

Trial Examination 2007

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
9	9	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are NOT permitted to bring into the examination room: notes of any kind, a calculator of any type, blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 8 pages with a detachable sheet of miscellaneous formulas in the centrefold.

Working space is provided throughout the booklet.

Instructions

Detach the formula sheet from the centre of this book during reading time.

Write **your name** and your **teacher's name** in the space provided above on this page.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2007 VCE Specialist Mathematics Units 3 & 4 Written Examination 1.

Instructions

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

Consider the relation $xy - y^2 - 4 = 0$, where $y > -4$.

Find the gradient of the curve when $x = -5$.

4 marks

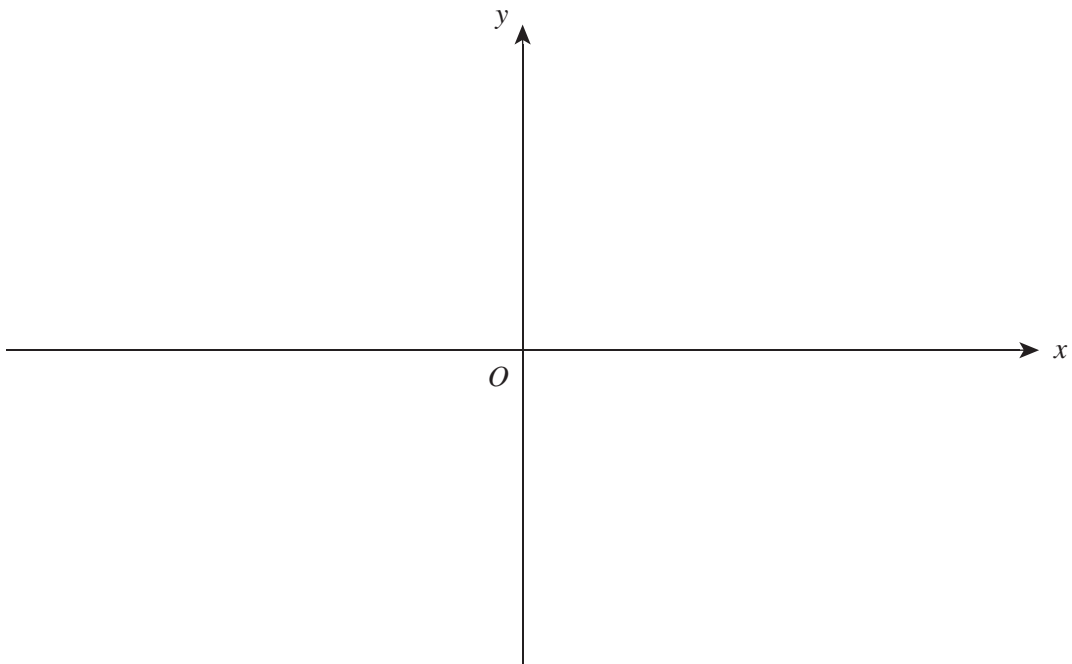
Question 2

Solve the differential equation $\frac{dy}{dx} - \cos(2x)e^{-\sin(2x)} = 0$ given that $y = 1$ when $x = 0$.

4 marks

Question 3

- a. Sketch the graph of the function $y = \frac{1}{x^2 + 4x + 8}$, showing all asymptotes, stationary points and intercepts with the axes.



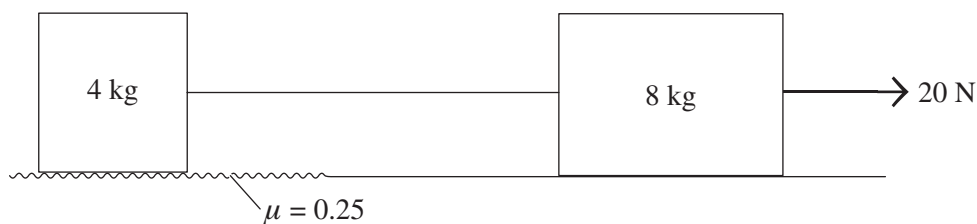
3 marks

- b. Find the exact value of the area bounded by the curve $y = \frac{1}{x^2 + 4x + 8}$, the x -axis and the lines $x = -4$ and $x = 0$.

3 marks
Total 6 marks

Question 4

The diagram below shows a horizontal force of 20 N being applied to an 8 kg mass which sits on a frictionless section of a horizontal surface. The 8 kg mass is joined by a light, inextensible string to a 4 kg mass which sits on the same horizontal surface but on a section which has a coefficient of friction of 0.25.



- a. On the diagram above, label all the forces acting on each mass.

