The Mathematical Association of Victoria

SPECIALIST MATHEMATICS

Trial written examination 1

2008

Reading time: 15 minutes
Writing time: 1 hour

Student's Name: ....

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are NOT permitted to bring the examination room: notes of any kind, a calculator or any type, blank sheets of paper and/or white-out liquid/tape

These questions have been written and published to assist students in their preparations for the 2008 Specialist Mathematics Examination 1. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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Instructions

Answer **all** questions in the spaces provided.
A decimal approximation will not be accepted if an **exact** answer is required to a question.
In questions where more than one mark is available, appropriate working **must** be shown.
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
Take the **acceleration due to gravity** to have magnitude g m/s^2, where $g = 9.8$.

Question 1

Given $z = 6\text{cis}\left(\dfrac{5\pi}{6}\right)$ is a solution of the equation $\left\{z : z^3 = m\right\}$.

a. Show that $m = 216i$.

1 mark

b. Find the other solutions of $\left\{z : z^3 = m\right\}$ in polar form.

2 marks

Question 2

Let $u = 1 + i$ and $v = 2\text{cis}\left(-\frac{\pi}{6}\right)$.

a. Express u in polar form.

1 mark

b. Express the product uv in simplest polar form.

1 mark

c. Express v in Cartesian form.

1 mark

d. Express the product uv in Cartesian form.

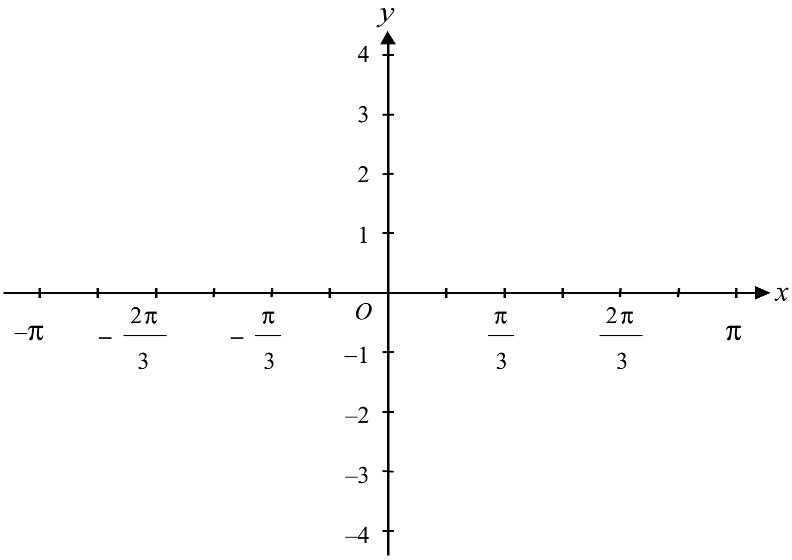
1 mark

e. Hence determine the exact value of $\sin\left(\frac{\pi}{12}\right)$.

1 mark

Question 3

Sketch the graph of $f:[-\pi,\pi]\rightarrow R,\ f(x)=\operatorname{cosec}\left(2x+\frac{\pi}{3}\right)$ on the axes below, labelling endpoint coordinates, equations of asymptotes and axial intercepts in exact form where they exist.



3 marks

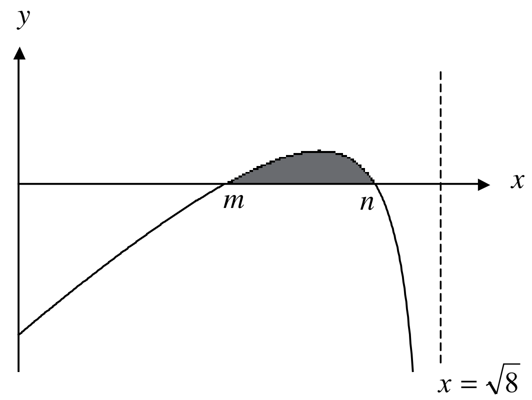
Question 4

Find the gradient of the tangent to the curve with equation $x\sin(y) = 1$ at the point where $y = \dfrac{\pi}{6}$.

