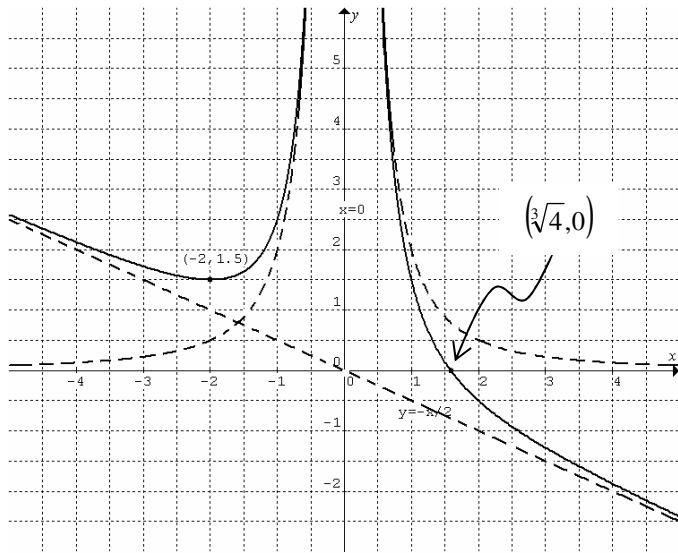


Q1 Sketch $y = \frac{2}{x^2}$ and $y = -\frac{x}{2}$, then use addition of ordinates to sketch $y = \frac{2}{x^2} - \frac{x}{2}$.

Straight line asymptotes are $x = 0$ and $y = -\frac{x}{2}$.

x -intercept: Let $y = 0$, $\frac{2}{x^2} - \frac{x}{2} = 0$, $4 - x^3 = 0$, $x = \sqrt[3]{4}$, $(\sqrt[3]{4}, 0)$

Turning point: $\frac{dy}{dx} = 0$, $-\frac{4}{x^3} - \frac{1}{2} = 0$, $x^3 = -8$, $x = -2$ and $y = \frac{3}{2}$, $(-2, 1.5)$.



Q2 When $x = 1$, $3 + 2y + y^2 = 11$, $y^2 + 2y - 8 = 0$, $(y + 4)(y - 2) = 0$, $\therefore y = 2$ in the first quadrant. $(1, 2)$

Implicit differentiation: $6x + 2y + 2x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$,

$$\frac{dy}{dx} = -\frac{3x + y}{x + y}.$$

Gradient of tangent at $(1, 2) = -\frac{3(1) + 2}{1 + 2} = -\frac{5}{3}$, gradient of

normal at $(1, 2) = \frac{3}{5}$.

Q3 Linearly dependent: Let $\tilde{c} = p\tilde{a} + q\tilde{b}$,
 $m\tilde{i} + n\tilde{k} = (-3p - 2q)\tilde{i} + (2p - 2q)\tilde{j} + (3p + q)\tilde{k}$
 $\therefore m = -3p - 2q$, $2p - 2q = 0$ and $n = 3p + q$
 $\therefore p = q$, $m = -5p$ and $n = 4p$. $\therefore \frac{m}{n} = -\frac{5}{4}$.

$$\begin{aligned} \text{Q4 } \sec\left(\frac{\pi}{5}\right) &= \frac{1}{\cos\left(\frac{\pi}{5}\right)} = \frac{1}{\cos\left(2\left(\frac{\pi}{10}\right)\right)} = \frac{1}{1 - 2\sin^2\left(\frac{\pi}{10}\right)} \\ &= \frac{1}{1 - 2\left(\frac{6 - 2\sqrt{5}}{16}\right)} = \frac{4}{\sqrt{5} + 1} = \frac{4(\sqrt{5} - 1)}{4} = \sqrt{5} - 1 \end{aligned}$$

$$\text{Q5a } v = -x^2, \frac{1}{2}v^2 = \frac{1}{2}x^4, a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = 2x^3.$$

Initially $x = 1$ and $\therefore a = 2$.

$$\text{Q5b } v = \frac{dx}{dt} = -x^2, \frac{dt}{dx} = -\frac{1}{x^2}, t = \int -\frac{1}{x^2} dx, t = \frac{1}{x} + c.$$

When $t = 0$ (initially), $x = 1$, $\therefore c = -1$, $t = \frac{1}{x} - 1$.

$$\text{Hence } x = \frac{1}{t + 1}.$$

$$\text{Q6 } f''(x) = -\sec^2(2x), f'(x) = \int -\sec^2(2x) dx = -\frac{1}{2}\tan(2x) + c.$$

$$f'\left(\frac{\pi}{8}\right) = -\frac{1}{2}\tan\left(2\left(\frac{\pi}{8}\right)\right) + c = -1, \therefore -\frac{1}{2} + c = -1, c = -\frac{1}{2}.$$

$$\therefore f'(x) = -\frac{1}{2}\tan(2x) - \frac{1}{2}.$$

The gradient at $x = \frac{\pi}{12}$ is

$$f'\left(\frac{\pi}{12}\right) = -\frac{1}{2}\tan\left(2\left(\frac{\pi}{12}\right)\right) - \frac{1}{2} = -\frac{1}{2}\left(\frac{1}{\sqrt{3}} + 1\right) = -\frac{\sqrt{3} + 3}{6}.$$

Q7 Horizontal component: $-T \sin 45^\circ + 120 \sin 30^\circ = 0$,
 $\therefore T = 60\sqrt{2}$.

