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Student Name:

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 2

2009

Reading Time: 15 minutes

Writing time: 2 hours

Instructions to students

This exam consists of Section 1 and Section 2.
Section 1 consists of 22 multiple-choice questions and should be answered on the detachable answer sheet on page 33 of this exam. This section of the paper is worth 22 marks.
Section 2 consists of 5 extended-answer questions, all of which should be answered in the spaces provided. Section 2 begins on page 13 of this exam. This section of the paper is worth 58 marks.
There is a total of 80 marks available.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact value is required to a question a decimal approximation will not be accepted.
Unless otherwise stated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude $g \text{ m/s}^2$ where $g = 9.8$
Students may bring one bound reference into the exam.
Students may bring an approved graphics or CAS calculator into the exam.
Formula sheets can be found on pages 30 - 32 of this exam.

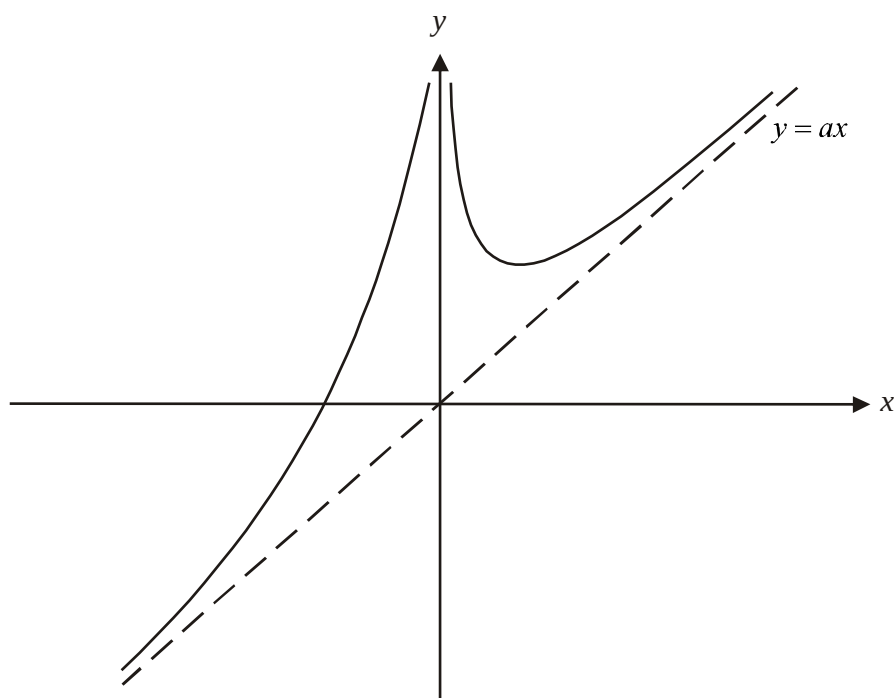
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SECTION 1

Question 1



The graph shown above could have the equation

- A. $y = \frac{ax^3 + 1}{x}, a < 0$
- B. $y = \frac{ax^3 + 1}{x^2}, a > 0$
- C. $y = \frac{x^3 + a}{x^2}, a > 0$
- D. $y = \frac{ax^4 + a}{x}, a > 0$
- E. $y = \frac{ax^4 + 1}{x^2}, a < 0$

Question 2

The graph of the function $f : [0, \infty) \rightarrow \mathbb{R}, f(x) = \operatorname{cosec}(ax), a > 0$ has asymptotes located at

- A. $x = 0, \frac{2\pi}{a}, \frac{4\pi}{a}, \frac{6\pi}{a}, \dots$
- B. $x = \frac{\pi}{2a}, \frac{3\pi}{2a}, \frac{5\pi}{2a}, \frac{7\pi}{2a}, \dots$
- C. $x = 0, \frac{\pi}{a}, \frac{2\pi}{a}, \frac{3\pi}{a}, \dots$
- D. $x = \frac{\pi}{a}, \frac{3\pi}{a}, \frac{5\pi}{a}, \frac{7\pi}{a}, \dots$
- E. $x = 0, \pi a, 2\pi a, 3\pi a, \dots$

Question 3

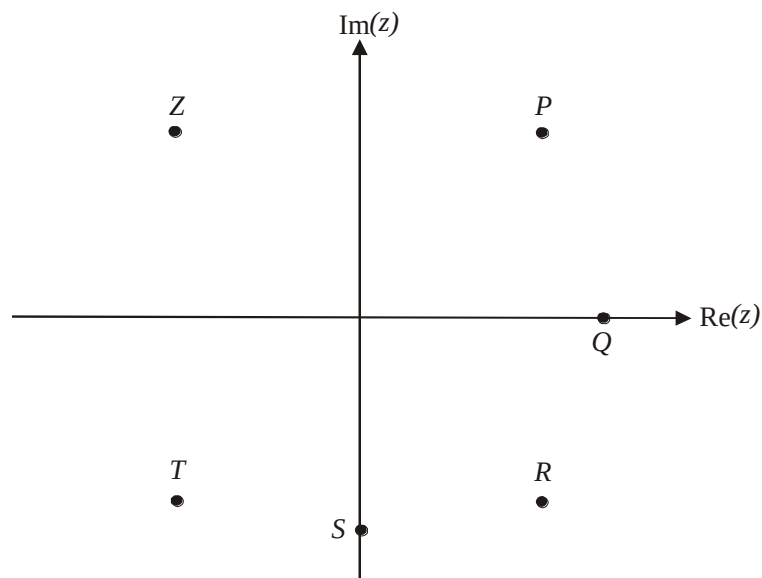
The domain and range of the function $y = 2\arcsin(2x + 1) - \pi$ are given respectively by

