



Online & home tutors Registered business name: mathlinE ABN: 35 631 847 853

Specialist Mathematics

2008

Trial Examination 1

Instructions

Answer **all** questions. Do **not** use calculators.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

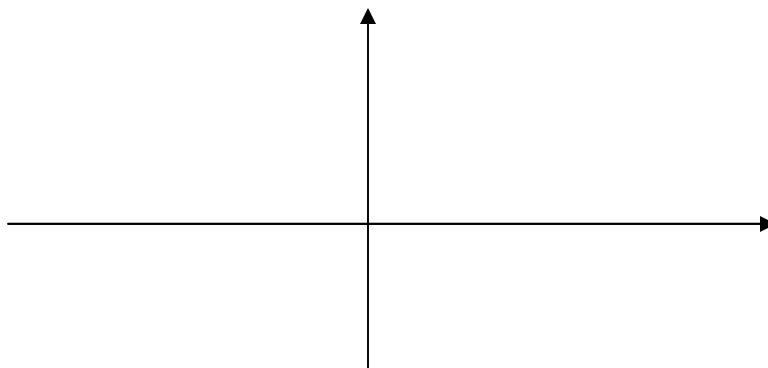
In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1 Consider the graph of $f(x) = \frac{7}{x^2 + 3x + 4}$.

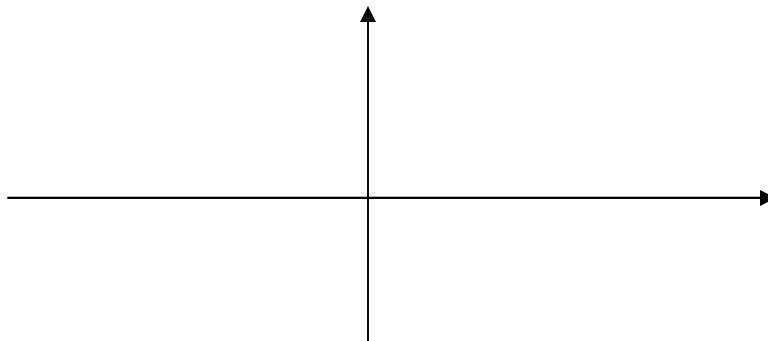
- State the coordinates of the y-intercept.
- Find the coordinates of the stationary point.
- Write down the equation of the asymptote.
- Sketch the graph of $f(x) = \frac{7}{x^2 + 3x + 4}$.



1 + 2 + 1 + 1 = 5 marks

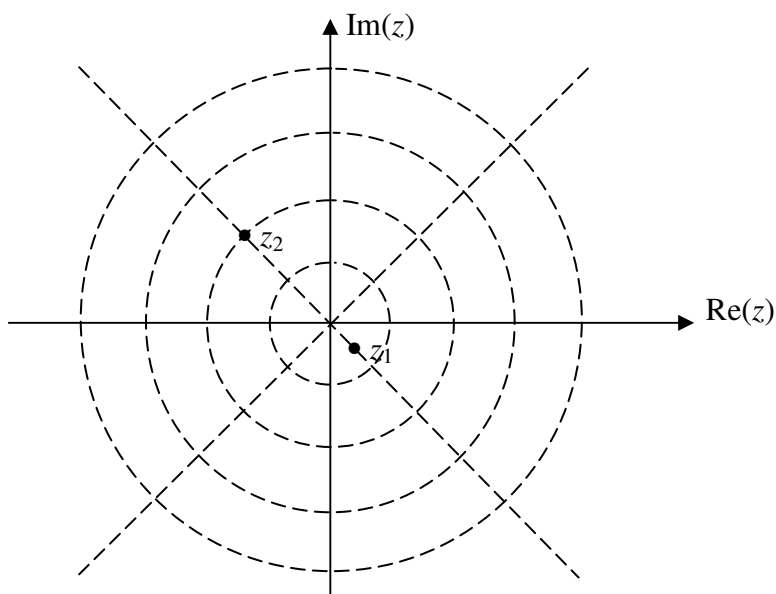
Question 2

- Express $\cos(2\sin^{-1} x)$ as a polynomial function of x .
- Hence sketch the graph of $\cos(2\sin^{-1} x)$.



2 + 2 = 4 marks

Question 3 z_1 and z_2 are shown in the following argand diagram. Plot $z_3 = z_1 z_2$ and $z_4 = \frac{z_2}{z_1}$ on the same argand diagram.



2 + 2 = 4 marks

Question 4

a. Given that $z = \frac{1}{\sqrt{2}}(1 - i)$ is a solution to $z^2 + i = 0$, find the other solution to the equation.

b. Hence or otherwise find all the solutions to $z^8 - 1 = 0$. Express the solutions in $x + yi$ form.

1 + 3 = 4 marks

Question 5

a. Let $F(x)$ be an anti-derivative of $f(x) = \frac{3}{2x^2 + 6}$ such that $F(3) = 0$. Find $F(x)$

