

Year 2008

VCE

Specialist Mathematics

Solutions

Trial Examination 1



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Question 1

$x^3 - 4x^2y^2 + 2y^2 = 7$ taking $\frac{d}{dx}$ of each term (implicit differentiation)

$$\frac{d}{dx}(x^3) - \frac{d}{dx}(4x^2y^2) + \frac{d}{dx}(2y^2) = \frac{d}{dx}(7)$$

product rule in the second term

$$3x^2 - \left(8xy^2 + 8x^2y \frac{dy}{dx} \right) + 4y \frac{dy}{dx} = 0 \quad \text{M1}$$

$$3x^2 - 8xy^2 = (8x^2y - 4y) \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2 - 8xy^2}{8x^2y - 4y} \quad \text{A1}$$

Question 2

$$y = \frac{3}{\sqrt{9+4x^2}} \quad V = \pi \int_a^b y^2 dx$$

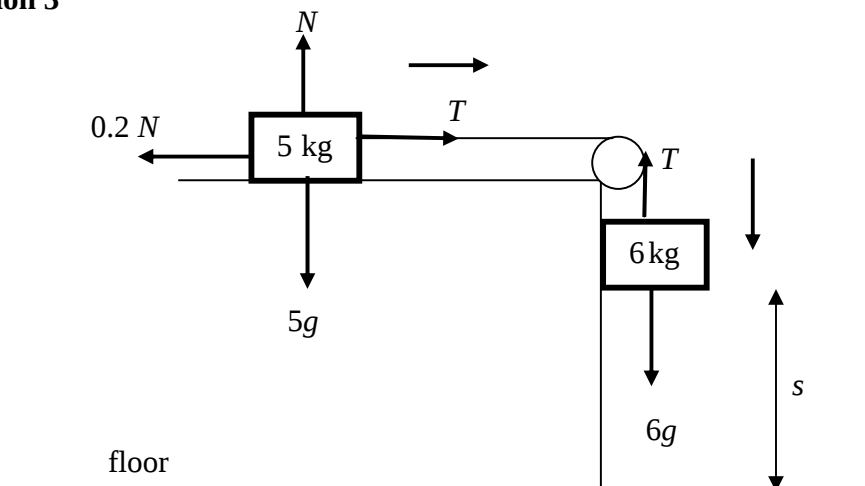
$$V = \pi \int_0^{\frac{3}{2}} \frac{9}{9+4x^2} dx \quad \text{let } u = 2x \quad \frac{du}{dx} = 2 \quad \text{terminals } x = \frac{3}{2} \quad u = 3 \quad \text{and } x = 0 \quad u = 0$$

$$V = \frac{9\pi}{2} \int_0^3 \frac{1}{9+u^2} du \quad \text{M1}$$

$$V = \frac{9\pi}{2} \left[\frac{1}{3} \tan^{-1} \left(\frac{u}{3} \right) \right]_0^3$$

$$V = \frac{3\pi}{2} \left[\tan^{-1}(1) - \tan^{-1}(0) \right]$$

$$V = \frac{3\pi^2}{8} \quad \text{A1}$$

Question 3

A1

let a be the acceleration of the system, and T the tension in the string.
resolving horizontally on the table around the 5 kg block.

$$(1) \quad T - 0.2N = 5a$$

resolving vertically on the table around the 5 kg block.

M1

$$(2) \quad N - 5g = 0 \quad \Rightarrow N = 5g \quad \text{substitute into (1) gives } T - 0.2 \times 5g = 5a$$

$$\text{or } T - g = 5a$$

resolving vertically downwards around the 6 kg block hanging vertically.

$$(3) \quad 6g - T = 6a \quad \text{adding these last two equations, to eliminate } T \text{ gives, } 5g = 11a$$

$$a = \frac{5g}{11} = \frac{5 \times 9.8}{11} = \frac{49}{11} \text{ m/s}^2 \quad s = ? \quad u = 0 \quad t = \frac{1}{2} \text{ sec} \quad \text{M1}$$

$$\text{using } s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2} \times \frac{49}{11} \times \left(\frac{1}{2}\right)^2 = \frac{49}{88} \text{ metres} \quad \text{A1}$$

