



Trial Examination 2007

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Suggested Solutions

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Question 1

$$xy - y^2 - 4 = 0$$

When $x = -5$, we obtain a quadratic equation in y , i.e. $-5y - y^2 - 4 = 0$.

Solve $-5y - y^2 - 4 = 0$ or equivalent for y .

M1

Hence $y = -4$ or -1 , but as $y > -4$ we obtain $y = -1$.

A1

Using implicit differentiation, we obtain $y + x \frac{dy}{dx} - 2y \frac{dy}{dx} = 0$.

M1

Substituting $x = -5$ and $y = -1$ to find $\frac{dy}{dx}$, we obtain $-1 - 5 \frac{dy}{dx} + 2 \frac{dy}{dx} = 0$.

Hence $\frac{dy}{dx} = -\frac{1}{3}$.

A1

Question 2

$$\frac{dy}{dx} = \cos(2x)e^{-\sin(2x)}$$

$$y = \int \cos(2x)e^{-\sin(2x)} dx$$

Let $u = \sin(2x)$, so $\frac{du}{dx} = 2 \cos(2x)$.

M1

$$y = \frac{1}{2} \int e^{-u} du$$

A1

Hence $y = -\frac{1}{2}e^{-u} + c$ or $y = -\frac{1}{2}e^{-\sin(2x)} + c$.

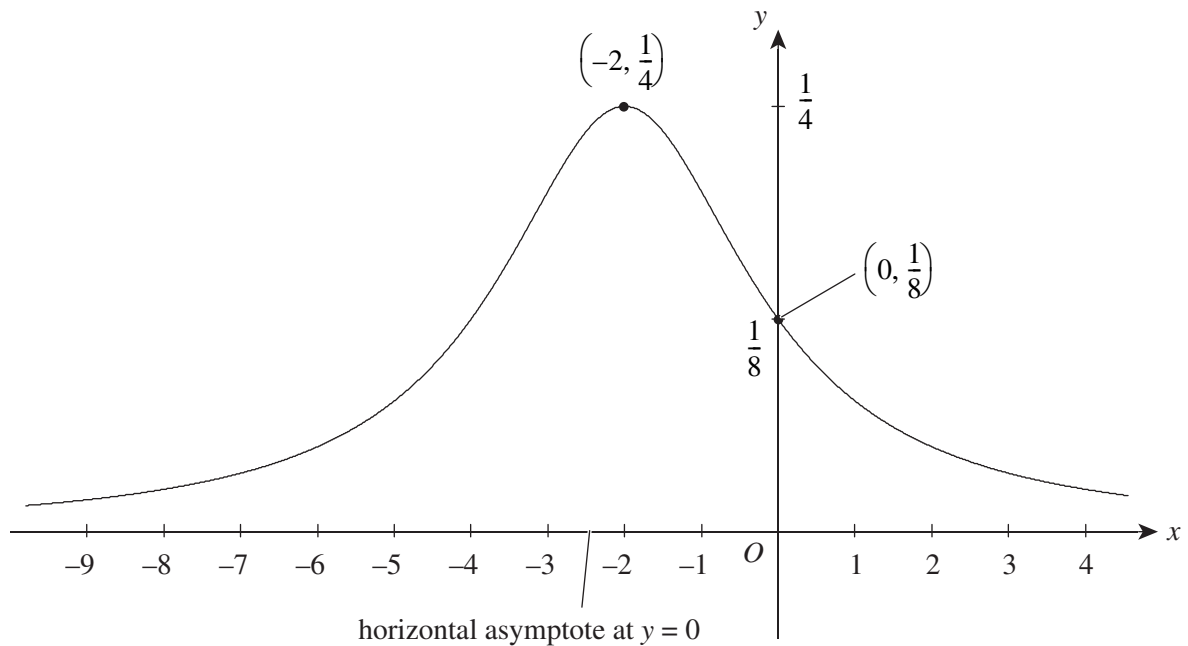
A1

We now apply the condition $y = 1$ when $x = 0$ to find the value of c using either of the above.

$$1 = -\frac{1}{2} + c, \text{ so } c = \frac{3}{2}.$$

Hence $y = \frac{3}{2} - \frac{1}{2}e^{-\sin(2x)}$.

