

Year 2008

VCE

Specialist Mathematics

Trial Examination 1



KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9817 5374
FAX: (03) 9817 4334
kilbaha@gmail.com
<http://kilbaha.googlepages.com>

IMPORTANT COPYRIGHT NOTICE

- This material is copyright. Subject to statutory exception and to the provisions of the relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Kilbaha Pty Ltd.
 - The contents of this work are copyrighted. Unauthorised copying of any part of this work is illegal and detrimental to the interests of the author.
 - For authorised copying within Australia please check that your institution has a licence from Copyright Agency Limited. This permits the copying of small parts of the material, in limited quantities, within the conditions set out in the licence.
 - Teachers and students are reminded that for the purposes of school requirements and external assessments, students must submit work that is clearly their own.
 - Schools which purchase a licence to use this material may distribute this electronic file to the students at the school for their exclusive use. This distribution can be done either on an Intranet Server or on media for the use on stand-alone computers.
 - Schools which purchase a licence to use this material may distribute this printed file to the students at the school for their exclusive use.
-
- **The Word file (if supplied) is for use ONLY within the school.**
 - **It may be modified to suit the school syllabus and for teaching purposes.**
 - **All modified versions of the file must carry this copyright notice.**
 - **Commercial use of this material is expressly prohibited.**

**Victorian Certificate of Education
2008**

STUDENT NUMBER

Figures	<table border="1"><tr><td></td><td></td></tr></table>			<table border="1"><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>								Letter	
Words	<table border="1"><tr><td></td><td></td></tr></table>			<table border="1"><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table>								<table border="1"><tr><td></td></tr></table>	

SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 12 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above on this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

Question 1

Consider the relation $x^3 - 4x^2y^2 + 2y^2 = 7$. Find an expression for $\frac{dy}{dx}$ in terms of x and y .

2 marks

Question 2

Find the volume generated when the region enclosed by the curve with the equation

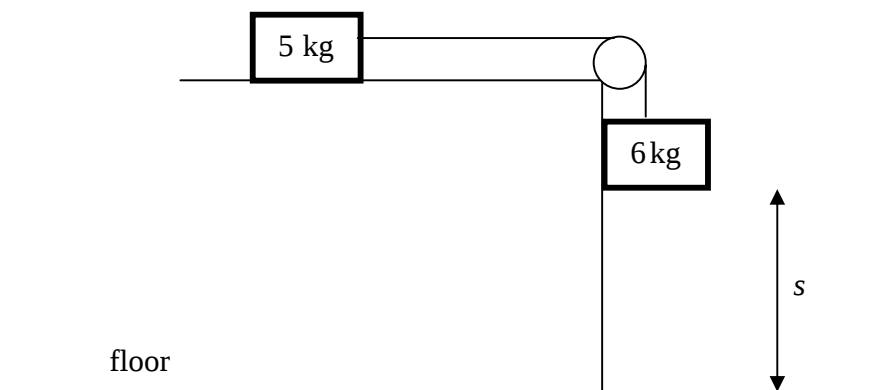
$y = \frac{3}{\sqrt{9+4x^2}}$, the x -axis, the y -axis and the line $x = \frac{3}{2}$ is rotated about the x -axis to

form a solid of revolution.

2 marks

Question 3

A block of mass 5 kg rests on a horizontal table. The coefficient of friction between the block and the table surface is 0.2. The block is connected by a light string which passes over a smooth pulley at the edge of the table to another block of mass 6 kg which is hanging vertically at the edge of the table. The system is released from rest, when the 6 kg block is s metres above the floor. After one half of a second, the 6 kg mass hits the floor. Find the value of s .



4 marks

Question 4

- a. Solve the quadratic equation $z^2 + 2zi - 4 = 0$, expressing your answers in exact cartesian form.

2 marks

- b. If $z = -\sqrt{3} - i$, express z in polar form and, hence, find z^6 giving your answer in exact cartesian form.