- A. $\left[-\frac{1}{2}, \frac{1}{2}\right]$ and $[-\pi, \pi]$
- B. $[-\pi, \pi]$ and $[-1, 0]$
- C. $[0, 1]$ and $[-\pi, 0]$
- D. $[-\pi, 0]$ and $\left[-\frac{1}{2}, 0\right]$
- E. $[-1, 0]$ and $[-2\pi, 0]$

Question 4

If $z = \sqrt{3}\text{cis}\left(-\frac{\pi}{2}\right)$ then $\text{Arg}(z^6)$ is equal to

- A. -3π
- B. $-\pi$
- C. $-\frac{\pi}{12}$
- D. $\frac{\pi}{2}$
- E. π

Question 5

The point Z on the Argand diagram above represents the complex number z .
 The complex number $\bar{z}i$ could be represented by the point

- A. P
- B. Q
- C. R
- D. S
- E. T

Question 6

The relation $|z - ai| = |z + a|$, $a > 0$, is represented on an Argand diagram. The graph shows a straight line. That line passes through the point representing the complex number

- A. $-a - ai$ and has a gradient of 1
- B. $-a + ai$ and has a gradient of -1
- C. ai and has a gradient of 1
- D. $a + ai$ and has a gradient of -1
- E. $a + ai$ and has a gradient of 1

Question 7

$p(z)$ is a cubic polynomial with real coefficients and $z \in \mathbb{C}$.

If $z = -ai$, $a \in \mathbb{R}$, is a root of $p(z) = 0$ then $p(z)$ could be given by

- A. $z^2 + a^2$
- B. $z^3 + a^3$
- C. $z^3 - a^3$
- D. $z^3 - z^2 + a^2z - a^2$
- E. $z^3 - z^2 + a^2z + a^2$

Question 8

Using a suitable substitution $\int_0^{\frac{\pi}{2}} \sin(x) \cos^3(x) dx$ can be expressed as

- A. $-\int_0^{\frac{\pi}{2}} u^3 du$
- B. $\int_0^{\frac{\pi}{2}} u^3 du$
- C. $-\int_0^1 u^3 du$
- D. $\int_1^0 u^3 du$
- E. $\int_0^1 u^3 du$

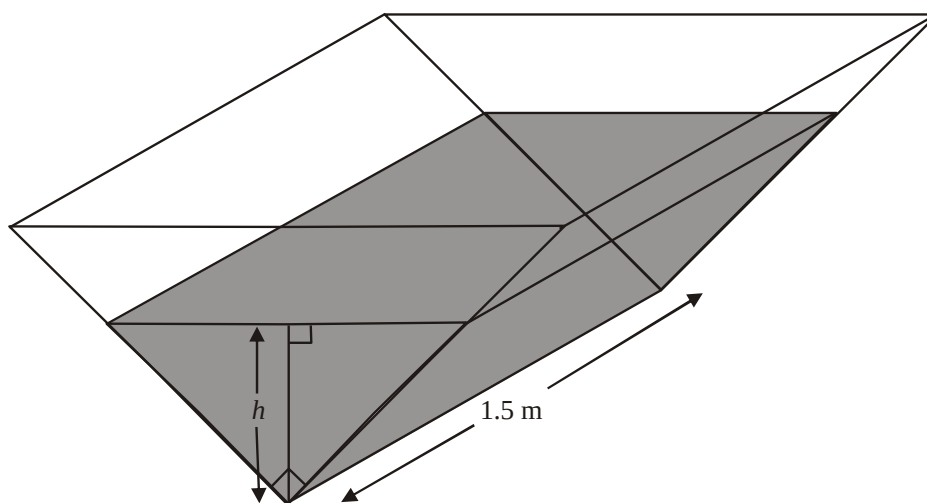
Question 9

If $f'(t) = \ln(t+1)$ and $f(0) = 2$ then the solution to this differential equation when $t = 1$ can be found by evaluating

- A. $\int_0^1 \ln(t+1) dt$
 B. $\int_0^1 \ln(t+1) dt - 2$
 C. $\int_0^1 \ln(t+1) dt + 2$
 D. $\int_0^1 (2 + \ln(t+1)) dt$
 E. $\int_0^1 (-2 + \ln(t+1)) dt$

Question 10

A water trough is in the shape of a triangular prism of length 1.5m. The cross-section of the trough is a right isosceles triangle.



The trough is being filled at the rate of $300\text{cm}^3/\text{min}$.

Let h cm be the height of the water in the trough t minutes after it has started to be filled.

The rate in cm/min, at which the height of the water is increasing when $h = 20$ is

- A. 0.0025
 B. 0.005
 C. 0.05
 D. 0.25
 E. 5

Question 11

Using a suitable substitution, $\int_0^2 \frac{2x-1}{\sqrt{3-x}} dx$ can be expressed as

- A. $\int_0^2 (7-2u) du$
- B. $\int_0^2 (5u^{-\frac{1}{2}} - 2u^{\frac{1}{2}}) du$
- C. $\int_1^3 (7u^{-\frac{1}{2}} - 2u) du$
- D. $-\int_1^3 (5u^{-\frac{1}{2}} - 2u^{\frac{1}{2}}) du$
- E. $\int_1^3 (5u^{-\frac{1}{2}} - 2u^{\frac{1}{2}}) du$

Question 12

A 200 litre gas cylinder contains 40% hydrogen by volume. To increase this level, pure hydrogen is pumped into the cylinder at a constant rate of 5 litres/minute whilst the mixture in the cylinder, which has been uniformly mixed, is pumped out at a rate of 5 litres/minute. Let H represent the volume of hydrogen in litres present in the cylinder t minutes after the pumping in and out begins.

