

SPECIALIST MATHEMATICS

Units 3 & 4 – Written examination 1

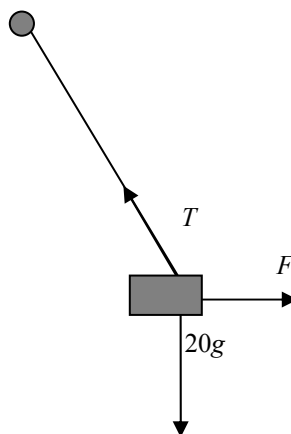


2007 Trial Examination

SOLUTIONS

Question 1

a.



A1

b. $\sin \theta = \frac{0.5}{2} = \frac{1}{4}$ so by triangle or Pythagoras, $\cos \theta = \frac{\sqrt{15}}{4}$.

A1

$$20g = T \cos \theta = \frac{T\sqrt{15}}{4} \quad (\text{downward})$$

A1

$$F = T \sin \theta = \frac{T}{4} \quad (\text{horizontally})$$

$$\text{Eliminating } T \text{ and rationalizing gives } F = \frac{4g\sqrt{15}}{3}$$

A1

Question 2

$$y = \int \frac{e^{2x} dx}{(e^{2x})^2 + 3}.$$

A1

Substitution is $u = e^{2x}$ so $\frac{du}{dx} = 2e^{2x}$.

A1

$$\begin{aligned} y &= \frac{1}{2} \int \frac{du}{u^2 + 3} = \frac{1}{2\sqrt{3}} \int \frac{\sqrt{3} du}{u^2 + 3} \\ &= \frac{1}{2\sqrt{3}} \tan^{-1} \frac{e^{2x}}{\sqrt{3}} + c \end{aligned}$$

M1, A1

If $y = 0$ when $x = 0$, $c = -\frac{1}{2\sqrt{3}} \tan^{-1} \frac{1}{\sqrt{3}} = \frac{-\pi}{12\sqrt{3}}$

$$y = \frac{1}{2\sqrt{3}} \tan^{-1} \frac{e^{2x}}{\sqrt{3}} - \frac{\pi}{12\sqrt{3}}$$

A1

Question 3

a. $-2 - 2\sqrt{3}i = 4 \operatorname{cis} \frac{-2\pi}{3}$

A1

b. $z - 2 + i = \pm 2 \operatorname{cis} \frac{-\pi}{3} = \pm (1 - \sqrt{3}i)$

M1

$$z = 3 - (1 + \sqrt{3})i \text{ or } 1 - (1 - \sqrt{3})i$$

A1 + A1

Question 4

a. Using implicit differentiation (product rule on LHS) gives

$$2x + 4xy' + 4y + 2y' = 0$$

M1

Regrouping $(4x + 2)y' = -2x - 4y$

So $\frac{dy}{dx} = -\frac{2x + 4y}{4x + 2}$

$$\frac{dy}{dx} = -\frac{x + 2y}{2x + 1}$$

A1

b. If $x = 1$, $1 + 4y + 2y = -11$ so $y = -2$.

A1

Substituting in derivative $\frac{dy}{dx} = -\frac{1 - 4}{2 + 1} = 1$.