1 mark

Trial Examination 2007

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Formula Sheet

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

SPECIALIST MATHEMATICS FORMULAS**Mensuration**

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc \sin A$

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (complex numbers)

$$z = x + yi = r(\cos\theta + i\sin\theta) = r\text{cis}\theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$-\pi < \text{Arg}(z) \leq \pi$$

$$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \text{cis}(n\theta) \text{ (de Moivre's theorem)}$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \quad n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e|x| + c, \text{ for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), \quad x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\underline{i} + \frac{dy}{dt}\underline{j} + \frac{dz}{dt}\underline{k}$$

Mechanics

momentum: $\underline{p} = m\underline{v}$

equation of motion: $\underline{R} = m\underline{a}$

sliding friction: $F \leq \mu N$

END OF FORMULA SHEET

- b. Let $a \text{ m/s}^2$ be the acceleration of the system for the length of time that the 4 kg mass remains on the section for which friction applies. Find the value of a , giving your answer in terms of g .

3 marks

Total 4 marks

Question 5

- a. Find the values of the integers a and b such that $\tan(15^\circ) = a + b\sqrt{3}$.

3 marks

- b. Prove that $\frac{\cos(2A)}{1 + \sin(2A)} = \frac{\cos(A) - \sin(A)}{\cos(A) + \sin(A)}$.

3 marks

Total 6 marks

Question 6

Find the volume of the solid created when the curve with equation $y = 4 - x^2$, where $x \in [0, 2]$, is rotated around the y-axis.

4 marks

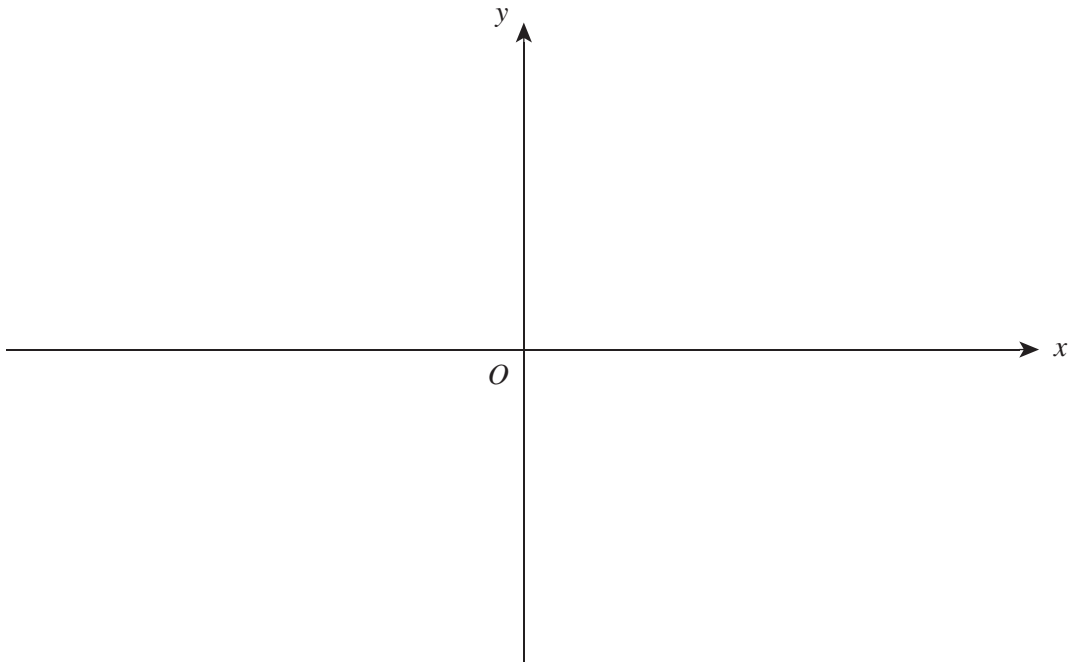
Question 7

The position vector of a moving particle is given by $\underline{r} = \cos^2(t)\underline{i} + 4\sin^2(t)\underline{j}$, $t \geq 0$.

- a.** Find the Cartesian equation of the path followed by the particle, expressing your answer in the form $y = f(x)$.

2 marks

- b.** Sketch the path of the particle on the axes provided.



2 marks

Total 4 marks

Question 8

Consider $P(z) = z^2 + bz + (1 + i)$, where $b \in \mathbb{C}$.

- a. If $z = i$ is a root of the equation $P(z) = 0$, find the value of b .

1 mark

- b. Hence solve the equation $P(z) = 0$.

3 marks

Total 4 marks

Question 9

Find an antiderivative of $\frac{x^2}{x^2 - 4}$.

4 marks

END OF QUESTION AND ANSWER BOOKLET