A differential equation for H in terms of t is

- A. $\frac{dH}{dt} = \frac{H-80}{200}, \quad t=0, H=80$
- B. $\frac{dH}{dt} = \frac{H-80}{200}, \quad t=0, H=200$
- C. $\frac{dH}{dt} = \frac{H}{40}, \quad t=0, H=200$
- D. $\frac{dH}{dt} = \frac{200-H}{40}, \quad t=0, H=80$
- E. $\frac{dH}{dt} = \frac{195-H}{40}, \quad t=0, H=80$

Question 13

The position vectors of two particles R and S at time t , $t \geq 0$, are given by

$$\underline{r} = t^2 \underline{i} + (3t - 4) \underline{j}$$

and $\underline{s} = (7t - 10) \underline{i} + 11 \underline{j}$

The particles are in the same position when

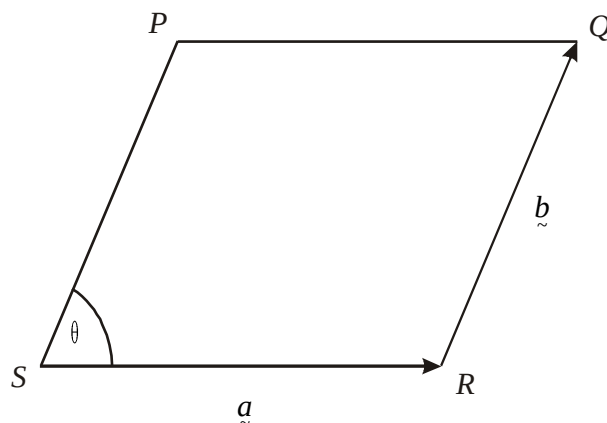
- A. $t = 2$
- B. $t = 5$
- C. $t = 2$ and $t = 5$
- D. $t = 2$ or $t = 10$
- E. $t = 10$

Question 14

If $\underline{a} = \underline{i} + 2 \underline{j} + 2 \underline{k}$ and $\underline{b} = 2 \underline{i} - \underline{j} + 2 \underline{k}$ then the angle between $\underline{a}$ and $\underline{b}$ is closest to

- A. 26°
- B. 27°
- C. 64°
- D. 76°
- E. 87°

Question 15



In the rhombus $PQRS$, $\vec{SR} = \underline{a}$, $\vec{RQ} = \underline{b}$, $\angle PSR = \theta$, $\theta \neq 90^\circ$ and $|\underline{a}| = 1$.

Which one of the following statements is false?

- A. $\underline{a} \cdot \underline{a} = \underline{b} \cdot \underline{b}$
- B. $|\underline{a}| = |\underline{b}|$
- C. $\underline{a} \cdot \underline{b} = \cos(\theta)$
- D. $\underline{a} \cdot \underline{b} = \cos(180 - \theta)$
- E. $\underline{a} \cdot \underline{b} \neq 0$

Question 16

The position vector of a particle at time t , $t \geq 0$, is given by $\underline{r} = 2\sqrt{t} \underline{i} + (5 - t) \underline{j}$.

The particle is closest to the origin when

- A. $t = 0$
- B. $t = 1$
- C. $t = 3$
- D. $t = 4$
- E. $t = 5$

Question 17

Three forces $\vec{P}$, $\vec{Q}$ and $\vec{R}$ act on a body of mass 8kg. The forces are all measured in newtons and $\vec{P} = 4\vec{i} - \vec{j}$, $\vec{Q} = -3\vec{i} + 2\vec{j}$ and $\vec{R} = 2\vec{i} + 3\vec{j}$.

The magnitude of the acceleration of the body in m/s^2 is

- A. $\frac{5}{8}$
- B. $\frac{3}{4}$
- C. $\frac{7}{8}$
- D. $\frac{4}{3}$
- E. $\frac{5}{2}$

Question 18

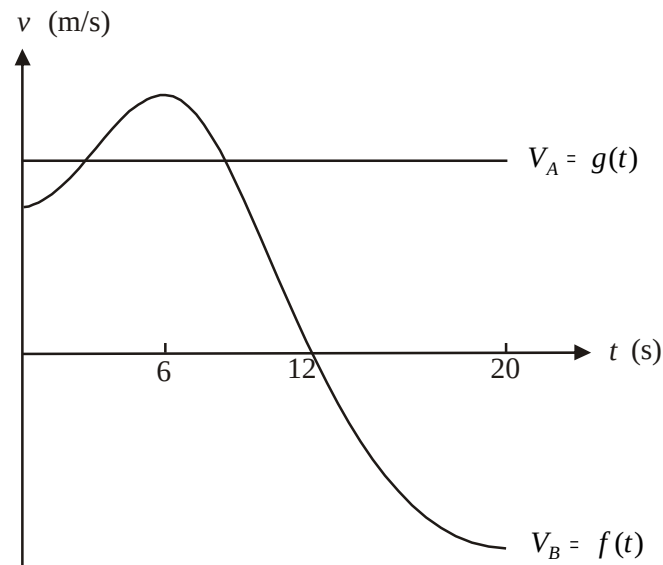
A mass travels in a straight line at 4m/s and has a momentum of 20kg m/s.