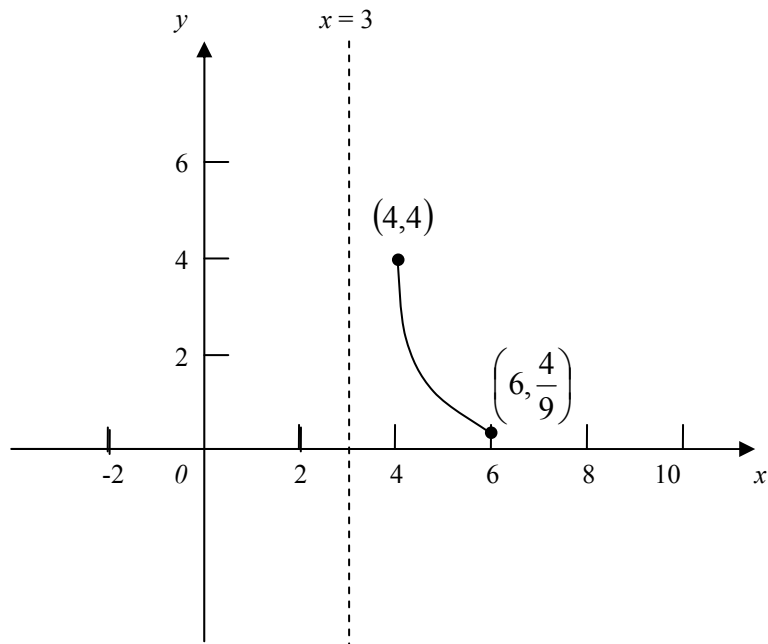
A1

Question 5

a. $x = t + 3$, so $t = x - 3$. $y = \frac{4}{t^2} = \frac{4}{(x-3)^2}$, A1

Domain: $1 \leq t \leq 3$ becomes $1 \leq x - 3 \leq 3$ or $4 \leq x \leq 6$. A1

b.



Shape A1

End points A1

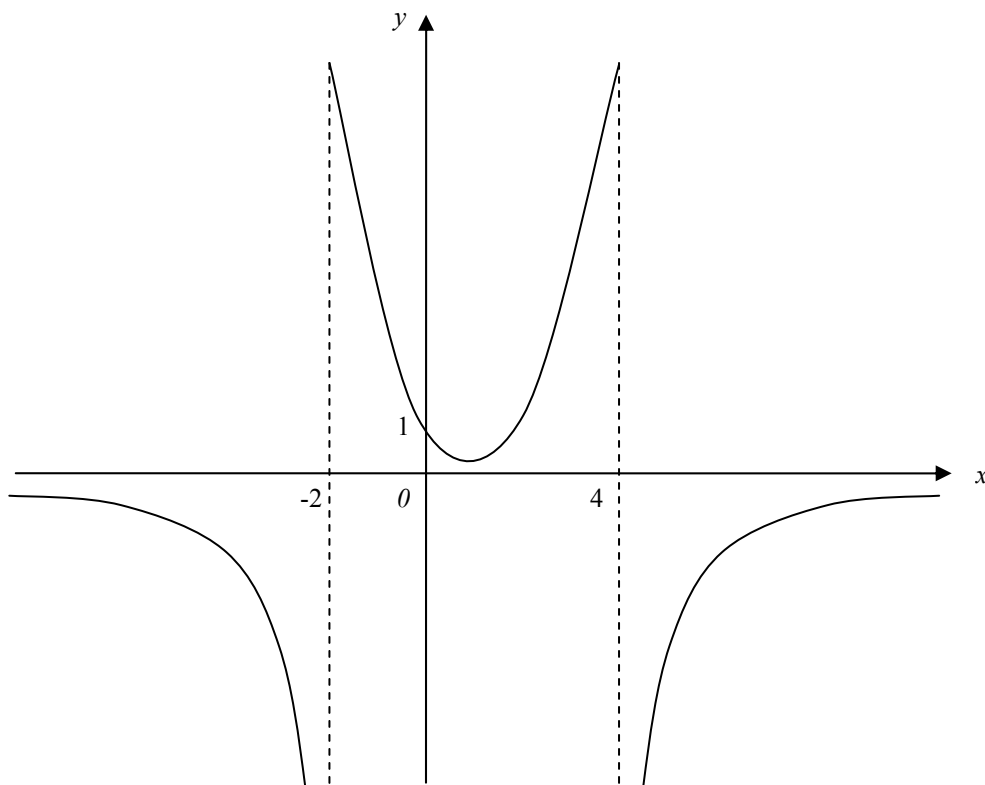
Question 6

a. Asymptotes $x = -2$, $x = 4$.

A1

Turning point $\left(1, \frac{8}{9}\right)$, intercept $(0, 1)$.

A1



A1

$$\text{b. } \int_0^2 \frac{8dx}{(4-x)(x+2)} = \frac{8}{6} \int_0^2 \left(\frac{1}{4-x} + \frac{1}{x+2} \right) dx$$