3 marks

Question 5

The graph of the function $f:\left[0, \sqrt{8}\right) \rightarrow R,\ f\left(x\right) = x - 2\sqrt{\dfrac{3}{8 - x^2}}$ is shown below.



a. Show that $m = \sqrt{2}$ and $n = \sqrt{6}$

2 marks

b. Determine the exact area of the shaded region.

3 marks

Question 6

Solve the differential equation $\dfrac{dy}{dx} = \dfrac{1+x}{(1-x)^2}$, $x \neq 1$, given $y = 0$ when $x = 0$.

4 marks

Question 7

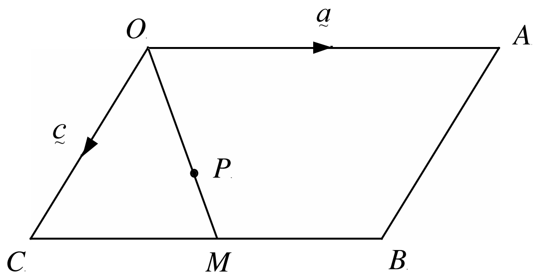
Let $\underset{\sim}{a} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = -\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$

Find the vector resolute of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$.

3 marks

Question 8

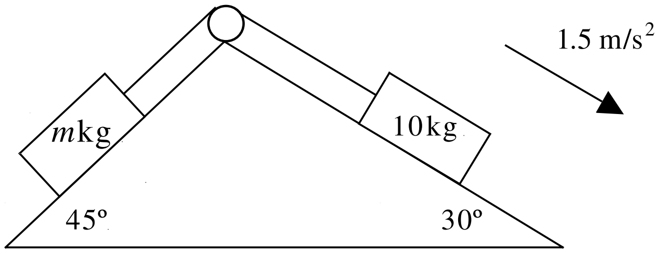
In parallelogram $OABC$, $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.
Point M is the midpoint of $\overrightarrow{CB}$. Point P is such that $\overrightarrow{OP} = \frac{2}{3}\overrightarrow{OM}$.
Prove that $\overrightarrow{AP} = 2\overrightarrow{PC}$.



3 marks

Question 9

A mass of m kg and a mass of 10 kg are connected on a smooth back-to-back plane by a light string passing over a smooth pulley. The back-to-back plane is inclined at angles of 45º and 30º to the horizontal level as shown below.



Assume the 10 kg mass is moving down the plane with an acceleration of 1.5 m/s^2.

a. Find the tension in the string connecting the two masses in newtons.

2 marks

b. Determine the exact value of m giving your answer in the form $\dfrac{a}{g\sqrt{b}+c}$ where a, b and c are positive integers and g is the acceleration due to gravity.

2 marks

Question 10

A curve is defined by the parametric equations $x = \dfrac{t}{t^2+1}$ and $y = \dfrac{1}{t^2+1}$, $t \geq 0$.

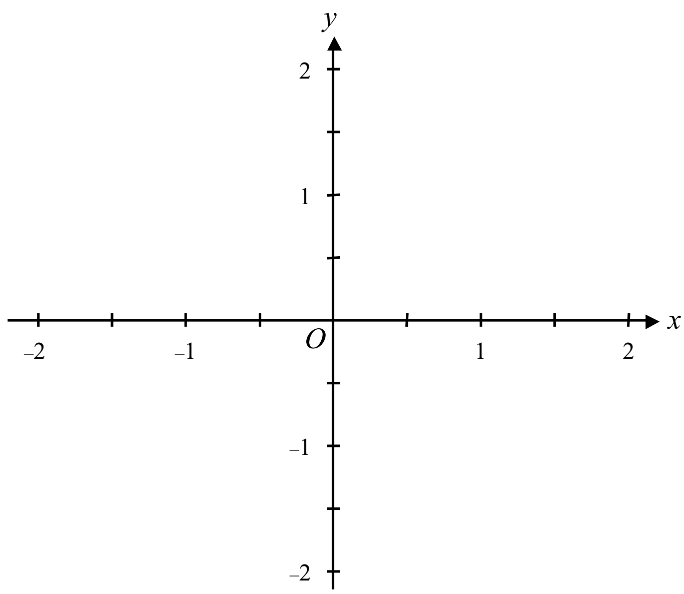
a. Find $\dfrac{dy}{dx}$ when $t = 2$.