A force acts on the mass in the same direction of motion so that the mass accelerates at 0.5m/s^2 until it has a momentum of 45kg m/s.

The distance in metres covered by the mass during this time is

- A. 25
- B. 26.25
- C. 47
- D. 65
- E. 125

Question 19



Particles A and B move from the same point in a straight line with velocities in m/s given respectively by $V_A = g(t)$ and $V_B = f(t)$ where t is in seconds and $t \geq 0$.

The velocity-time graphs for the particles are shown above.

At $t = 20$ seconds, the distance in metres between the particle A and particle B is given by

- A. $\left| \int_0^{20} f(t) dt - 20g(20) \right|$
- B. $\left| \int_0^{20} f(t) dt - g(t) \right|$
- C. $\left| \int_0^6 f(t) dt - \int_6^{20} f(t) dt - 20g(t) \right|$
- D. $\left| \int_0^{12} f(t) dt - \int_{12}^{20} f(t) dt - 20g(20) \right|$
- E. $\left| \int_0^{12} f(t) dt - \int_{12}^{20} f(t) dt - 20g(t) \right|$

Question 20

The acceleration $a \text{ m/s}^2$ of a body moving in a straight line when it is $x \text{ m}$ from a fixed origin is given by $a = \frac{1}{\sqrt{4-x^2}}$.

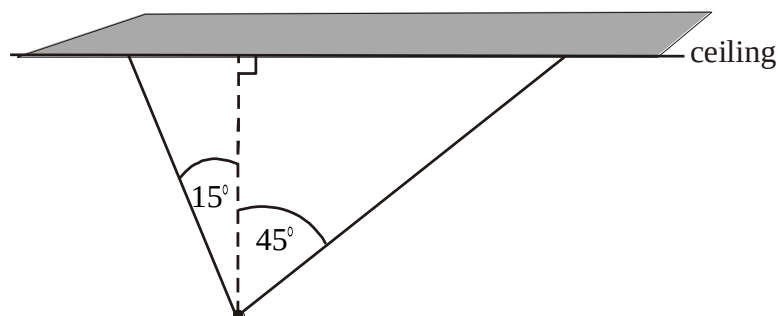
When the body passes through the origin its velocity $v \text{ m/s}$ is 4 m/s .

The function x expressed in terms of v is given by

- A. $x = 2 \sin(v - 4)$
- B. $x = 2 \cos(v^2 - 4)$
- C. $x = 2 \sin\left(\frac{v^2 - 8}{2}\right)$
- D. $x = 2 \cos\left(\frac{v^2 - 8}{2}\right)$
- E. $x = 2 \sin\left(\frac{v^2 - 16}{2}\right)$

Question 21

A mass of 12 kg is suspended from a horizontal ceiling by two light inextensible strings of different lengths. The shorter string makes an angle of 15° with the vertical and the longer string makes an angle of 45° with the vertical as shown below in the diagram.



The magnitude in newtons of the tension in the shorter string is

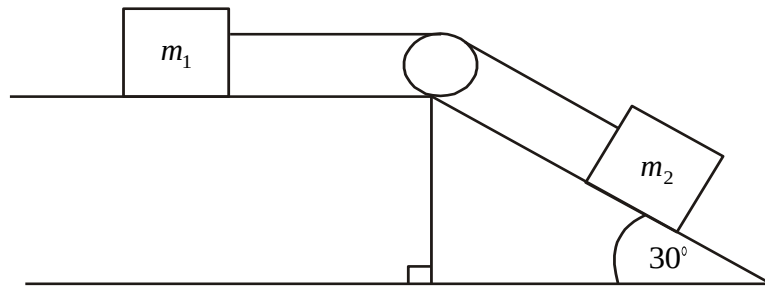
- A. $\sqrt{6}$
- B. $4\sqrt{6}$
- C. $\sqrt{6g}$
- D. $4\sqrt{6g}$
- E. $6\sqrt{6g}$

Question 22

A mass of m_1 kg rests on a rough horizontal surface and is connected by a light string that passes over a smooth pulley and is connected to a second mass m_2 kg.

This second mass rests on a plane with a rough surface inclined at an angle of 30° to the horizontal.

The coefficient of friction between the m_1 kg mass and the horizontal surface is μ as is the coefficient of friction between the m_2 kg mass and the inclined plane. The m_2 kg mass is on the point of slipping down the plane.



The ratio $\frac{m_1}{m_2}$ is equal to

- A. $\frac{1}{2\mu}$
- B. $\frac{1 - \sqrt{3}\mu}{2\mu}$
- C. $\frac{1 + \sqrt{3}\mu}{2\mu}$
- D. $\frac{\sqrt{3} - \mu}{2\mu}$
- E. $\frac{\sqrt{3} + \mu}{2\mu}$

SECTION 2**Question 1**

A red light moves around a closed shape in a lighting display. It has a position vector given by

