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Specialist Mathematics

2008

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 For any defined value of x , the value of $(\sin^{-1} x)^2 - (\sin x)^{-2}$ is

- A. 0
- B. less than 1
- C. in $(-\infty, 10]$
- D. in $[0, 1]$
- E. in $[-4820, 4820]$

Question 2 $\cos(x - y + z)$ can be expressed as

- A. $\cos(x)\cos(y + z) + \sin(x)\sin(y + z)$
- B. $\cos(x + z)\cos(y) - \sin(x + z)\sin(y)$
- C. $\cos(x)\cos(y + z) - \sin(x)\sin(y + z)$
- D. $\cos(x - y)\cos(z) + \sin(x - y)\sin(z)$
- E. $\cos(x)\cos(y - z) + \sin(x)\sin(y - z)$

Question 3 The inverse function of $2 \arctan\left[\frac{1}{2}(x+1)\right] - 1$ is

- A. $\frac{1}{2} \tan[2(x+1)] - 1$, where $-\frac{\pi}{4} < x < \frac{\pi}{4}$
- B. $\frac{1}{2} \tan[2(x+1)] - 1$, where $-\frac{\pi}{4} - 1 < x < \frac{\pi}{4} - 1$
- C. $\frac{1}{2} \tan[2(x+1)] - 1$
- D. $2 \tan\left[\frac{1}{2}(x+1)\right] - 1$, where $-\pi - 1 < x < \pi - 1$
- E. $2 \tan\left[\frac{1}{2}(x+1)\right] - 1$

Question 4 The graph of $y = \left(x - a + \frac{1}{\sqrt{x-a}}\right)\left(x - a - \frac{1}{\sqrt{x-a}}\right) + b$ has straight-line asymptote(s)

- A. $x = a$
- B. $x = a$ and $y = b$
- C. $x = -a$ and $y = 0$
- D. $y = b$
- E. $x = a$ and $y = -b$

Question 5 The graph of $\frac{(x+1)^2}{4} + (y-2)^2 = 1$ undergoes the following transformations in the order shown.

Translate to the right by 1 unit, translate downward by 1 unit, dilate from the x -axis by a factor of 2. The transformed equation is

- A. $\frac{x^2}{4} + \frac{(y-3)^2}{6} = 1$
- B. $x^2 + (y-1)^2 = 4$
- C. $x^2 + (y-2)^2 = 4$
- D. $(x+2)^2 + (y-2)^2 = 4$
- E. $2x^2 + (y-1)^2 = 1$

Question 6 If $z = i\left(\cos\left(\frac{\pi}{2} + \theta\right) - i\sin\left(\frac{\pi}{2} - \theta\right)\right)$ and $0 < \theta < \frac{\pi}{2}$, $\text{Arg}(z)$ is

- A. $-\theta$
- B. θ
- C. $\frac{\pi}{2} - \theta$
- D. $\frac{\pi}{2} + \theta$
- E. $\theta - \frac{\pi}{2}$

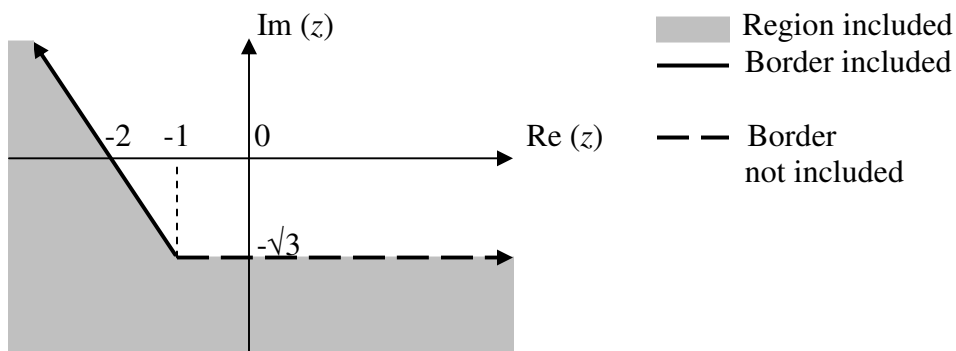
Question 7 $3 - 2i$ is a root of $2z^2 - (5-i)z + 3 + 11i = 0$, the other root is