M1

$$= \left[\frac{4}{3} \log_e(x+2) - \frac{4}{3} \log_e(4-x) \right]_0^2$$

A1

$$= \frac{4}{3} (\log_e 4 - \log_e 2 - \log_e 2 + \log_e 4)$$

M1

A1

$$= \frac{4}{3} \log_e 4 = \frac{8}{3} \log_e 2$$

Question 7

a. $y = \int \frac{dx}{\sqrt{4-x^2}} = \sin^{-1} \frac{x}{2} + c$

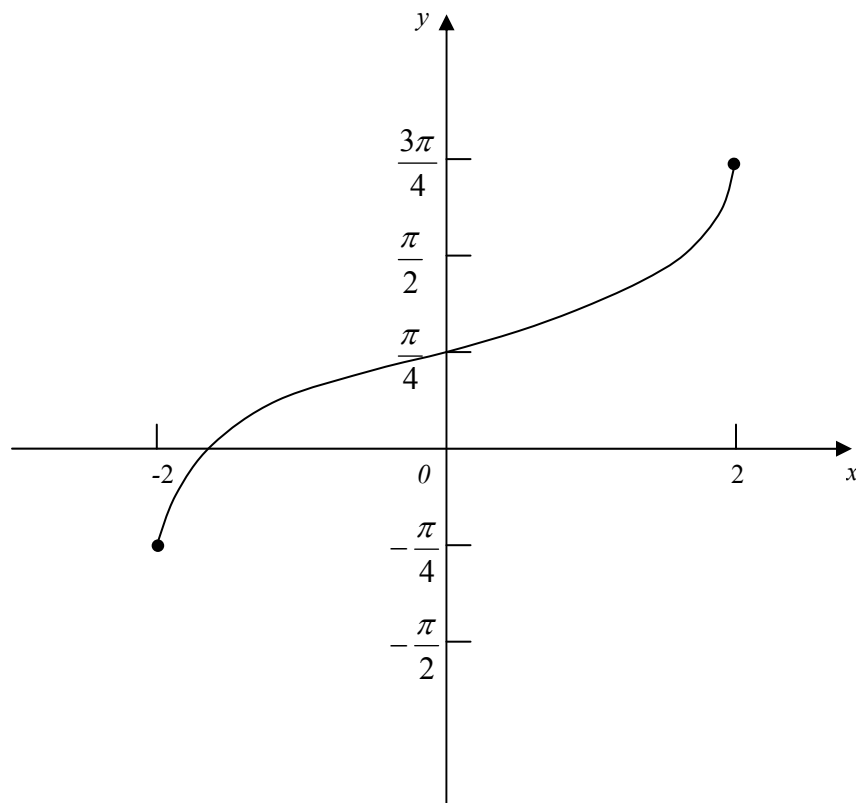
A1

If $x = 0, y = \frac{\pi}{4}$ so $c = \frac{\pi}{4}$.

$y = \sin^{-1} \frac{x}{2} + \frac{\pi}{4}$

A1

b.



Shape A1
End points A1

Question 8

$$u = \sin 3x, \text{ so } \frac{du}{dx} = 3 \cos 3x, \quad dx = \frac{du}{3 \cos 3x} \quad \text{A1}$$

$$y = \int \frac{dx}{\cos 3x} = \frac{1}{3} \int \frac{du}{\cos^2 3x} = \frac{1}{3} \int \frac{du}{1-u^2} \quad \text{M1}$$

$$= \frac{1}{6} \int \left(\frac{1}{1-u} + \frac{1}{1+u} \right) du = \frac{1}{6} \log_e \frac{1+u}{1-u} + c, \quad \text{M1}$$

$$= \frac{1}{6} \log_e \frac{1+\sin 3x}{1-\sin 3x} + c. \text{ So } a = \frac{1}{6}, \quad f(x) = \frac{1+\sin 3x}{1-\sin 3x}$$

A1

Question 9

$$\text{a. } V = \pi \int_0^a \cos^2 \frac{x}{4} dx.$$

A1

$$\text{b. } V = \frac{\pi}{2} \int_0^a \left(1 + \cos \frac{x}{2} \right) dx = \frac{\pi}{2} \left[x + 2 \sin \frac{x}{2} \right]_0^a$$

M1

$$= \frac{\pi}{2} \left(a + 2 \sin \frac{a}{2} \right)$$

A1

$$\text{c. We require } \frac{\pi}{2} \left(\frac{\pi}{3} + 1 \right) = \frac{\pi}{2} \left(a + 2 \sin \frac{a}{2} \right).$$

$$\text{If } a = \frac{\pi}{3}, \text{ it is easy to check that } 2 \sin \frac{a}{2} = 2 \sin \frac{\pi}{6} = 2 \times \frac{1}{2} = 1.$$

A1