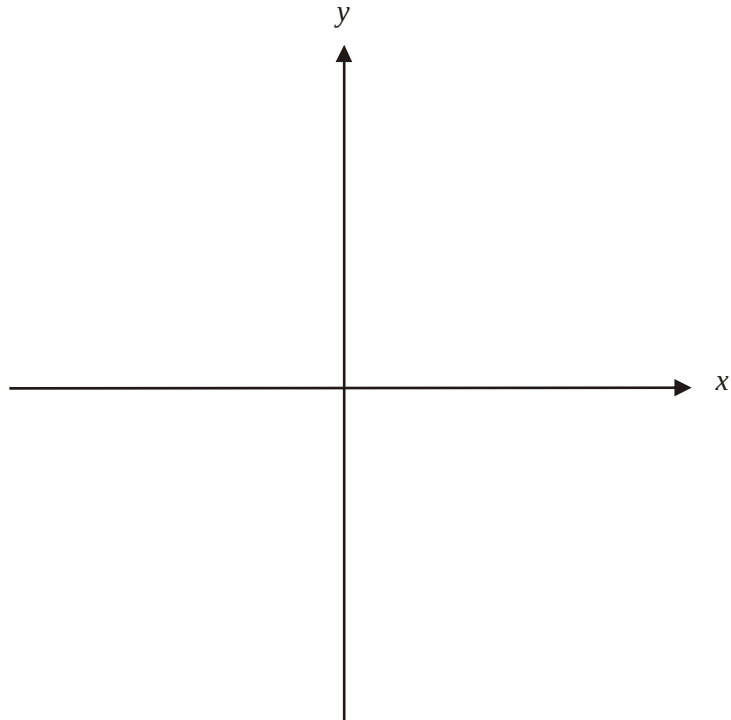
$$\underline{r}(t) = 2\cos(t)\underline{i} + 3\sin(t)\underline{j}, \quad t \geq 0$$

where t represents time in seconds and the displacement components are measured in metres.

- a.** Find the Cartesian equation of the path of the red light.

1 mark

- b.** Sketch the path of the red light on the set of axes below.



1 mark

- c. Describe the path of the red light including its starting point and the direction it travels in.

2 marks

- d. How long does it take the red light to complete one complete circuit of the closed shape? Express your answer in exact form.

1 mark

- e. Show that the speed of the red light is given by $\sqrt{4\sin^2(t) + 9\cos^2(t)}$.

2 marks

f. It is known that $\frac{d}{dt} \left(\sqrt{4 \sin^2(t) + 9 \cos^2(t)} \right) = \frac{-5 \sin(t) \cos(t)}{\sqrt{5 \cos^2(t) + 4}}.$

- i. Hence or otherwise find the time(s) when the speed of the red light is a maximum for $0 < t \leq 2\pi$.

- ii. What is the maximum speed of the red light?

2+1 = 3 marks
Total 10 marks

Question 2

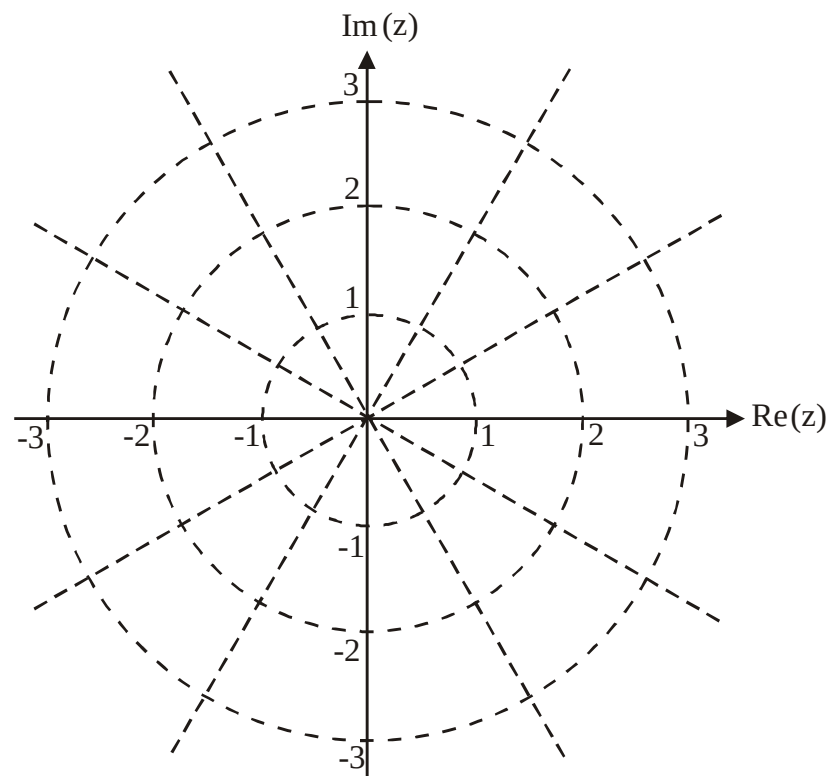
Let $z_1 = \sqrt{3} + i$.

- a. Express z_1 in polar form.

1 mark

z_1 is a root of the equation $z^3 = 8i$.

- b. Plot z_1 and the other roots of this equation on the Argand diagram below. Label each one clearly, expressing them in polar form.