- A. $\frac{3}{2} - i$
- B. $-\frac{3}{2} - i$
- C. $-\frac{1}{2} + \frac{3}{2}i$
- D. $-\frac{1}{2} - \frac{3}{2}i$
- E. $2 - 3i$

Question 8 $3\text{cis}\left(\frac{\pi}{3}\right) - i\text{cis}\left(-\frac{\pi}{6}\right) =$

- A. $2\text{cis}\left(\frac{\pi}{3}\right)$
- B. $2\text{cis}\left(\frac{\pi}{6}\right)$
- C. $2i\text{cis}\left(\frac{\pi}{3}\right)$
- D. $-2i\text{cis}\left(\frac{\pi}{6}\right)$
- E. $4\text{cis}\left(\frac{\pi}{6}\right)$

Question 9



The shaded region in the complex plane C represents

- A. $C \setminus \left\{ z : 0 \leq \text{Arg}(z + 1 + i\sqrt{3}) < \frac{2\pi}{3} \right\}$
- B. $C \setminus \left\{ z : 0 < \text{Arg}(z + 1 + i\sqrt{3}) \leq \frac{2\pi}{3} \right\}$
- C. $\left\{ z : -\frac{4\pi}{3} \leq \arg(z + 1 - i\sqrt{3}) < \frac{2\pi}{3} \right\}$
- D. $C \setminus \left\{ z : 0 < \text{Arg}(z - 1 - i\sqrt{3}) \leq \frac{2\pi}{3} \right\}$
- E. $C \setminus \left\{ z : 0 < \text{Arg}(z - 1 - i\sqrt{3}) \leq \frac{\pi}{3} \right\}$

Question 10 For constants $a, b \in \mathbb{R}$, the graph of $y = 1000(x + a)^2(x - b)^{\frac{1}{3}}$ has

- A. 3 stationary points and 2 inflection points
- B. 2 stationary points and 2 inflection points
- C. 3 stationary points and 1 inflection point
- D. 2 stationary points and 1 inflection point
- E. 3 stationary points and no inflection points

Question 11 For constant $a \in \mathbb{R}$, $\frac{d}{dx} \left(\frac{-1}{\sqrt{a - x^2}} \right) =$

- A. $\cos^{-1}\left(\frac{x}{a}\right)$
- B. $\arcsin\left(\frac{x}{a}\right)$
- C. $\arccos\left(\frac{x}{\sqrt{a}}\right)$
- D. $\sin^{-1}\left(\frac{x}{\sqrt{a}}\right)$
- E. $-\frac{x}{(a - x^2)\sqrt{a - x^2}}$