3 marks

b. Find the Cartesian equation of the curve.

2 marks

c. Sketch a graph of the curve on the axes below.



2 marks

END OF QUESTION AND ANSWER BOOK

Working space

SPECIALIST MATHEMATICS

Trial written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time This formula sheet is provided for your reference

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$\cos^2(x) + \sin^2(x) = 1$

$1 + \tan^2(x) = \sec^2(x)$

$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\sin(x - y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

$\cos(x - y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$

$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\sin(2x) = 2\sin(x)\cos(x)$

$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + yi = r(\cos\theta + i\sin\theta) = r\,\text{cis}\,\theta$

$|z| = \sqrt{x^2 + y^2} = r$ $\quad -\pi < \text{Arg}\, z \le \pi$

$z_1 z_2 = r_1 r_2\, \text{cis}(\theta_1 + \theta_2)$ $\quad \dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\text{cis}(\theta_1 - \theta_2)$

$z^n = r^n\, \text{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$\dfrac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\quad \int x^n\,dx = \dfrac{1}{n+1}x^{n+1} + c, n \ne -1$

$\dfrac{d}{dx}\left(e^{ax}\right) = ae^{ax}$ $\quad \int e^{ax}\,dx = \dfrac{1}{a}e^{ax} + c$

$\dfrac{d}{dx}\left(\log_e(x)\right) = \dfrac{1}{x}$ $\quad \int \dfrac{1}{x}\,dx = \log_e|x| + c$

$\dfrac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$ $\quad \int \sin(ax)\,dx = -\dfrac{1}{a}\cos(ax) + c$

$\dfrac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$ $\quad \int \cos(ax)\,dx = \dfrac{1}{a}\sin(ax) + c$

$\dfrac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$ $\quad \int \sec^2(ax)\,dx = \dfrac{1}{a}\tan(ax) + c$

$\dfrac{d}{dx}\left(\sin^{-1}(x)\right) = \dfrac{1}{\sqrt{1-x^2}}$ $\quad \int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\cos^{-1}(x)\right) = \dfrac{-1}{\sqrt{1-x^2}}$ $\quad \int \dfrac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\dfrac{x}{a}\right) + c, a > 0$

$\dfrac{d}{dx}\left(\tan^{-1}(x)\right) = \dfrac{1}{1+x^2}$ $\quad \int \dfrac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\dfrac{x}{a}\right) + c$

product rule: $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

quotient rule: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

chain rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

Euler's method: If $\dfrac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \dfrac{d^2x}{dt^2} = \dfrac{dv}{dt} = v\dfrac{dv}{dx} = \dfrac{d}{dx}\left(\dfrac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \dfrac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \dfrac{1}{2}(u + v)t$

TURN OVER

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1x_2 + y_1y_2 + z_1z_2$

$\dot{\underset{\sim}{r}} = \dfrac{d\underset{\sim}{r}}{dt} = \dfrac{dx}{dt}\underset{\sim}{i} + \dfrac{dy}{dt}\underset{\sim}{j} + \dfrac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum: $\underset{\sim}{p} = m\underset{\sim}{v}$

equation of motion: $\underset{\sim}{R} = m\underset{\sim}{a}$

friction: $F \le \mu N$

END OF FORMULA SHEET