THE HEFFERNAN GROUP

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Students Name:……………………………………

SPECIALIST MATHEMATICS

TRIAL EXAMINATION 1

2008

Reading Time: 15 minutes
Writing time: 1 hour

Instructions to students

This exam consists of 10 questions.
All questions should be answered.
There is a total of 40 marks available.
The marks allocated to each of the ten questions are indicated throughout.
Students may not bring any notes or calculators into the exam.
Where more than one mark is allocated to a question, appropriate working must be shown.
Where an exact answer is required to a question, a decimal approximation will not be accepted.
Unless otherwise indicated, diagrams in this exam are not drawn to scale.
The acceleration due to gravity should be taken to have magnitude g m/s^2 where $g = 9.8$
Formula sheets can be found on pages 12-14 of this exam.

Question 1

OAB is a triangle with $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$. The midpoint of $\overrightarrow{OA}$ is M and the midpoint of $\overrightarrow{OB}$ is N.

Prove that $\overrightarrow{MN}$ is parallel to $\overrightarrow{AB}$.

2 marks

Question 2

A box of mass 5kg rests on a rough horizontal floor. The coefficient of friction between the box and the floor is $0\cdot 1$. A boy applies a horizontal dragging force of D newtons to the box in an attempt to move it.

a. Find the values of D if the box is **not** at the point of moving across the floor.

2 marks

b. If the boy now applies a dragging force of 10 newtons at an angle of 60° to the horizontal, find the acceleration of the box across the floor.

2 marks

Question 3

a. Find the values of a and b given that $a, b \in R$ and that -1 is a solution to the equation

$$z^2 + (a - i)z + b(1 - i) = 0 .$$

2 marks

b. Hence find the other solution to the equation.

2 marks

Question 4

Given that $1-i$ is a solution to the equation

$$z^4-4z^3+9z^2-10z+6=0,$$

find the other solutions.

3 marks

Question 5

Find the equation of the normal to the curve with equation $2x^2 + 3xy^2 - 4y - 6 = 0$ at the point $(1, 2)$.

3 marks

Question 6

The region enclosed by the graph of the function $y=\frac{1}{x^2+1}$, the y-axis, and the line $y=\frac{1}{2}$ is rotated about the y-axis to form a solid of revolution.

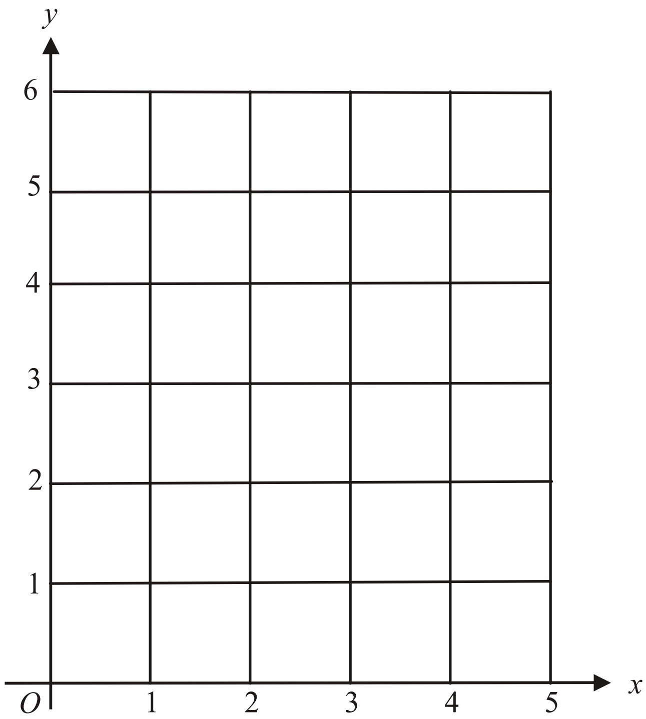
Find the volume generated.