Vertical component: $T \cos 45^\circ + F - 120 \cos 30^\circ = 0$,
 $\therefore F = 60(\sqrt{3} - 1)$.

$$\text{Q8a } \overrightarrow{OA} = \tilde{i} + 5\tilde{k}, \overrightarrow{OB} = -\tilde{i} + 2\tilde{j} + 4\tilde{k}, \therefore \overrightarrow{AB} = -2\tilde{i} + 2\tilde{j} - \tilde{k}$$

$$\text{Q8b Let } D \text{ be } (p, q, r), \overrightarrow{OD} = p\tilde{i} + q\tilde{j} + r\tilde{k}.$$

$$\overrightarrow{OC} = 3\tilde{i} + 5\tilde{j} + 2\tilde{k}.$$

$$\therefore \overrightarrow{DC} = \overrightarrow{OC} - \overrightarrow{OD} = (3 - p)\tilde{i} + (5 - q)\tilde{j} + (2 - r)\tilde{k}.$$

$ABCD$ is a parallelogram, $\therefore \overrightarrow{DC} = \overrightarrow{AB}$.

Hence $3 - p = -2$, $5 - q = 2$ and $2 - r = -1$.

$\therefore p = 5$, $q = 3$ and $r = 3$. $\therefore D(5, 3, 3)$.

$$\text{Q8c } \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 4\tilde{i} + 3\tilde{j} - 2\tilde{k}$$

$$\overrightarrow{AB} \cdot \overrightarrow{BC} = -8 + 6 + 2 = 0, \overrightarrow{AB} \neq \vec{0} \text{ and } \overrightarrow{BC} \neq \vec{0},$$

$\therefore \overrightarrow{AB} \perp \overrightarrow{BC}$. $\therefore ABCD$ is a rectangle.

Q9a Area = $\pi \times 1 = \pi$ square units.

Q9b $y = \cos^{-1}(x)$, $\therefore x = \cos y$, $x^2 = \cos^2 y = \frac{1}{2}(\cos(2y) + 1)$

$$V = 2 \int_0^{\frac{\pi}{2}} \pi x^2 dy = \pi \int_0^{\frac{\pi}{2}} (\cos(2y) + 1) dy$$

$$= \pi \left[\frac{1}{2} \sin(2y) + y \right]_0^{\frac{\pi}{2}} = \frac{\pi^2}{2} \text{ cubic units.}$$

Q10a $w = 1 + ai$, $|w| = (1 + a^2)^{\frac{1}{2}}$,

$$|w^3| = |w|^3 = \left((1 + a^2)^{\frac{1}{2}} \right)^3 = (1 + a^2)^{\frac{3}{2}}.$$

Q10b $|w^3| = 8$, $|w|^3 = 8$, $|w| = 2$, $(1 + a^2)^{\frac{1}{2}} = 2$,

$$\therefore 1 + a^2 = 4, a^2 = 3, a = \pm\sqrt{3}.$$

Q10c z that satisfy $|z^3| = 8$ are $z = 1 + i\sqrt{3}$, $z = 1 - i\sqrt{3}$, $z = 2$
and $z = -2$.

The 3 roots of $P(z) = 0$ must be 3 of the 4 above.

Since $P(z)$ has real coefficients, $\therefore P(z) = 0$ has a pair of complex conjugate roots and a real root, i.e.

$$P(z) = (z - 1 - i\sqrt{3})(z - 1 + i\sqrt{3})(z - p), \text{ where } p = -2 \text{ or } 2.$$

Expand and collect like terms to obtain:

$$P(z) = z^3 - (p + 2)z^2 + (2p + 4)z - 4p.$$

Since b, c and d are non-zero real constants, $\therefore p + 2 \neq 0$,

$2p + 4 \neq 0$ and $p \neq 0$, i.e. $p \neq -2$ and $p \neq 0$.

$\therefore p = 2$ is the only remaining possibility.

Hence $P(z) = z^3 - 4z^2 + 8z - 8$, $\therefore b = -4$, $c = 8$ and $d = -8$.

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