Question 4

a. $z^2 + 2zi - 4 = 0$

using the quadratic formulae with $a = 1$ $b = 2i$ $c = -4$

$$\Delta = b^2 - 4ac = (2i)^2 + 16 = -4 + 16 = 12$$

$$z = \frac{-b \pm \sqrt{\Delta}}{2a} \quad \text{M1}$$

$$z = \frac{-2i \pm \sqrt{12}}{2} = \frac{-2i \pm 2\sqrt{3}}{2}$$

$$z = \sqrt{3} - i \quad \text{and} \quad -\sqrt{3} - i \quad \text{A1}$$

b. $z = -\sqrt{3} - i \quad |z| = \sqrt{(-\sqrt{3})^2 + (-1)^2} = 2$

$$\text{Arg}(z) = -\pi + \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = -\pi + \frac{\pi}{6} = -\frac{5\pi}{6}$$

$$z = 2\text{cis}\left(-\frac{5\pi}{6}\right) \quad \text{A1}$$

$$z^6 = \left(2\text{cis}\left(-\frac{5\pi}{6}\right)\right)^6 = 2^6 (\text{cis}(-5\pi)) \quad \text{M1}$$

$$z^6 = 64\text{cis}(-\pi)$$

$$z^6 = -64 \quad \text{A1}$$

Question 5

a. Let $y = \sin^{-1}\left(\frac{3}{\sqrt{x}}\right) = \sin^{-1}(u)$ where $u = \frac{3}{\sqrt{x}} = 3x^{-\frac{1}{2}}$

$$\frac{dy}{du} = \frac{1}{\sqrt{1-u^2}} \quad \frac{du}{dx} = -\frac{3}{2}x^{-\frac{3}{2}} = \frac{-3}{2\sqrt{x^3}} \quad \text{chain rule} \quad \text{M1}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \frac{-3}{2\sqrt{x^3} \sqrt{1-\frac{9}{x}}}$$

$$\frac{dy}{dx} = \frac{-3}{2\sqrt{x^3} \sqrt{\frac{x-9}{x}}} \quad \text{since } x > 9$$

$$\frac{dy}{dx} = \frac{-3}{2x\sqrt{x-9}}$$

so shown $\frac{d}{dx}\left(\sin^{-1}\left(\frac{3}{\sqrt{x}}\right)\right) = \frac{-3}{2x\sqrt{x-9}} \quad \text{for } x > 9 \quad \text{A1}$

b. $\int_{12}^{18} \frac{1}{x\sqrt{x-9}} dx.$

$$-\frac{2}{3} \left[\sin^{-1}\left(\frac{3}{\sqrt{x}}\right) \right]_{12}^{18} \quad \text{M1}$$

$$= -\frac{2}{3} \left(\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) - \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \right) \quad \text{M1}$$

$$= -\frac{2}{3} \left(\frac{\pi}{4} - \frac{\pi}{3} \right)$$

$$= \frac{\pi}{18} \quad \text{A1}$$

Question 6

- a. Using Euler's method $\frac{dy}{dx} = \log_e(2x-3)$ $y(2)=1$ $h=0.5$

$$x_0 = 2 \quad y_0 = 1 \quad h = \frac{1}{2} \quad f(x) = \log_e(2x-3)$$

$$y_1 = y_0 + h f(x_0)$$

$$y_1 = 1 + \frac{1}{2} \log_e(1) = 1 \quad x_1 = x_0 + h = \frac{5}{2} \quad \text{M1}$$

$$y_2 = y_1 + h f(x_1)$$

$$y_2 = 1 + \frac{1}{2} \log_e(2) = \log_e(e) + \log_e(\sqrt{2}) = \log_e(\sqrt{2}e)$$

$$p = \sqrt{2}e \quad \text{A1}$$

- b. $\frac{d}{dx}[(2x-3)\log_e(2x-3)] = (2x-3) \times \frac{2}{(2x-3)} + 2\log_e(2x-3)$

$$\frac{d}{dx}[(2x-3)\log_e(2x-3)] = 2 + 2\log_e(2x-3) \quad \text{A1}$$

$$\text{Hence } \int (2 + 2\log_e(2x-3)) dx = 2x + 2 \int \log_e(2x-3) dx = (2x-3)\log_e(2x-3)$$

$$\int \log_e(2x-3) dx = \frac{1}{2} [(2x-3)\log_e(2x-3) - 2x]$$

$$\frac{dy}{dx} = \log_e(2x-3) \quad y = \int \log_e(2x-3) dx$$

$$y = \frac{1}{2} [(2x-3)\log_e(2x-3)] - x + C \quad \text{to find } C \text{ use } x=2 \text{ when } y=1 \quad \text{M1}$$