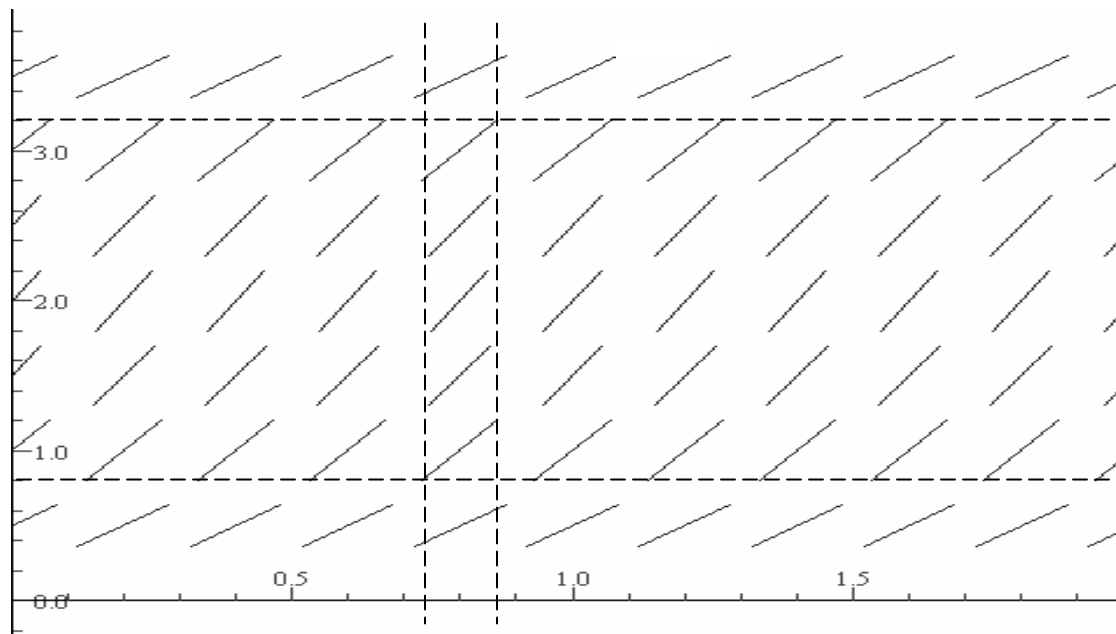
Question 12 For $2x^2y^2 = x^2 - y^2$, at $x=1$, $\frac{dy}{dx} =$

- A. $\frac{1}{\sqrt{3}}$ B. $-\frac{\sqrt{3}}{9}$ C. $\pm\frac{1}{3\sqrt{3}}$ D. $\frac{1}{3}$ E. $\pm\frac{1}{3}$

Question 13 An anti-derivative of $\frac{1}{1-x^2}$ with respect to x^2 for $x \in \mathbb{R} \setminus [-1,1]$ is

- A. $-\log_e(x^2 - 1) + 2$
 B. $\log_e|1 - x^2| - 3$
 C. $\frac{1}{2}\log_e\left|\frac{1+x}{1-x}\right|$
 D. $\frac{1}{2}\log_e\left|\frac{1-x}{1+x}\right|$
 E. $\log_e\sqrt{\frac{x+1}{x-1}} - 1$

Question 14



A slope field for $y' = y(a - y)$ is shown above, the approximate value of constant a is closest to

- A. 2 B. 3 C. 4 D. 5 E. 6

Question 15 For $\frac{dy}{dx} = \tan(x^2)$, $y = -1$ when $x = -\frac{1}{2}$. When $x = -1$, the value of y is closest to

- A. -1.4 B. -0.64 C. 1.4 D. 0.64 E. -0.5

Question 16 The angle between vector $\mathbf{p} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ and the zx -plane is

- A. $\tan^{-1}\left(\frac{b}{\sqrt{c^2 + a^2}}\right)$
B. $\cos^{-1}\left(\frac{b}{\sqrt{a^2 + b^2 + c^2}}\right)$
C. $\tan^{-1}\left(\frac{c}{\sqrt{a^2 + b^2}}\right)$
D. $\tan^{-1}\left(\frac{a}{\sqrt{b^2 + c^2}}\right)$
E. $\sin^{-1}\left(\frac{c + a}{\sqrt{a^2 + b^2 + c^2}}\right)$

Question 17 For vectors $\mathbf{a} = 2\mathbf{i} - \mathbf{j}$, $\mathbf{b} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{c} = 4\mathbf{i} - 9\mathbf{j}$, which one of the following statements is **false**?

- A. $\mathbf{a}$ and $\mathbf{b}$ are independent
B. $\mathbf{b}$ and $\mathbf{c}$ are independent
C. $\mathbf{c}$ and $\mathbf{a}$ are independent
D. $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are independent
E. $\frac{1}{2}\mathbf{c} + \mathbf{b}$ and $\mathbf{a}$ are dependent

Question 18 The position vector of a moving particle is given by $\mathbf{r}(t) = \tan(t)\mathbf{i} + \frac{1}{2}\sec^2(t)\mathbf{j} + \mathbf{k}$.

