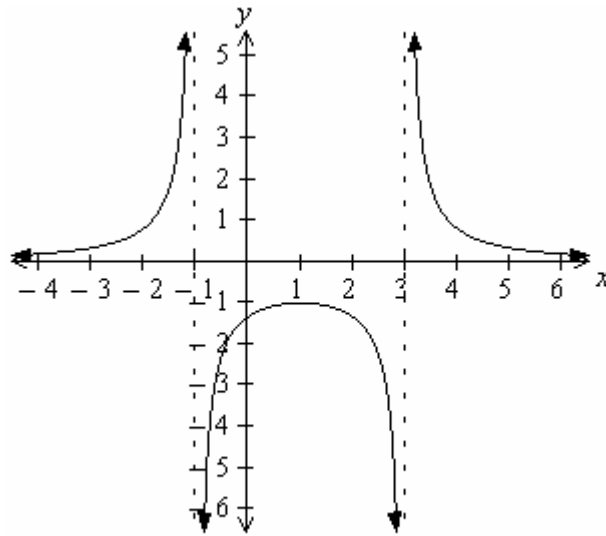


E.



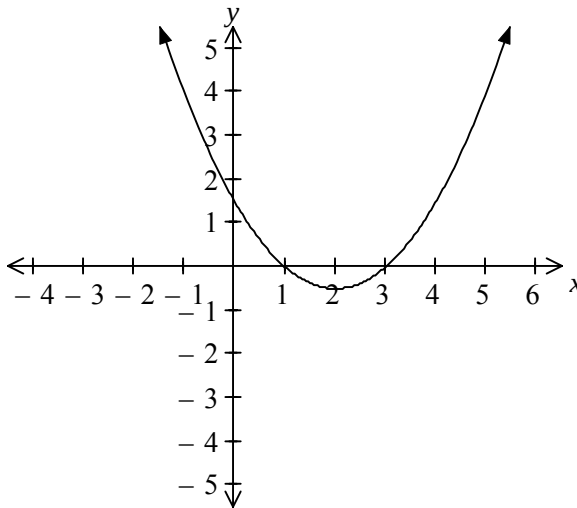
Question 6

The graph of $y = \frac{1}{f(x)}$ is shown below.

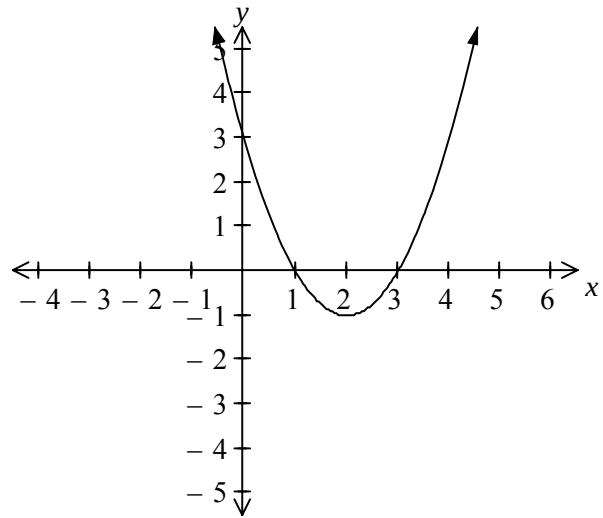


The graph of $y = f(x)$ could be

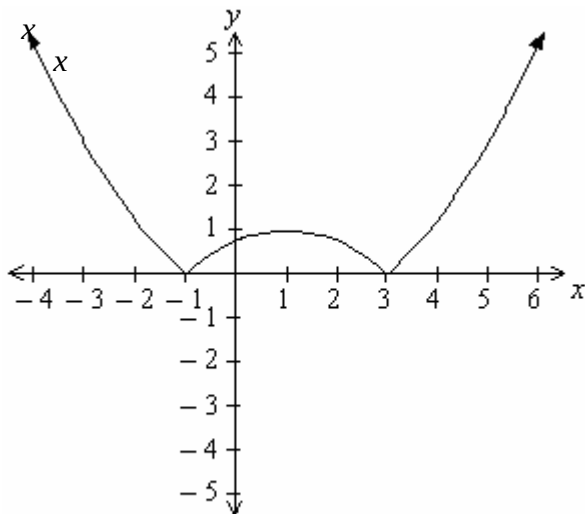
A.