$$1 = \frac{1}{2} [(4-3)\log_e(4-3)] - 2 + C \quad \Rightarrow C = 3$$

$$y(x) = \frac{1}{2} [(2x-3)\log_e(2x-3)] - x + 3 \quad \text{A1}$$

$$y(3) = \frac{3}{2} \log_e(3)$$

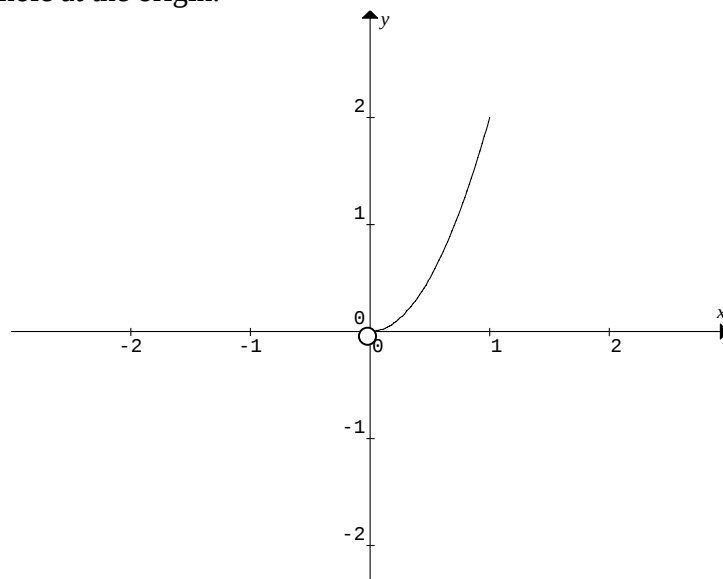
$$y(3) = \log_e(\sqrt{27}) \quad q = \sqrt{27} \quad \text{A1}$$

Question 7

- a. $\vec{r}(t) = e^{-t}\vec{i} + 2e^{-2t}\vec{j}$ for $t \geq 0$ vector equation,
the parametric equations are $x = e^{-t}$ and $y = 2e^{-2t}$
now $y = 2e^{-2t} = 2(e^{-t})^2 = 2x^2$
 $y = 2x^2$ is the Cartesian equation of the path, but!

A1

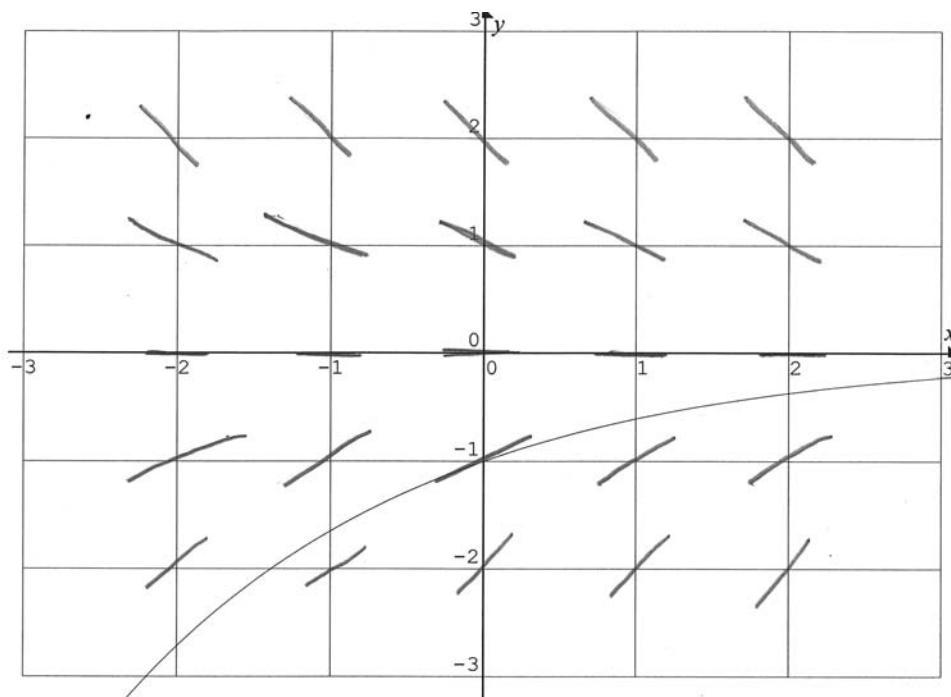
- b. since $t \geq 0 \Rightarrow 0 < x \leq 1$ and $0 < y \leq 2$, the graph is not the whole parabola it has a hole at the origin. A1



Question 8

a.

$\frac{dy}{dx} = -\frac{y}{2}$	y	-2	-1	0	1	2
slope		1	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1



A2

b. $2\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{2} \quad y(0) = -1$

$$-\frac{1}{2} \int dx = \int \frac{1}{y} dy = \log_e(|y|)$$

M1

$$-\frac{x}{2} + C = \log_e(|y|) \Rightarrow y = \exp\left(-\frac{x}{2} + C\right) = Ae^{-\frac{x}{2}}$$

$$x = 0 \quad y = -1 \Rightarrow A = -1$$

$$y = -e^{-\frac{x}{2}}$$

A1

c. The graph of $y = -e^{-\frac{x}{2}}$ on the above diagram passing through $(0, -1)$ A1