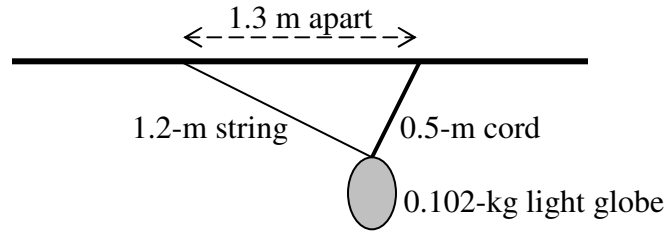
At $t = \frac{3\pi}{4}$, the direction of motion of the particle is given by

- A. $\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}$ B. $-\mathbf{i} + \mathbf{j} + \mathbf{k}$ C. $\frac{1}{\sqrt{3}}(-\mathbf{i} + \mathbf{j} + \mathbf{k})$ D. $\frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$ E. $\frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$

Question 19 The momentum of a 0.20-kg particle changes from $(3\mathbf{i} - \mathbf{j} + \mathbf{k}) \text{ kg ms}^{-1}$ to $(\mathbf{i} - \mathbf{k}) \text{ kg ms}^{-1}$. The change of speed (in ms^{-1}) of the particle is closest to

- A. 10 B. -10 C. 15 D. -15 E. 3

Question 20

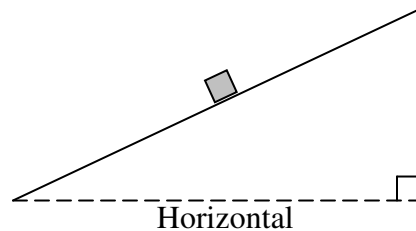


The diagram above shows a 0.102-kg light globe suspended from the ceiling by a 1.2-m string and a 0.5-m cord. The tension in the string is T_s newtons and the tension in the cord is T_c newtons.

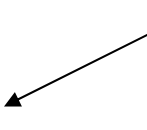
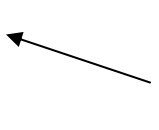
The value of the ratio $\frac{T_s}{T_c}$ is closest to

- A. 2.4 B. 0.38 C. 1.08 D. 0.92 E. 0.42

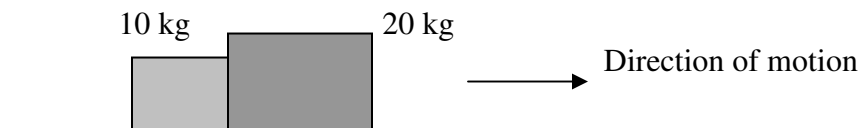
Question 21



The diagram above shows a particle sliding down a slope at constant speed. The direction of the reaction force of the slope on the particle is best indicated by arrow

- A.  B.  C.  D.  E. 

Question 22



The diagram above shows two boxes in contact sliding along a rough horizontal floor. The sliding friction is 0.2 newtons per kg. The force (in newtons) exerted by the 20-kg box on the 10-kg box is closest to

- A. 4 B. 2 C. 0 D. 1 E. 3

SECTION 2 Extended-answer questions

Instructions for Section 2

Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 The acceleration a of a particle is a function of velocity v , where $a^2 = 1 - v^2$.

At time $t = 0$, the particle is at position $x = 1$.

The particle is at rest at positions $x = \pm 1$. **Note** that $0 \leq t \leq \pi$.

a. In terms of variables x and v , set up a differential equation to the rectilinear motion of the particle.

1 mark

b. i. Show that the solution to the differential equation is $v = A\sqrt{B + Cx^2}$. Find the values of A , B and C .

3 marks

b. ii. Hence show that $x = D \cos(nt)$. Find the values of D and n .

2 marks

c. Find the value of $\int_0^{\pi} v dt$.