3 marks

Question 5

- a. Show that $\frac{d}{dx}\left(\sin^{-1}\left(\frac{3}{\sqrt{x}}\right)\right) = \frac{-3}{2x\sqrt{x-9}}$ for $x > 9$.

2 marks

- b. Hence, find the exact value of $\int_{12}^{18} \frac{1}{x\sqrt{x-9}} dx$.

3 marks

Question 6

- a.** Use Euler's method to find y_2 if $\frac{dy}{dx} = \log_e(2x-3)$, given that $y_0 = y(2) = 1$ and $h = 0.5$. Express your answer in the form $\log_e(p)$, where p is a real positive constant.

2 marks

- b.** Differentiate $(2x-3)\log_e(2x-3)$ and, hence, solve the differential given in part **a.** to find the value of y which is estimated by y_2 . Express your answer in the form $\log_e(q)$, where q is a real positive constant.

4 marks

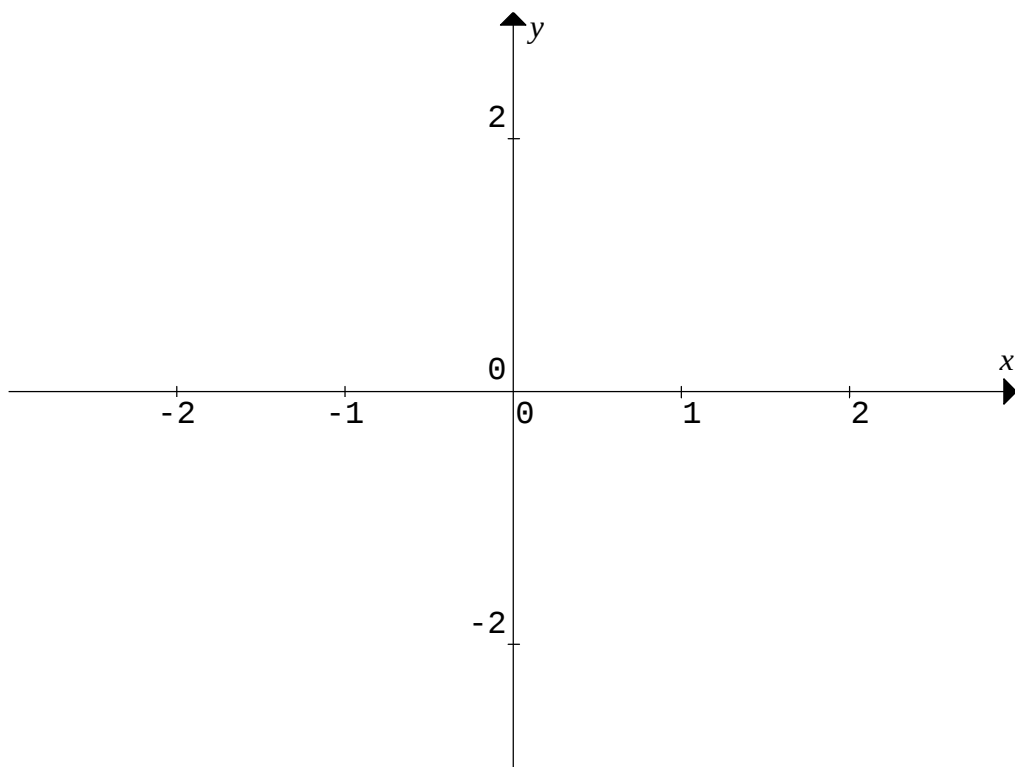
Question 7

The position vector of a moving particle is given by $\mathbf{r}(t) = e^{-t}\mathbf{i} + 2e^{-2t}\mathbf{j}$ for $t \geq 0$.

- a.** Find the Cartesian equation of the path.