3 marks

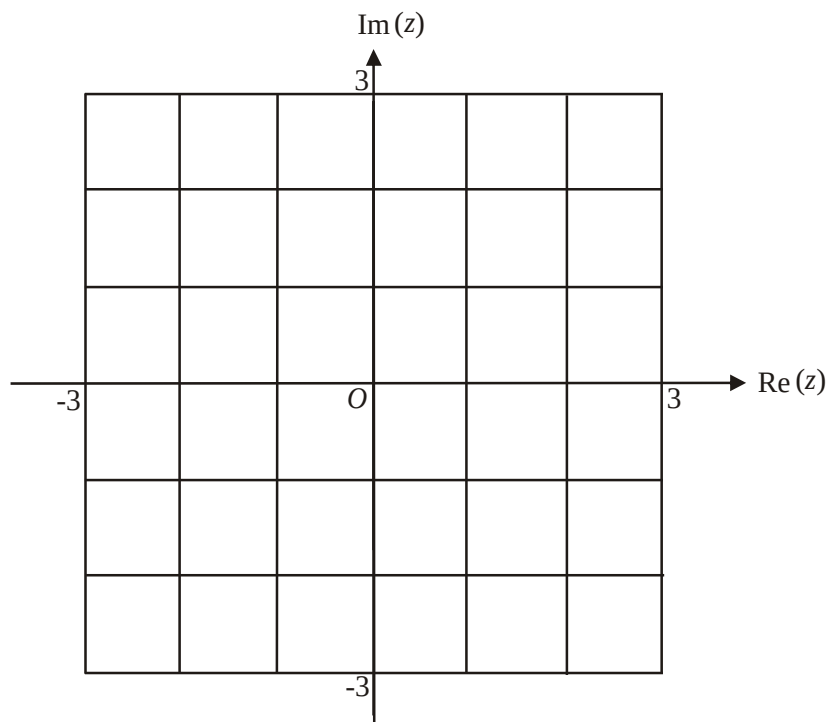
Let $z = x + iy$, $x, y \in \mathbb{R}$ and $z_1 = \sqrt{3} + i$ (from part a.)

- c. Show that the Cartesian equation for the relation $z \bar{z} + |z_1| \times \operatorname{Re}(i^2 z) - 2\operatorname{Im}(z) = -1$ is given by $(x-1)^2 + (y-1)^2 = 1$.

2 marks

Let $S = \{z : z \bar{z} + |z_1| \times \operatorname{Re}(i^2 z) - 2\operatorname{Im}(z) \leq -1 \text{ and } 0 \leq \operatorname{Arg}(z) \leq \frac{\pi}{4}\}$.

- d. Sketch the region described by S on the Argand diagram below.



3 marks

- e. If $z_2 \in S$, find the maximum and minimum values of $|z_2|$. Express your values in exact form.

3 marks
Total 12 marks

Question 3

A hot air balloon of mass 700kg lifts off the ground with a lift force of L newtons acting vertically upwards. The only other force acting on the hot air balloon during this time is the gravitational force. The balloon accelerates at 0.5m/s^2 .

- a.** Find the value of L .

1 mark

- b.** How long does it take for the balloon to be 50m above the ground?

1 mark

- c.** What is the velocity of the balloon at this altitude?

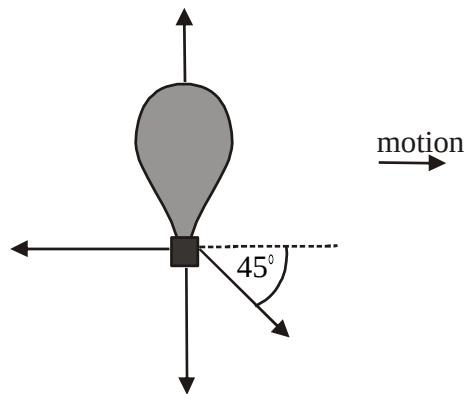
1 mark

After the hot air balloon has completed its flight it prepares to land. A rope is dropped from the balloon to the crew below who anchor the rope to a vehicle. The pilot of the balloon maintains the balloon at a constant altitude whilst it is towed in a straight line by the vehicle to a safe landing spot. The horizontal velocity of the balloon is v m/s.

During this phase, the forces acting on the balloon are a lift force L newtons, the towing force of 1200 newtons, the resistance force of $\frac{v}{50}$ newtons and the gravitational force.

The rope used to tow the balloon makes an angle of 45° to the horizontal.

- d. On the diagram below label the forces acting on the hot air balloon.



1 mark

Whilst the pilot of the balloon maintains a constant altitude, the balloon is towed horizontally at a m/s².

- e. Write down the equation of motion for the balloon during this phase and express a in terms of v .

2 marks

- f.** Find the distance covered by the balloon when $v = 5\text{m/s}$ given that $v = 0$ when the vehicle began towing the balloon. Express your answer in metres correct to 2 decimal places.

2 marks

In order to land safely the balloon needs to be towed 100m.

- g.** Using an appropriate integral, verify that when $v = 15 \cdot 5685\text{ m/s}$, the balloon has been towed 100m; to the nearest whole number, by the vehicle.

1 mark

- h.** Hence how long will it take to tow the balloon to safety? Express your answer correct to 2 decimal places.