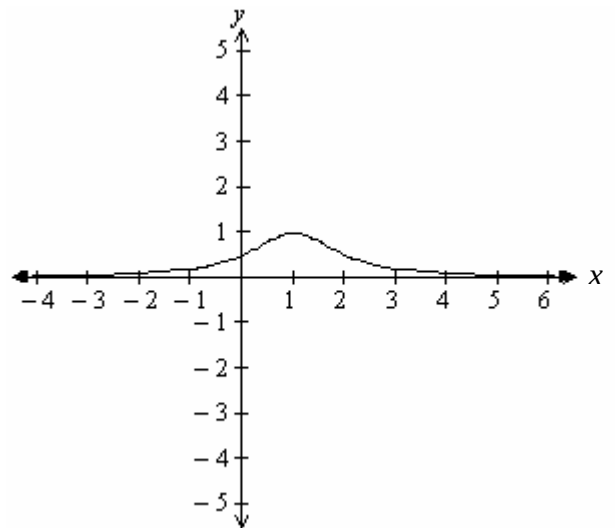
B.



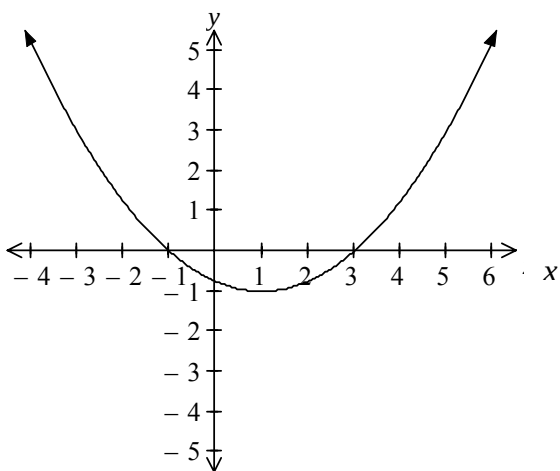
C.



D.



E.



Question 7

The solutions to $z^2 = a + \sqrt{3}ai$, where $z \in \mathbb{C}$ and $a \in \mathbb{R}^+$, are

- A. $\pm \frac{\sqrt{2}}{2}(\sqrt{3} + i)$
- B. $\pm \frac{\sqrt{2a}}{2}(\sqrt{3} + i)$
- C. $\pm \frac{\sqrt{2}}{2}(1 + \sqrt{3}i)$
- D. $\pm \frac{\sqrt{2a}}{2}(1 + \sqrt{3}i)$
- E. $\pm \frac{\sqrt{2}}{2}(1 - \sqrt{3}i)$

Question 8

For the vectors $\underline{a} = 3\underline{i} - \underline{j} + 2\underline{k}$, $\underline{b} = \underline{i} + 2\underline{j} - 2\underline{k}$ and $\underline{c} = x\underline{i} - 7\underline{j} + 10\underline{k}$ to be linearly dependent, the value of x must be