3 marks

Question 7

Consider the differential equation $\frac{dy}{dx} = \sqrt{x}$, $x \geq 0$.

a. Sketch the slope field for this differential equation for $x = 0, 1, 2, 3$ and 4 at each of the values $y = 0, 1, 2, 3, 4, 5$ on the axes below.



2 marks

b. Given that $y = 0$ when $x = 0$, solve the differential equation.

1 mark

c. Sketch the graph of the function found in part **b.** on the slope field in part **a.**

1 mark

Question 8

a. Find $\int \frac{x}{3+x}\,dx$.

2 marks

b. Find an antiderivative of $\frac{x+2}{1+x^2}$.

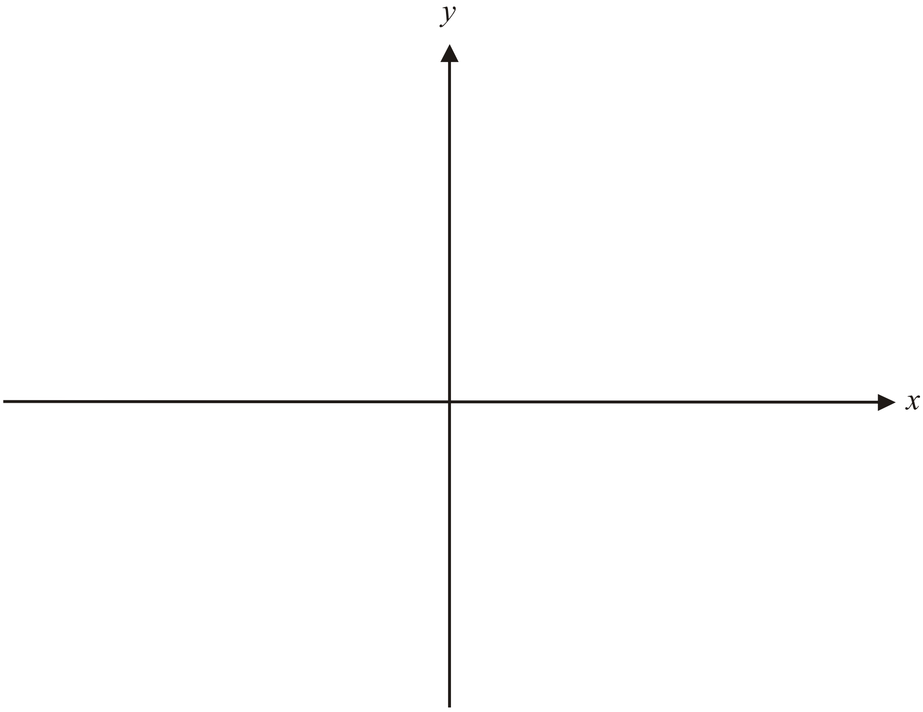
2 marks

c. Evaluate $\int_0^4 \frac{-5}{\sqrt{16-x^2}}\,dx$.

2 marks

Question 9

a. Sketch the graph of $y = \dfrac{1}{x^2 - x - 2}$, indicating clearly any asymptotes or axis intercepts.



2 marks

b. Find the area enclosed by the graph of $y = \dfrac{1}{x^2 - x - 2}$, the x-axis, the y-axis and the line $x = 1$.

4 marks

Question 10

At time t seconds the velocity of a moving particle is given by

$$\underset{\sim}{v}(t) = \cos(t)\underset{\sim}{i} - 2\sin(2t)\underset{\sim}{j}, \quad 0 \le t \le \pi$$