3 marks

Total 12 marks

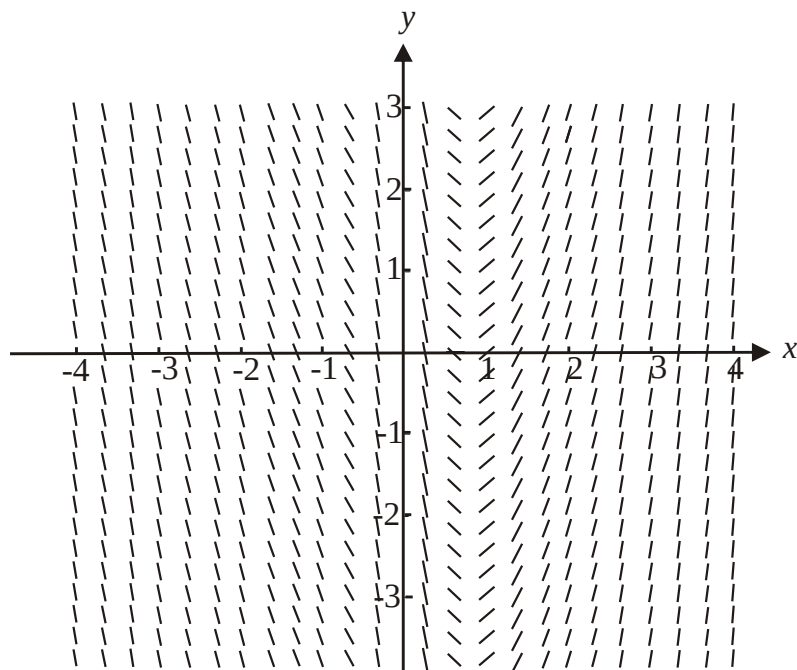
Question 4

Consider the differential equation

$$\frac{dy}{dx} = 2x - \frac{1}{x^2}.$$

One of the branches of the graph of one of the solutions to this differential equation passes through the point $(-1, -2)$.

- a. On the direction (slope) field for this differential equation below, sketch both branches of the graph of this particular solution.



3 marks

- b.** For the particular solution to the differential equation that passes through the point $(-1, -2)$ use Eulers method with a step size of 0.25 to find an approximate value of y when $x = -\frac{1}{2}$. Express your answer correct to 2 decimal places.

2 marks

- c.** Verify that one of the branches of the graph of each of the solutions to this differential equation has a point of inflection and find the x -coordinate of the point of inflection.

3 marks

Let c be a constant of antidifferentiation obtained when the differential equation is solved.

- d.** For $x > 0$, find the value of c for which the graph of the solution to the differential equation touches but does not cut the x -axis. Express your answer correct to 4 decimal places.

2 marks
Total 10 marks

Question 5

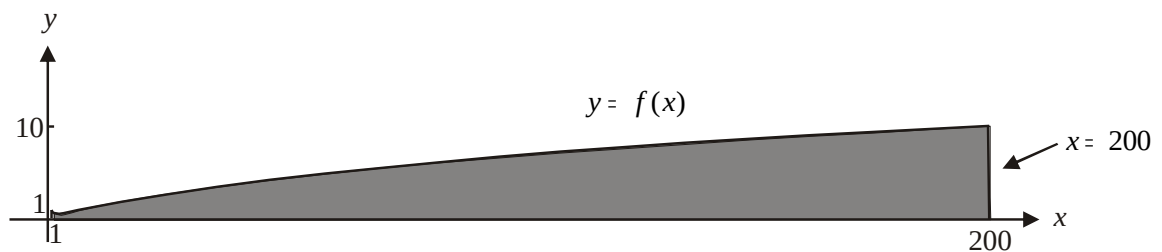
A function f has a rule given by $f(x) = \sqrt{\frac{x^3}{2x^2 - 1}}$.

- a.** Write down the maximal domain of the function f .

1 mark

A bridge that spans a harbour is constructed with pylons. Each pylon is constructed by rotating the region enclosed by the graph of the function $y = f(x)$, the x -axis and the lines $x = 1$ and $x = 200$ around the x -axis.

This region is indicated by the shaded area shown below.



The unit of measurement is the metre.

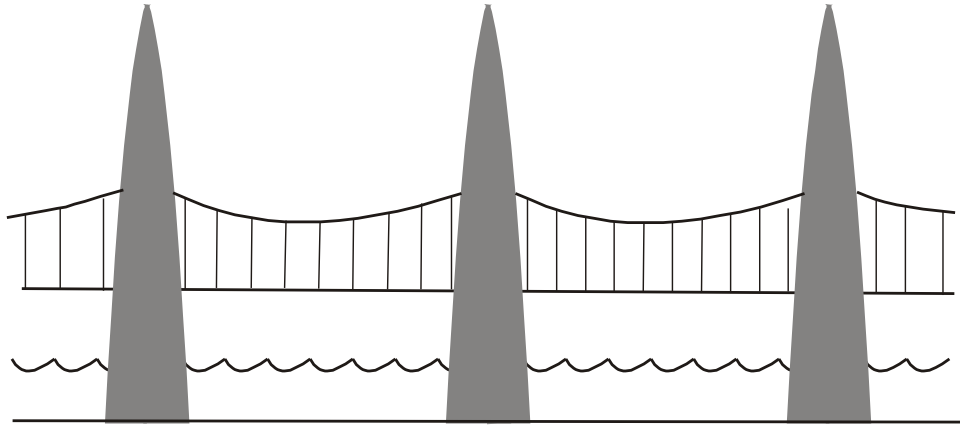
- b. i.** Write down an expression for V , the volume of the solid of rotation formed.

- ii. Use a suitable substitution of $u = g(x)$ to write an integral expression for V in terms in u .

- iii. Hence use calculus to find V in exact form.

1+2+2=5 marks

A cross-section of the bridge is shown below with the pylons standing vertically.



- c. What is the height of each pylon?

_____ 1 mark

- d. What is the diameter of each pylon at its base? Express your answer to the nearest whole metre.

_____ 1 mark

For the function $f(x) = \sqrt{\frac{x^3}{2x^2-1}}$, the first and second derivatives are given by

$$f'(x) = \frac{x^2(2x^2-3)}{2(2x^2-1)^2 \sqrt{\frac{x^3}{2x^2-1}}} \text{ and } f''(x) = \frac{-x(4x^4-20x^2-3)}{4(2x^2-1)^3 \sqrt{\frac{x^3}{2x^2-1}}}$$

- e.** Use $f'(x)$ to find the minimum radius of each of the pylons. Express your answer in metres correct to 2 decimal places.