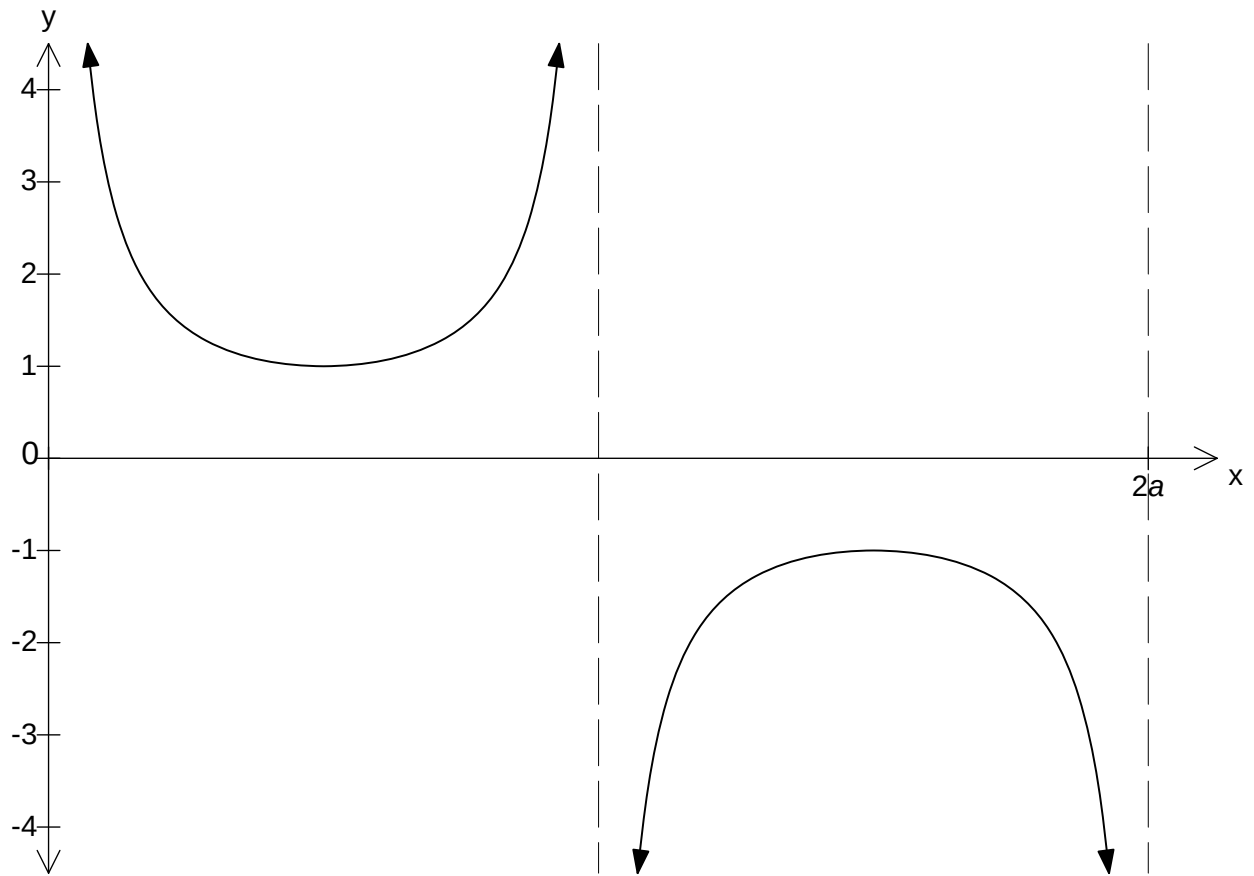
- A. 4
- B. 7
- C. 2
- D. -7
- E. -2

Question 9

The graph of the relation $\{z : z\bar{z} - 2\operatorname{Re}(z) = 8, z \in \mathbb{C}\}$ would be

- A. a circle with centre $(0, 0)$ and radius $2\sqrt{2}$.
- B. a circle with centre $(-1, 0)$ and radius 3.
- C. a straight line with gradient 2 and y-intercept of 8.
- D. a straight line with gradient 1 and y-intercept of 8.
- E. a circle with centre $(1, 0)$ and radius 3.

Question 10



The rule for the function graphed above, where $a > 0$, could be

- A. $y = \operatorname{cosec}\left(\frac{\pi x}{a}\right)$
- B. $y = \sec\left(\frac{\pi x}{a}\right)$
- C. $y = \sec\left(\frac{\pi}{a}\left(x - \frac{a}{2}\right)\right)$
- D. $y = -\operatorname{cosec}\left(\frac{\pi x}{a}\right)$
- E. $y = \operatorname{cosec}\left(\frac{\pi}{a}\left(x + \frac{a}{2}\right)\right)$

Question 11

$\int_0^1 \left(\frac{2-3x}{4-x^2} \right) dx$ is equal to

- A. $\log_e \left(\frac{9}{2} \right)$
- B. $\log_e 18$
- C. 0
- D. $\log_e 72$
- E. $\log_e \left(\frac{9}{8} \right)$

Question 12

The gradient of the tangent to the curve $2x \log_e(y) - x = y$ at the point where $y = e$ is