Question 9

a. $y = \frac{32}{x^2 - 16}$

vertical asymptotes at $x = 4$ and $x = -4$

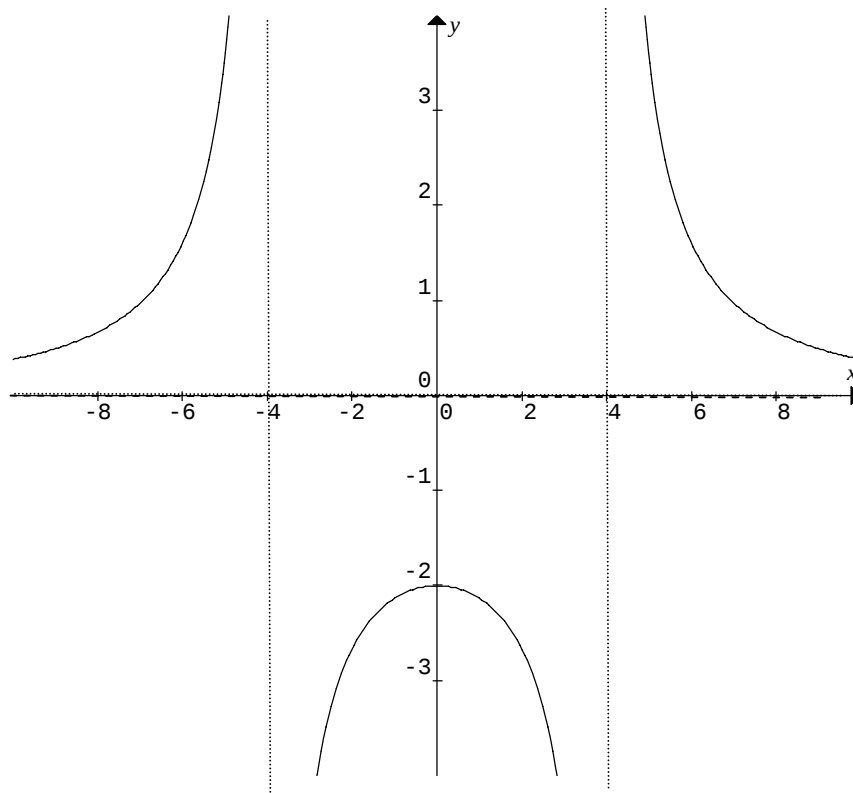
horizontal asymptotes at $y = 0$ (the x-axis)

A1

the turning point is a maximum turning point at $(0, -2)$ also the y-intercept

correct graph and turning point

A1



b. the area is $\int_{-2}^0 \frac{32}{x^2 - 16} dx$ but this is below the x-axis and negative, so the area is

$$A = \int_{-2}^0 \frac{32}{16 - x^2} dx \quad \text{A1}$$

by partial fractions $\frac{32}{16 - x^2} = \frac{B}{4 + x} + \frac{C}{4 - x}$ adding the partial fractions

$$= \frac{B(4 - x) + C(4 + x)}{(4 + x)(4 - x)} = \frac{x(C - B) + 4B + 4C}{16 - x^2} \quad \text{M1}$$

$$(1) \quad 4(B + C) = 32 \quad \text{and} \quad (2) \quad C - B = 0 \quad \text{so that} \quad B = C = 4$$

$$A = \int_{-2}^0 \frac{32}{16 - x^2} dx = 4 \int_{-2}^0 \left(\frac{1}{4 + x} + \frac{1}{4 - x} \right) dx$$

$$A = 4 \left[\log_e(4 + x) - \log_e(4 - x) \right]_{-2}^0 = 4 \left[\log_e \left(\frac{4 + x}{4 - x} \right) \right]_{-2}^0 \quad \text{M1}$$

$$A = 4 \left[\log_e(1) - \log_e \left(\frac{1}{3} \right) \right] = 4 \log_e(3)$$

$$A = \log_e(81) \quad a = 81 \quad \text{A1}$$

Question 10

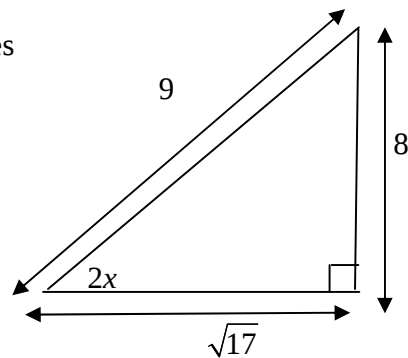
$\cos(x) - \sin(x) = \frac{1}{3}$ squaring both sides gives

$$\cos^2(x) - 2\sin(x)\cos(x) + \sin^2(x) = \frac{1}{9}$$

$$2\sin(x)\cos(x) = \sin(2x) = 1 - \frac{1}{9} = \frac{8}{9}$$

since $0 < 2x < \frac{\pi}{2}$ in the first quadrant

$$\cot(2x) = \frac{\sqrt{17}}{8}$$



M2

A1

END OF SUGGESTED SOLUTIONS