1 mark

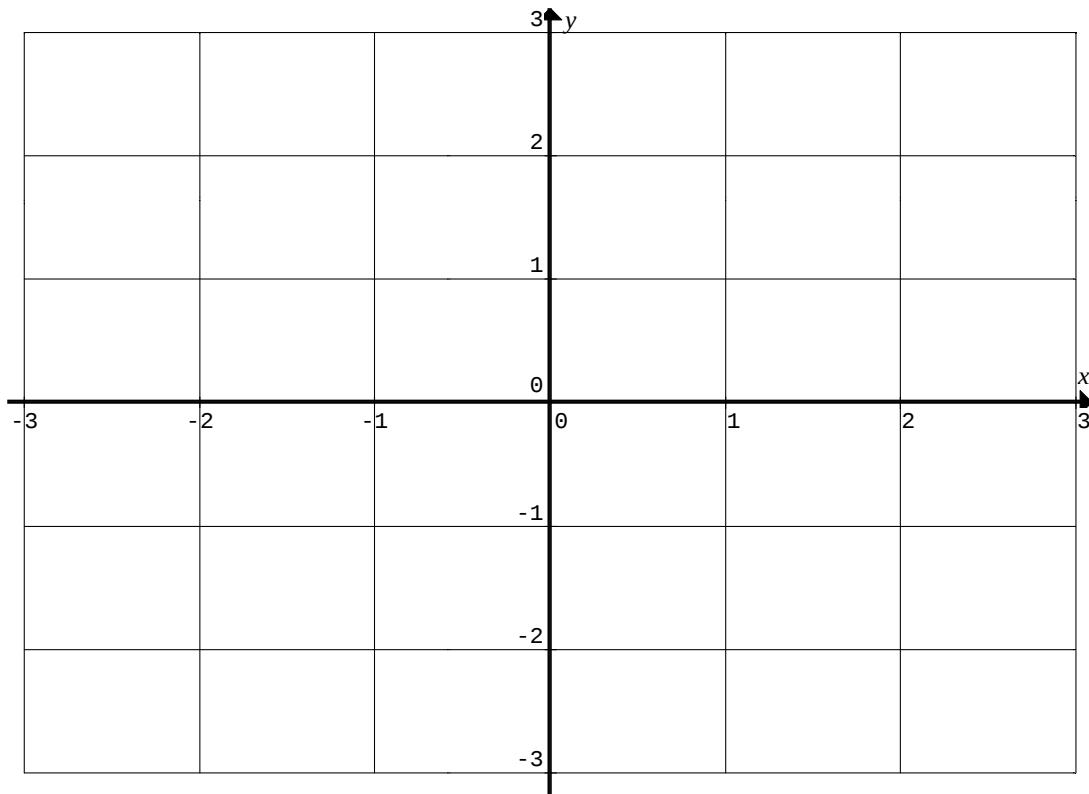
- b.** Sketch the path of the particle on the axes provided.



1 mark

Question 8

- a.** Sketch the slope field of the differential equation $2\frac{dy}{dx} + y = 0$ for $y = -2, -1, 0, 1, 2$ at each of the values $x = -2, -1, 0, 1, 2$ on the axes below.



2 marks

- b.** If $y = -1$ when $x = 0$, solve the differential equation given in part **a.** to find y in terms of x .