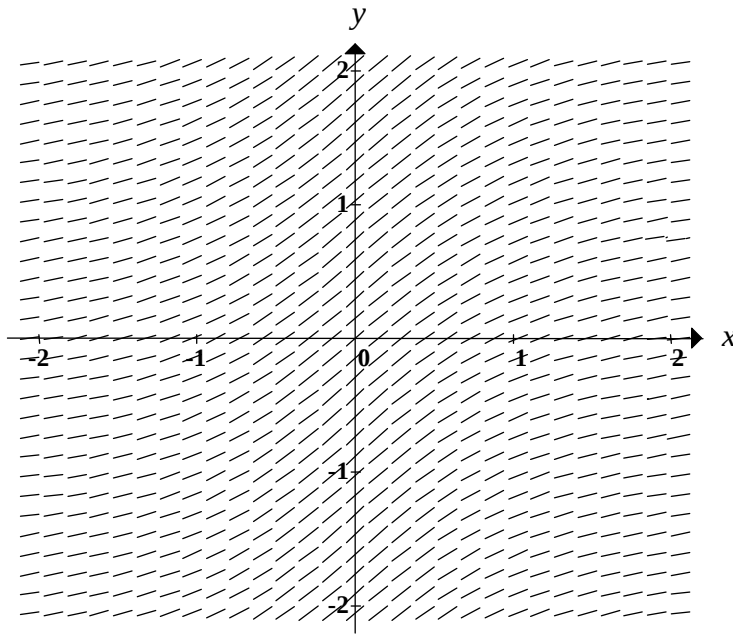
- A. 3
- B. -1
- C. -3
- D. 1
- E. $\frac{1}{2}$

Question 13

Using a suitable substitution, $\int_0^1 \frac{\cos^{-1}\left(\frac{x}{2}\right)}{\sqrt{4-x^2}} dx$ is equal to

- A. $2 \int_2^0 u \, du$
- B. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} u \, du$
- C. $\frac{1}{2} \int_0^2 u \, du$
- D. $2 \int_0^{\frac{\pi}{4}} u \, du$
- E. $\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{u} \, du$

Question 14



The direction (slope) field for a certain first-order differential equation is shown above.

The differential equation could be

- A. $\frac{dy}{dx} = \frac{1}{1+x^2}$
- B. $\frac{dy}{dx} = \tan^{-1} x$
- C. $\frac{dy}{dx} = 1 + x^2 + y^2$
- D. $\frac{dy}{dx} = |x+1|$
- E. $\frac{dy}{dx} = \frac{1}{|x+y+1|}$

Question 15

If $\frac{dy}{dx} = \log_e(x)$ and $y(1) = 2$, then the value of y when $x = 3$ can be found by evaluating

- A. $1 + \int_2^3 \log_e(t) dt$
- B. $2 + \int_1^3 \frac{1}{t} dt$
- C. $2 + \int_1^3 \log_e(t) dt$
- D. $1 - \int_2^3 \log_e(t) dt$
- E. $3 + \int_1^2 \log_e(t) dt$

Question 16

The position vectors of two moving particles, R and S , at any time t seconds are given by

$$\underline{r} = at\underline{i} - 4\underline{j} \text{ and } \underline{s} = t^2\underline{i} + 2t\underline{j}, t \geq 0, a \in \mathbb{R}, \text{ respectively.}$$

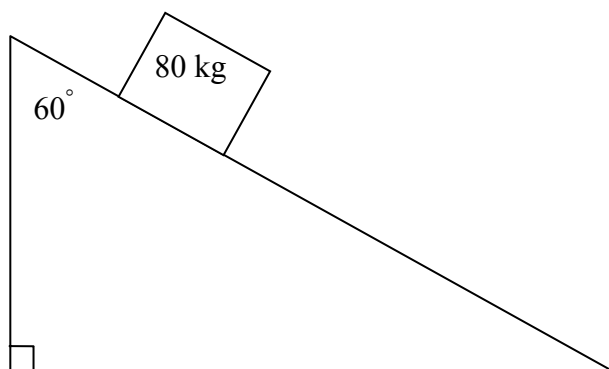
The angle between the directions of the two particles at $t = 1$ is

- A. 69.3°
- B. 45°
- C. 35.3°
- D. 19.5°
- E. dependent on the value of a .