A1

Question 3**a.**

Correct shape ("camel's hump") and horizontal asymptote at $y = 0$ is shown.

A1

Maximum stationary point at $(-2, \frac{1}{4})$ is shown.

A1

y-intercept at $(0, \frac{1}{8})$ is shown.

A1

b. The area between $x = -4$ and $x = 0$ is given by $\int_{-4}^0 \frac{dx}{x^2 + 4x + 8}$.

A1

$$= \int_{-4}^0 \frac{dx}{(x+2)^2 + 4}$$

$$= \frac{1}{2} \int_{-4}^0 \frac{2}{(x+2)^2 + 2^2} dx$$

A1

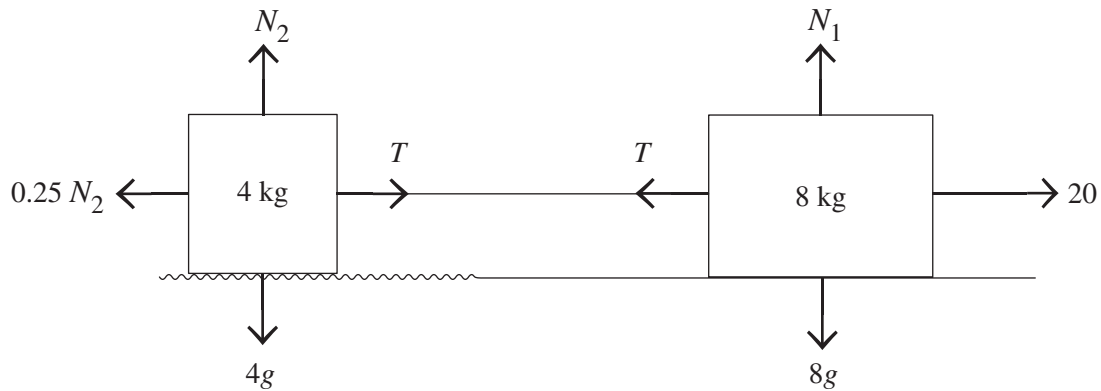
$$= \frac{1}{2} \left[\tan^{-1} \left(\frac{x+2}{2} \right) \right]_{-4}^0$$

$$= \frac{1}{2} [\tan^{-1}(1) - \tan^{-1}(-1)]$$

$$= \frac{1}{2} \left[\frac{\pi}{4} + \frac{\pi}{4} \right]$$

$$= \frac{\pi}{4} \text{ square units}$$

A1

Question 4**a.**

All forces are correctly shown.

A1

b. Resolving horizontally for the 8 kg mass, $20 - T = 8a$ (1)

A1

Resolving vertically for the 4 kg mass, $N_2 = 4g$ (2)Resolving horizontally for the 4 kg mass, $T - 0.25N_2 = 4a$ (3)Substituting (2) into (3) gives $T - g = 4a$ (4)

A1

Adding (1) and (4) gives $20 - g = 12a$

$$\text{Thus } a = \frac{20 - g}{12}.$$

A1

Question 5**a.** $\tan(15^\circ) = \tan(45^\circ - 30^\circ)$

$$= \frac{\tan(45^\circ) - \tan(30^\circ)}{1 + \tan(45^\circ)\tan(30^\circ)} \quad \left(\text{using } \tan(x - y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)} \right)$$

M1

$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + (1)\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \times \frac{\sqrt{3} - 1}{\sqrt{3} - 1}$$

A1

$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3}$$

A1

Hence if $\tan(15^\circ) = a + b\sqrt{3}$, $a = 2$ and $b = -1$.