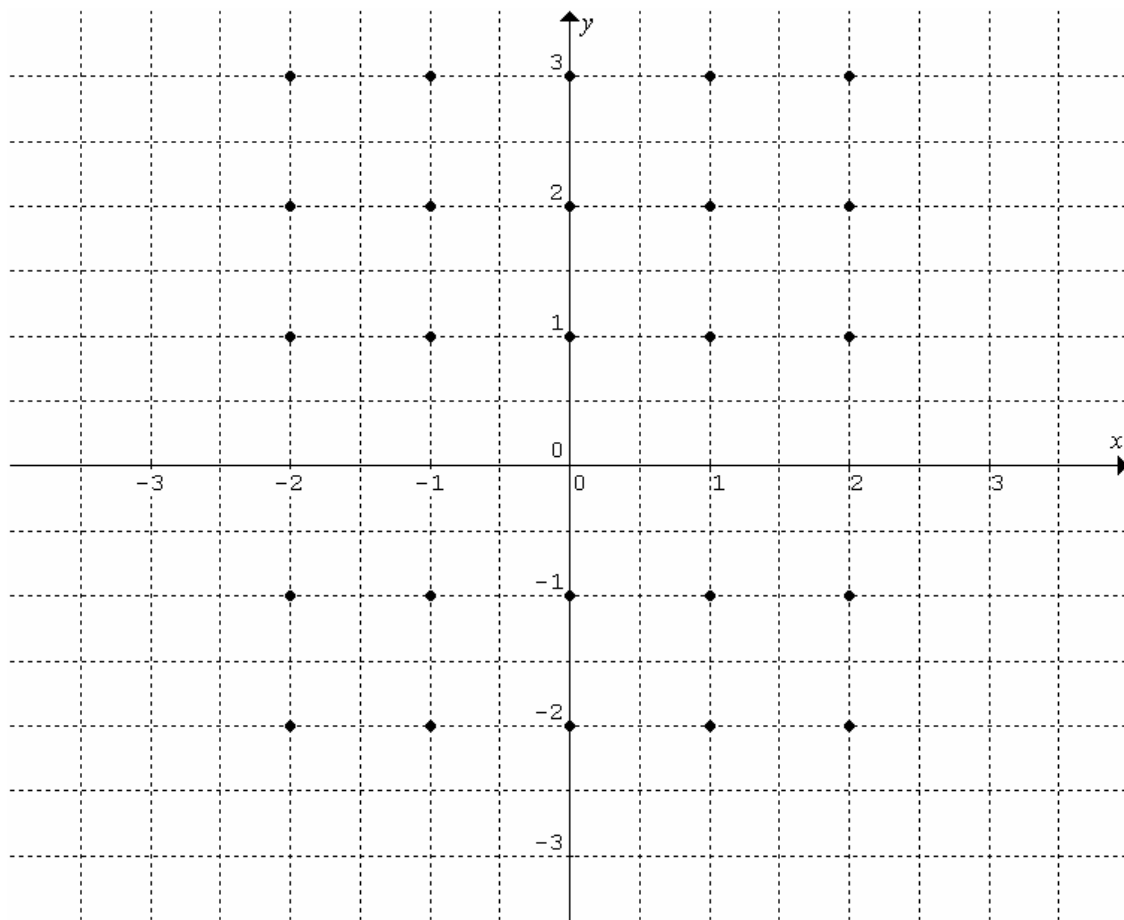
b. Find $\frac{d}{dx}(\sin^{-1} \sqrt{1-x^2})$ and state the domain.

2 + 3 = 5 marks

Question 6

a. Find $\frac{dy}{dx}$, given $4x^2 + y^2 = 16$.

b. Sketch the slope field of the differential equation $\frac{dy}{dx} = -\frac{4x}{y}$ at the points shown in the grid below.



c. Sketch a solution curve to the differential equation $\frac{dy}{dx} = -\frac{4x}{y}$ on the grid in part b.

2 + 2 + 1 = 5 marks

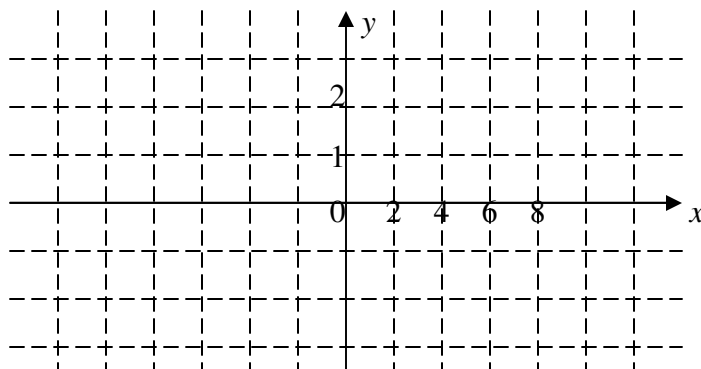
Question 7 Consider vector $3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}$ where $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$ are orthogonal unit vectors.

- Find a unit vector in the direction of vector $3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}$.
- Find $\cos \theta$ where θ is the angle between vector $3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}$ and unit vector $\mathbf{k}$.
- Find the value(s) of p such that vector $p\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and vector $3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}$ are independent.

1 + 1 + 1 = 3 marks

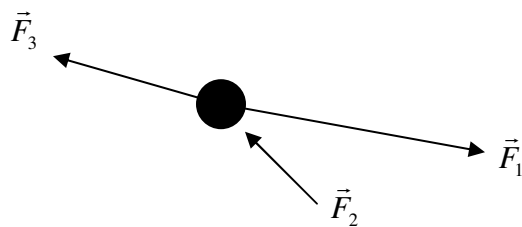
Question 8 A particle moves so that its position at time t , $0 \leq t < \frac{\pi}{2}$, is given by $\mathbf{r} = 2\sec(t)\mathbf{i} + \tan(t)\mathbf{j}$.

- Find the initial position of the particle.
- Find the velocity $\mathbf{v}$ of the particle at time t .
- Sketch the path of the particle. Hint: Find the cartesian equation of the path first.



1 + 2 + 3 = 6 marks

Question 9 A 0.5-kg particle is acted on by three forces: $\vec{F}_1 = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\vec{F}_2 = -\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ and $\vec{F}_3$. The acceleration of the particle is $4\mathbf{i} + 2\mathbf{k}$. Forces are in newtons, and acceleration is in ms^{-2} .



a. Find $\vec{F}_3$.

b. Find the exact distance travelled by the particle in the first second if it is initially at rest.

2 + 2 = 4 marks

End of Exam 1