Question 17

The volume of a tank is given by $V = 0.4\pi h^{\frac{5}{2}}$, where h cm is the depth of water in the tank at time t minutes. Water leaks from the tank at a rate of $16 \text{ cm}^3/\text{minute}$. The depth of water in the tank when the height is decreasing at a rate of $\frac{2}{\pi} \text{ cm/minute}$ is

- A. 16 cm
- B. 8 cm
- C. 4π cm
- D. 4 cm
- E. 8π cm

Question 18

A skier of mass 80 kilograms slides from rest down a straight slope inclined at 60° to the vertical. Assuming it is a smooth slope, the speed of the skier after moving 100 metres down the slope is nearest to

- A. 41.2 m/s
- B. 22.1 m/s
- C. 31.3 m/s
- D. 44.3 m/s
- E. 10 m/s

Question 19

A mass of 4 kilograms is at rest when two forces, $\vec{F}_1 = (\vec{i} - 3\vec{j})$ newtons and

$\vec{F}_2 = (2\vec{i} - \vec{j})$ newtons, act on it. The time taken for the mass to travel 10 metres is

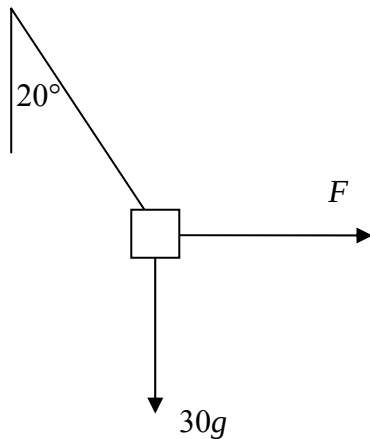
- A. 1 s
- B. 2 s
- C. 4 s
- D. 5 s
- E. 8 s

Question 20

The velocity of a particle moving in a straight line is given by $v(x) = \cos(x^2)$, where x is the displacement from the origin O .

The acceleration of the particle is

- A. $a(x) = -2x \sin(x^2)$
- B. $a(x) = \cos(2x)$
- C. $a(x) = -2x \tan(x^2)$
- D. $a(x) = -x \sin(2x^2)$
- E. $a(x) = -2x \tan(x^2) \sec(x^2)$

Question 21

The magnitude of the horizontal force, F newtons, required to hold a 30 kilogram child in equilibrium on a swinging rope, as shown in the diagram above, is

- A. $\frac{30g}{\tan 70^\circ}$
- B. $\frac{30g \sin 70^\circ}{\sin 20^\circ}$
- C. $30g \sin 20^\circ$
- D. $\frac{30g}{\sin 70^\circ}$
- E. $30g \tan 20^\circ$

Question 22

A lift travelling upwards accelerates at $a \text{ m/s}^2$ ($a > 0$) with a person of mass 100 kilograms standing on a set of weight scales in the lift. It then decelerates at twice the magnitude of the acceleration. The magnitude of the change in the reading on the scales will be

- A. $100a \text{ kg}$
- B. $200g \text{ kg}$
- C. $300a \text{ kg}$
- D. $100(g + a) \text{ kg}$
- E. $-100a \text{ kg}$

SECTION 2

Instructions for Section 2

Answer all the questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, approximate working must be shown.

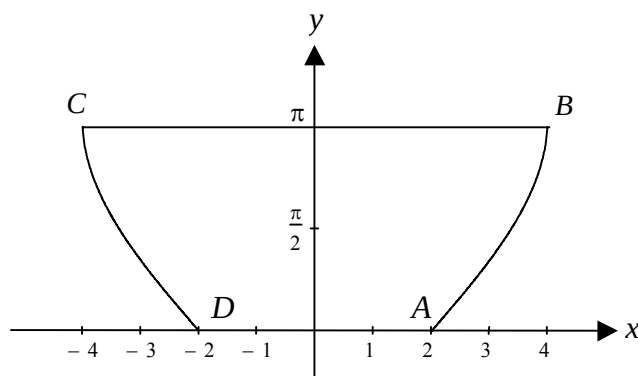
Unless otherwise indicated, the diagrams in this book have not been drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

Question 1

The diagram below shows the profile of a symmetrical small bowl $ABCD$. The bowl is generated by rotating the area between the curve AB and the y -axis about the y -axis. The top and base of the bowl have radii of 4 cm and 2 cm, respectively, and the height of the bowl is π cm.