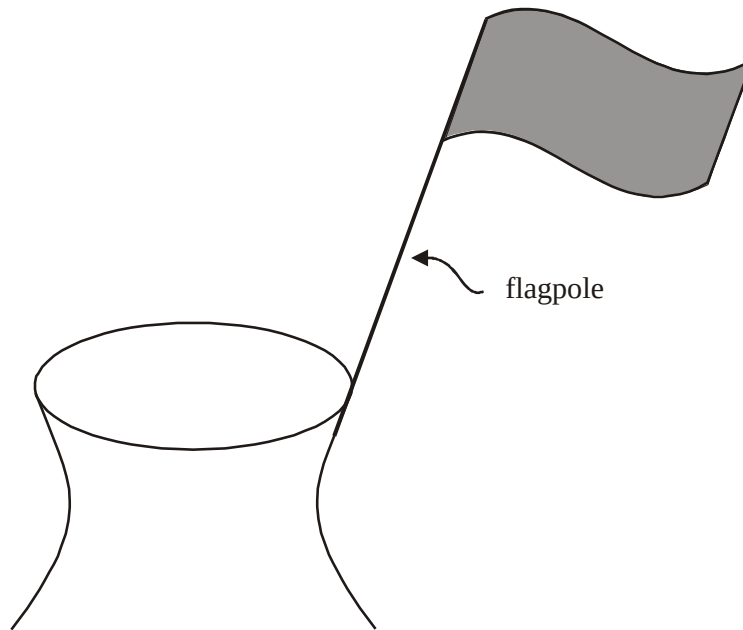
2 marks

Aircraft hazard lights are installed on the pylons at a point that coincides with the point of inflection on the graph of $y = f(x)$.

- f.** Find the vertical distance of these lights from the top of one of the pylons. Express your answer in metres correct to 2 decimal places.

2 marks

A flagpole is attached to the side of one of the pylons. The flagpole is a tangent to the outside of the pylon at the pylon's highest point.



- g. Find the angle that the flagpole makes with the vertical. Express your answer in degrees correct to one decimal place.

2 marks
Total 14 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\begin{aligned} \cos^2(x) + \sin^2(x) &= 1 \\ 1 + \tan^2(x) &= \sec^2(x) & \cot^2(x) + 1 &= \operatorname{cosec}^2(x) \\ \sin(x+y) &= \sin(x)\cos(y) + \cos(x)\sin(y) & \sin(x-y) &= \sin(x)\cos(y) - \cos(x)\sin(y) \\ \cos(x+y) &= \cos(x)\cos(y) - \sin(x)\sin(y) & \cos(x-y) &= \cos(x)\cos(y) + \sin(x)\sin(y) \\ \tan(x+y) &= \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)} & \tan(x-y) &= \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)} \\ \cos(2x) &= \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x) \\ \sin(2x) &= 2\sin(x)\cos(x) & \tan(2x) &= \frac{2\tan(x)}{1 - \tan^2(x)} \end{aligned}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$\begin{aligned} z &= x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta \\ |z| &= \sqrt{x^2 + y^2} = r & -\pi < \operatorname{Arg} z \leq \pi \\ z_1 z_2 &= r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2) & \frac{z_1}{z_2} &= \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2) \\ z^n &= r^n \operatorname{cis}(n\theta) & & \text{(de Moivre's theorem)} \end{aligned}$$

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Calculus

$$\frac{d}{dx} x^n = nx^{n-1}$$

$$\frac{d}{dx} e^{ax} = ae^{ax}$$

$$\frac{d}{dx} \log_e(x) = \frac{1}{x}$$

$$\frac{d}{dx} \sin(ax) = a \cos(ax)$$

$$\frac{d}{dx} \cos(ax) = -a \sin(ax)$$

$$\frac{d}{dx} \tan(ax) = a \sec^2(ax)$$

$$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b,$$

$$\text{then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

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Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Mechanics

momentum: $\vec{p} = m\vec{v}$

equation of motion: $\vec{R} = m\vec{a}$

friction: $F \leq \mu N$

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SPECIALIST MATHEMATICS

TRIAL EXAMINATION 2

MULTIPLE- CHOICE ANSWER SHEET

STUDENT NAME:.....

INSTRUCTIONS

Fill in the letter that corresponds to your choice. Example: ☐ A ☒ B ☐ C ☐ D ☐ E

The answer selected is B. Only one answer should be selected.

- | | |
|---|---|
| 1. ☐ A ☐ B ☐ C ☐ D ☐ E | 12. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 2. ☐ A ☐ B ☐ C ☐ D ☐ E | 13. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 3. ☐ A ☐ B ☐ C ☐ D ☐ E | 14. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 4. ☐ A ☐ B ☐ C ☐ D ☐ E | 15. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 5. ☐ A ☐ B ☐ C ☐ D ☐ E | 16. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 6. ☐ A ☐ B ☐ C ☐ D ☐ E | 17. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 7. ☐ A ☐ B ☐ C ☐ D ☐ E | 18. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 8. ☐ A ☐ B ☐ C ☐ D ☐ E | 19. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 9. ☐ A ☐ B ☐ C ☐ D ☐ E | 20. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 10. ☐ A ☐ B ☐ C ☐ D ☐ E | 21. ☐ A ☐ B ☐ C ☐ D ☐ E |
| 11. ☐ A ☐ B ☐ C ☐ D ☐ E | 22. ☐ A ☐ B ☐ C ☐ D ☐ E |