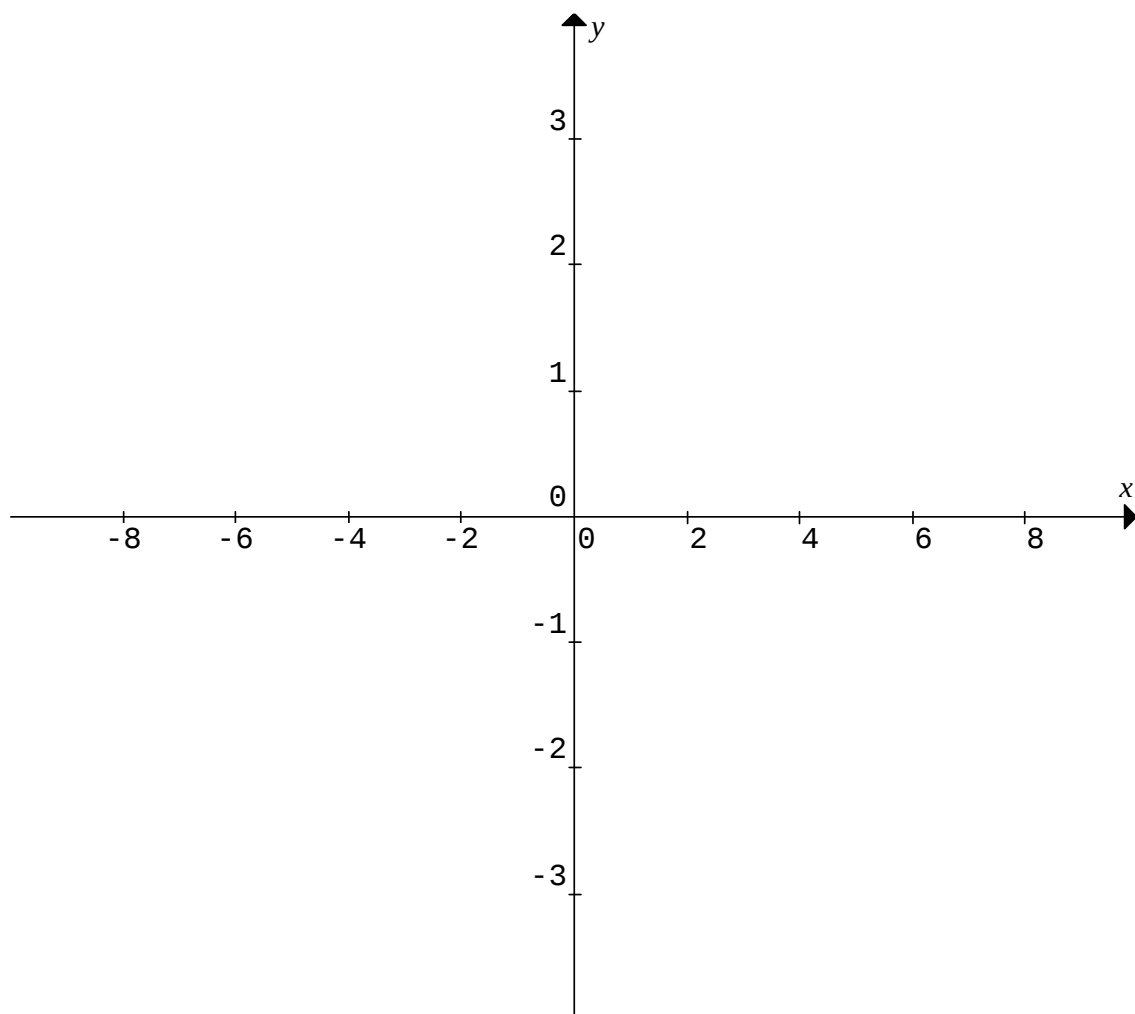
2 marks

- c.** Sketch the graph of the solution curve found in part **b.** on the slope field in part **a.**

1 mark

Question 9

- a. Sketch the graph with the equation $y = \frac{32}{x^2 - 16}$, on the axes below, clearly indicating the location of all asymptotes, any turning points and axial intercepts.



2 marks

- b. Find the area bounded by $y = \frac{32}{x^2 - 16}$, the x -axis, the y -axis and the line $x = -2$.
Express your answer in the form $\log_e(a)$, where a is a real positive constant.

4 marks

Question 10

Given that $\cos(x) - \sin(x) = \frac{1}{3}$ and $0 < x < \frac{\pi}{4}$, find the exact value of $\cot(2x)$.

3 marks

END OF EXAMINATION**© KILBAHA PTY LTD 2008**

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of triangle: $\frac{1}{2}bc \sin(A)$

sine rule: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$-\pi < \operatorname{Arg}(z) \leq \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$$

Mechanics

momentum: $\underline{p} = m\underline{v}$

equation of motion: $\underline{R} = m\underline{a}$

sliding friction: $F \leq \mu N$

constant (uniform) acceleration:

$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u + v)t$$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2}v^2 \right)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \quad \text{for } x > 0$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

Euler's method If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x)$

END OF FORMULA SHEET