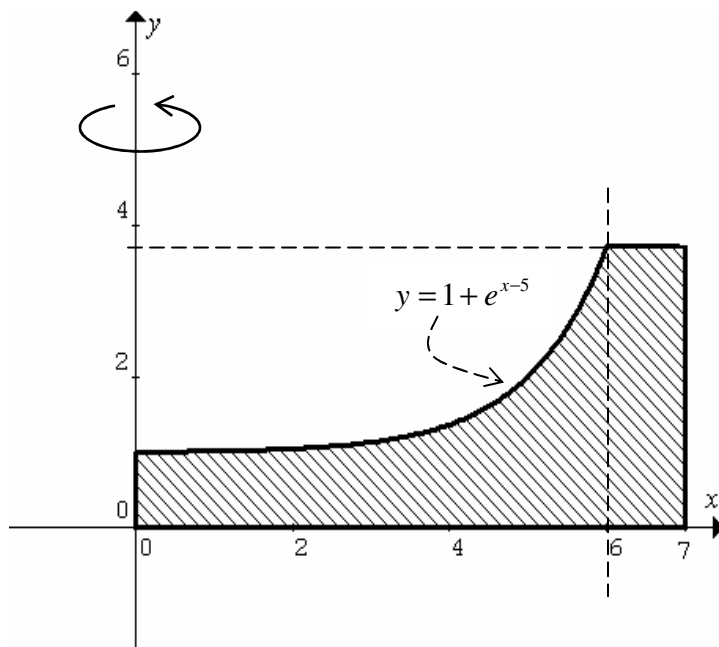
1 mark

d. Show that the direction of the acceleration of the particle is opposite to the direction of its displacement from the origin.

1 mark

Total 8 marks

Question 2 The following diagram shows the right-half of the upright cross-section of a large circular concrete structure for water storage. The structure is modelled by the solid of revolution of the shaded region in the y -axis. Length is measured in metres.



a. Find the exact area of the shaded region.

2 marks

b. i. Set up a definite integral for finding the **maximum** volume of water that the structure can hold.

b. ii. Find the volume of concrete (nearest m^3) required to build the structure.

3 + 3 = 6 marks

Water is filling the structure at a constant rate of 12 m^3 per minute.

c. Find the **average** rate (2 decimal places in mm per second) at which the water level is rising.

2 marks

d. i. When the structure is filled to a depth of h m, express the volume of water $V(h) \text{ m}^3$ as a definite integral. Integration is not required.

d. ii. Hence find an expression for $\frac{dh}{dt}$.

1 + 1 = 2 marks

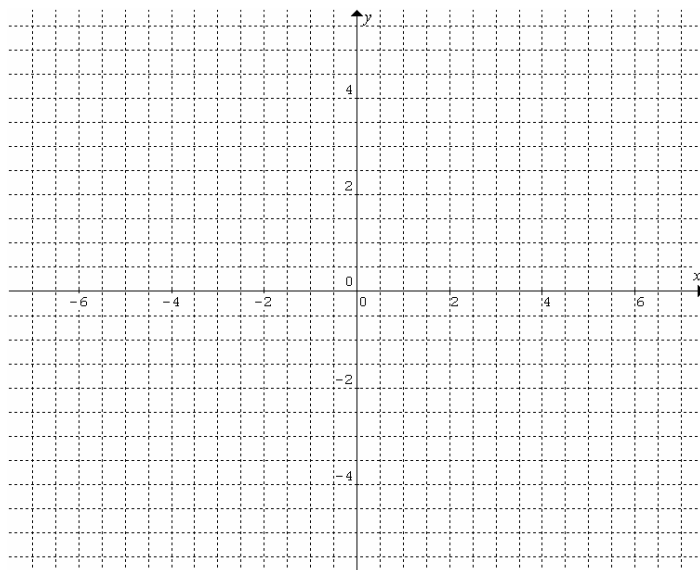
Total 12 marks

Question 3 The position of a particle is given by $\mathbf{r}(t) = \left(t + \frac{1}{t+1} + 1\right)\mathbf{i} + \left(t - \frac{1}{t+1} + 1\right)\mathbf{j}$, $t \geq 0$.

a Find the Cartesian equation of the locus of the particle. Include domain and range in the answer.

4 marks

- b.** Sketch the graph of the locus. Indicate the direction of motion with an arrow head. Show asymptote(s) and coordinates of intercept(s).