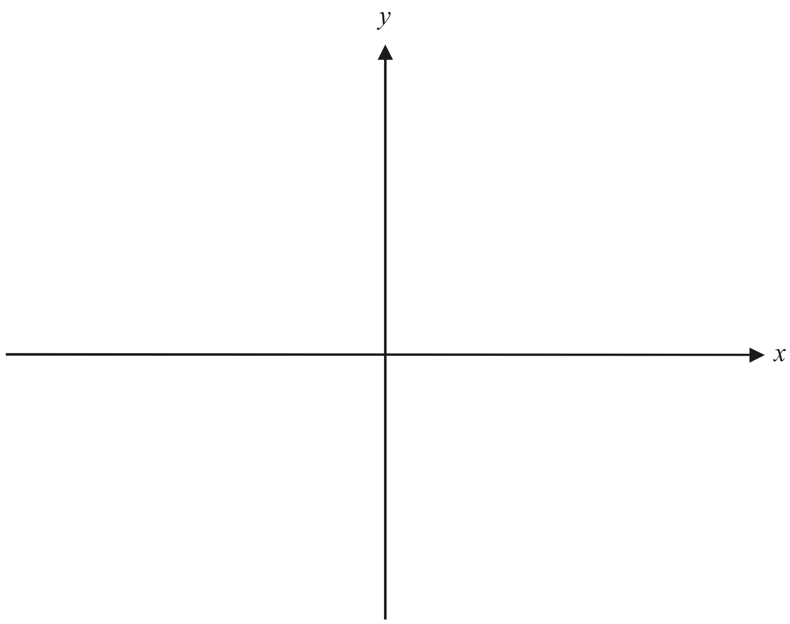
a. Find the position vector $\underset{\sim}{r}(t)$ of the particle at time t given that $\underset{\sim}{r}(0) = \underset{\sim}{j}$.

2 marks

b. Hence find the Cartesian equation of the path of the particle.

1 mark

c. Sketch the path of the particle on the axes below indicating clearly the coordinates of any endpoints.



2 marks

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc\sin A$
sine rule:	$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
cosine rule:	$c^2=a^2+b^2-2ab\cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ hyperbola: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Circular (trigonometric) functions

$\cos^2(x)+\sin^2(x)=1$

$1+\tan^2(x)=\sec^2(x)$ $\qquad$ $\cot^2(x)+1=\operatorname{cosec}^2(x)$

$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$ $\qquad$ $\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$

$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$ $\qquad$ $\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$

$\tan(x+y)=\frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$ $\qquad$ $\tan(x-y)=\frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$

$\cos(2x)=\cos^2(x)-\sin^2(x)=2\cos^2(x)-1=1-2\sin^2(x)$

$\sin(2x)=2\sin(x)\cos(x)$ $\qquad$ $\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$[0,\pi]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Algebra (Complex numbers)

$z=x+yi=r(\cos\theta+i\sin\theta)=r\operatorname{cis}\theta$

$|z|=\sqrt{x^2+y^2}=r$ $\qquad$ $-\pi<\operatorname{Arg} z\le\pi$

$z_1 z_2=r_1 r_2\operatorname{cis}(\theta_1+\theta_2)$ $\qquad$ $\frac{z_1}{z_2}=\frac{r_1}{r_2}\operatorname{cis}(\theta_1-\theta_2)$

$z^n=r^n\operatorname{cis}(n\theta)$ (de Moivre's theorem)

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c,\ n \neq -1$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\int \frac{1}{x}\,dx = \log_e|x| + c$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\cos(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax) + c$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\sin(ax)$$

$$\int \cos\,(ax)\,dx = \frac{1}{a}\sin(ax) + c$$

$$\frac{d}{dx}\left(\tan(ax)\right) = a\sec^2(ax)$$

$$\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + c$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}}\,dx = \cos^{-1}\left(\frac{x}{a}\right) + c,\ a > 0$$

$$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2}\,dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Euler's method: If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$,
then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

constant (uniform) acceleration: $v = u + at$ $\quad s = ut + \frac{1}{2}at^2$ $\quad v^2 = u^2 + 2as$ $\quad s = \frac{1}{2}(u+v)t$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$|\underset{\sim}{r}| = \sqrt{x^2 + y^2 + z^2} = r$

$\underset{\sim}{r}_1 . \underset{\sim}{r}_2 = r_1 r_2 \cos\theta = x_1 x_2 + y_1 y_2 + z_1 z_2$

$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$

Mechanics

momentum:	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion:	$\underset{\sim}{R} = m\underset{\sim}{a}$
friction:	$F \le \mu N$