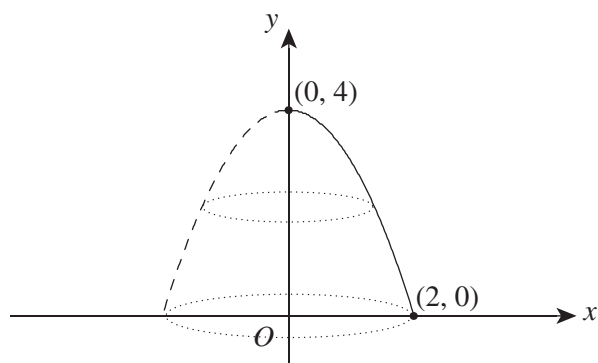
b. $\text{LHS} = \frac{\cos(2A)}{1 + \sin(2A)}$

$$= \frac{\cos^2(A) - \sin^2(A)}{\cos^2(A) + \sin^2(A) + 2\sin(A)\cos(A)} \quad \text{M1}$$

$$= \frac{(\cos(A) - \sin(A))(\cos(A) + \sin(A))}{(\cos(A) + \sin(A))^2} \quad \text{A1}$$

$$= \frac{\cos(A) - \sin(A)}{\cos(A) + \sin(A)} \quad \text{A1}$$

$$= \text{RHS}$$

Question 6

When $x = 0$, $y = 4$ and when $x = 2$, $y = 0$.

Since $y = 4 - x^2$, $x^2 = 4 - y$ over the interval $x \in [0, 2]$. M1

The volume is given by $\pi \int_0^4 x^2 dy = \pi \int_0^4 (4 - y) dy$ A1

$$= \pi \left[4y - \frac{y^2}{2} \right]_0^4 \quad \text{A1}$$

$$= \pi [(16 - 8) - (0 - 0)]$$

$$= 8\pi \text{ cubic units} \quad \text{A1}$$

Question 7

a. The parametric equations are:

$$x = \cos^2(t) \quad (1)$$

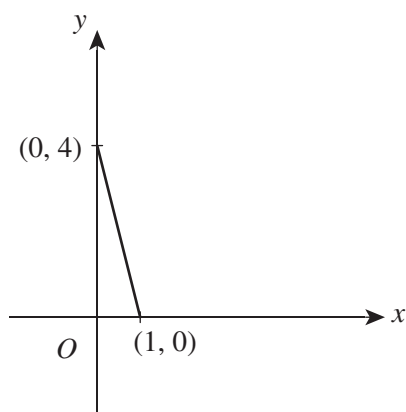
$$y = 4\sin^2(t) \quad (2)$$

$$\frac{y}{4} = \sin^2(t) \quad (3)$$

$$(1) + (3) \text{ gives } x + \frac{y}{4} = 1. \quad \text{M1}$$

Hence $y = 4 - 4x$, or equivalent. A1

- b. The path is a straight line with equation $y = 4 - 4x$ for $0 \leq x \leq 1$.



Straight line through (0, 4) and (1, 0).

A1

Line ends at the intercepts.

A1

Question 8

a.
$$P(i) = i^2 + bi + 1 + i$$
$$= (b + 1)i$$

Given that $P(i) = 0$, we obtain $b = -1$.

A1

- b. Let the roots of $z^2 - z + (1 + i) = 0$ be α and β .

$$(z - \alpha)(z - \beta) = z^2 - (\alpha + \beta)z + \alpha\beta$$

M1

By equating coefficients, $\alpha + \beta = 1$ and $\alpha\beta = 1 + i$.

A1

From a., $\alpha = i$, and from $\alpha + \beta = 1$ we obtain $\beta = 1 - i$.

A1

Question 9

$$\frac{x^2}{x^2 - 4} = \frac{4}{x^2 - 4} + 1 \text{ (by division)}$$

A1

Using partial fractions on $\frac{4}{x^2 - 4}$ gives

M1

$$\frac{4}{x^2 - 4} \equiv \frac{A}{x - 2} + \frac{B}{x + 2}$$

$$4 \equiv A(x + 2) + B(x - 2)$$

When $x = 2$, $A = 1$ and when $x = -2$, $B = -1$.

$$\text{Hence } \int \frac{x^2}{x^2 - 4} dx = \int \left(\frac{1}{x - 2} - \frac{1}{x + 2} + 1 \right) dx.$$

A1

$$\int \left(\frac{1}{x - 2} - \frac{1}{x + 2} + 1 \right) dx = \log_e \left| \frac{x - 2}{x + 2} \right| + x$$

A1