3 marks

Point $P(\sqrt{2}, 3)$ is away from the locus of the particle.

- c. i.** Find a vector from point P to the particle at time t .
- c. ii.** Find the shortest distance between the particle and point P.
- c. iii.** Find the time when the particle is closest to point P.

2 + 3 + 1 = 6 marks

- d.** Find the speed (1 decimal place) of the particle at $t = \sqrt{2}$.

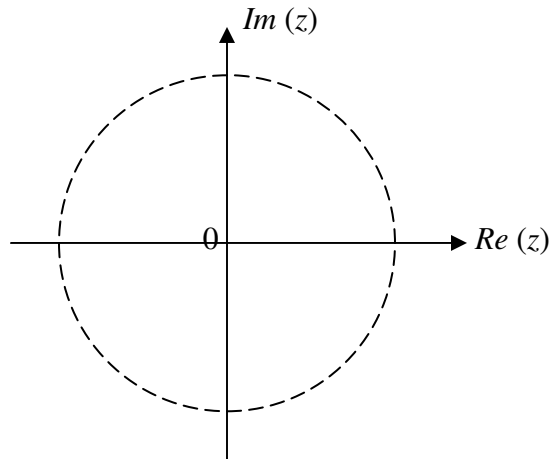
2 marks

Total 15 marks

Question 4 Let $z \in \mathbb{C}$.

- a.** $\frac{z^7 + 1}{z + 1}$ can be expressed as a polynomial of z , $P(z) = Az^6 + Bz^5 + Cz^4 + Dz^3 + Ez^2 + Fz + G$. Find the values of A, B, C, D, E, F and G . 2 marks

- b.** Plot accurately the roots of $P(z) = 0$ in the complex plane. 2 marks



- c.** Express the roots of $P(z) = 0$ in polar form. 2 marks

- d.** Find the products of the roots of $P(z) = 0$. 1 mark

- e.** Show that $\cos\left(\frac{\pi}{7}\right) + \cos\left(\frac{3\pi}{7}\right) + \cos\left(\frac{5\pi}{7}\right) = \frac{1}{2}$. 3 marks

Total 10 marks

Question 5 The coefficient of sliding friction between a 2.5-kg object and a plank is 0.7.
(Round all answers to 1 decimal place)

a. What is the force of friction if the object is at rest on the horizontal plank? 1 mark

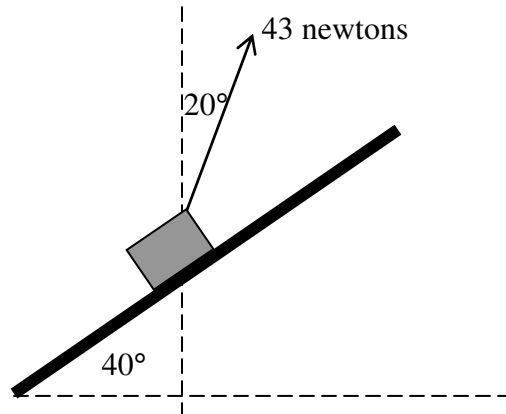
b. What is the force of friction (2 decimal places) if the plank is tilted, making a 30° angle with the horizontal? 2 marks

Now the angle is increased to 40° with the horizontal.

c i. Calculate the acceleration (2 decimal places) of the object down the plank.
2 marks

c. ii. The object starts from rest 3.0 m from the lower end of the plank. Find the magnitude of the momentum (2 decimal places) of the object when it reaches the lower end of the plank. 2 marks

The plank is left tilted at 40° with the horizontal. The object is placed on the plank, and a rope is attached to the object at an angle of 20° with the vertical. A student pulls the rope with a force of 43 newtons.



- d.** Calculate the magnitude of the resultant force (1 decimal place) on the object. 2 marks
- e.** Determine the direction (nearest degree) of motion of the object. 2 marks
- f.** Describe the path of the object and its motion in terms of direction, acceleration. 2 marks

Total 13 marks

End of Exam 2