The curve AB can be modelled by the function $y = a \sin^{-1}(bx - c)$, $x \in [2, 4]$.



- a. Show that $a = 2$, $b = \frac{1}{2}$ and $c = 1$.

3 marks

SECTION 2 – Question 1 – continued

TURN OVER

4 marks

- 1 mark

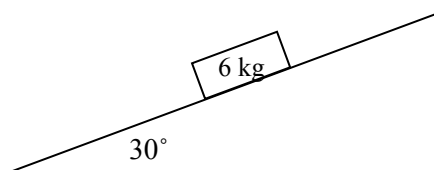
- d. To the nearest millimetre, what would be the height of the water when the bowl is filled to half its capacity?

2 marks

Total 3 + 4 + 1 + 2 = 10 marks

Question 2

A miniature racing car of mass 6 kilograms is propelled from rest up a rough ramp 19.6 metres long and inclined at an angle of 30° to the horizontal. The car is powered up the ramp by a constant force of $10g$ newtons. This causes the car to accelerate at 9.8 m/s^2 .



- a. Label the forces acting on the car as it moves up the ramp.
- 1 mark
- b. Show that at the top of the ramp the car is g metres above the ground and its speed is $2g \text{ m/s}$ when it leaves the ramp.

2 marks

SECTION 2 – Question 2 – continued
TURN OVER

- c. Calculate the exact value of the coefficient of friction.

2 marks

When the car leaves the ramp it is only subject to the force of gravity.

Take $\underline{i}$ as the unit vector in the horizontal direction and $\underline{j}$ as the unit vector in the vertical direction from the point on the ground, directly below the top of the ramp.

- d. Determine the velocity vector $\underline{v}$ and the position vector $\underline{r}$ of the car at any time t seconds.

2 marks

- e. Find the exact Cartesian equation of the path of the car after it leaves the ramp.

2 marks

- f. Find the exact magnitude of the momentum of the car when it hits the ground.

3 marks

Total $1 + 2 + 2 + 2 + 2 + 3 = 12$ marks

SECTION 2 – continued
TURN OVER

Question 3

- a.** Given $w = a + bi$, where $a, b \in \mathbb{R}$ and $b > 0$.

If $w + \bar{w} = 2$ and $w\bar{w} = 2$, show that $w = 1 + i$.

2 marks

- b.** If $v = 1 + \sqrt{3}i$,

- i.** Find $\frac{v}{w}$ in simplest exact Cartesian form.

1 mark

- ii.** Find $\frac{v}{w}$ in polar form.

2 marks

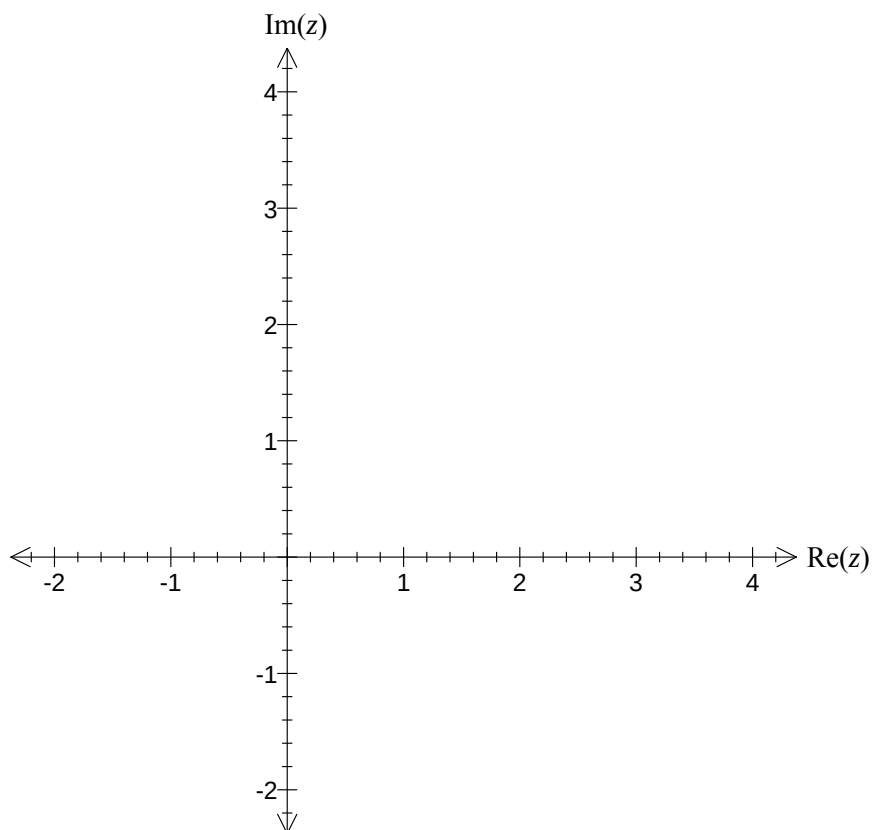
- c. Hence, express $\tan\left(\frac{\pi}{12}\right)$ in the form $a - \sqrt{b}$, where a and b are positive integers.

3 marks

- d. S is a subset of the complex plane, which is defined by

$$S = \{z : |z - w| = 1, \quad z \in \mathbb{C}\}$$

Plot the points v and w and sketch the relation defined by S on the Argand diagram below.



2 marks

SECTION 2 – Question 3 – continued
TURN OVER

Question 4

A tank contains 100 litres of sugar solution with a concentration of 0.05 kg/L. A sugar solution of concentration 0.1 kg/L flows into the tank at a rate of 2 L/min. The mixture, which is kept uniform by stirring, flows out of the tank at a rate of 2 L/min. After t minutes the tank contains x kilograms of sugar.

- a. Show that the differential equation for x in terms of t is $\frac{dx}{dt} = \frac{10-x}{50}$ kg/min.

1 mark

- b. Solve this differential equation to give x as a function of t .

3 marks

- c. Calculate the amount of sugar in the tank after 2 minutes. Give your answer correct to three decimal places.

1 mark

- d.** If this situation continued for a long period of time, how much sugar would be present in the tank?

1 mark

- e.** If the outflow from the tank was 1 L/min instead of 2 L/min, set up the new differential equation for x in terms of t .

1 mark

- f.** For the differential equation from part **e.** use Euler's method, with increments of 1 minute, to predict the amount of sugar in the tank after 2 minutes. Give your answer correct to three decimal places.

2 marks

Total $1 + 3 + 1 + 1 + 1 + 2 = 9$ marks

Question 5

At 10 a.m. an aircraft is flying at an altitude of $(e^2 - e)$ km, 500 km north and 440 km east of a point $T(0, 0, 0)$, which is its touchdown point on a horizontal runway.

The position of the aircraft relative to the point T is given by the vector

$$\underline{r}(t) = \left(a + \frac{2420}{t+5} \right) \underline{i} + (500 - 24t + 0.28t^2) \underline{j} + (e^{c-0.02t} - e) \underline{k}, \text{ where } a, c \in \mathbb{R}.$$

$\underline{r}$ is in kilometres and t is the time in minutes after 10 a.m.

$\underline{i}$ is the unit vector in an easterly direction, $\underline{j}$ is the unit vector in a northerly direction and

$\underline{k}$ is the unit vector representing the altitude of the aircraft.

(Treat the aircraft as a point in this problem.)

- a.** Show that $a = -44$ and $c = 2$.

1 mark

- b.** Show that the aircraft touches down at point T at 10.50 a.m.

1 mark

- c.** Show that the exact velocity of the aircraft at touchdown is $\underline{r}' = -0.8\underline{i} + 4\underline{j} - 0.02e\underline{k}$.

2 marks

SECTION 2 – Question 5 – continued
TURN OVER

- d. Find the vertical angle to the runway at which the aircraft lands. Give your answer to the nearest hundredth of a degree.

3 marks

- e. Relative to the point T , find the position vector $\underline{p}(t)$ of the aircraft on the runway when the aircraft stops.

3 marks

- f. A stationary fire engine is positioned 1.5 kilometres north and 200 metres west of T . Determine, to the nearest metre, the minimum distance between the fire engine and the aircraft on its path after touchdown.

3 marks

Total $1 + 1 + 2 + 3 + 3 + 3 = 13$ marks**END OF QUESTION AND ANSWER BOOK**