

Specialist Mathematics Exam 2: SOLUTIONS

Multiple-choice Answers

1. D	2. B	3. C	4. E	5. A
6. B	7. D	8. D	9. A	10. C
11. B	12. E	13. D	14. C	15. A
16. D	17. B	18. E	19. B	20. A
21. B	22. C			

Section 1: Multiple-choice Solutions

Question 1 Answer D

The range of $\cos^{-1}(3x)$ is $[0, \pi]$ and so the range of $y = a\cos^{-1}(3x) + \frac{\pi}{2}$ will be:

$$\left[a \times 0 + \frac{\pi}{2}, a \times \pi + \frac{\pi}{2} \right] = \left[\frac{\pi}{2}, (2a + 1)\frac{\pi}{2} \right]$$

Question 2 Answer B

The hyperbola is $\frac{(x-1)^2}{4} - \frac{y^2}{9} = 1$ which has centre at $(1, 0)$. The vertices are 2 units either side of the centre, given by $\sqrt{4}$ in the equation. Hence they are 4 units apart.

Equations of asymptotes: $y = \pm \frac{3}{2}(x-1)$ which can be written as: $3x - 2y - 3 = 0$ and $-3x - 2y + 3 = 0$.

Question 3 Answer C

$$\begin{aligned} z &= \left[8\text{cis}\left(-\frac{5\pi}{6}\right) \right]^3 \\ &= 512\text{cis}\left(-\frac{5\pi}{2}\right) \\ &= 512\text{cis}\left(-\frac{\pi}{2}\right) \end{aligned}$$

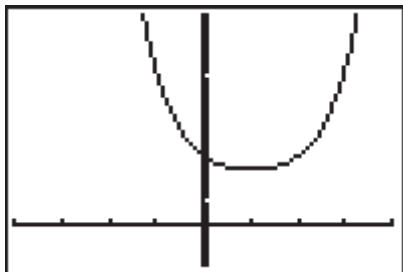
and so $|z|$ and $\text{Arg}(z)$ are 512 and $-\frac{\pi}{2}$

Question 4 Answer E

$$z^4 - 4z^3 + 8z^2 - 8z + 16 = 0, z \in \mathbb{C}$$

If there are any complex solutions then they will be in pairs as all of the coefficients are real. This eliminates alternatives A and C.

Graphing this polynomial onto the graphics calculator shows no x-intercepts and so there are 4 complex solutions.

**Question 5 Answer A**

$\underline{a} = \sqrt{2}\underline{i} - \underline{j} + 7\underline{k}$ has a magnitude of $\sqrt{2 + 1 + 49} = \sqrt{52}$ or $2\sqrt{13}$.

$$\therefore \text{unit vector} = \frac{1}{2\sqrt{13}}(\sqrt{2}\underline{i} - \underline{j} + 7\underline{k})$$

Question 6 Answer B

From the graph, the gradient is clearly positive and so alternatives D and E are incorrect. Differentiate the equation:

$$x^2 - 4xy + 4y^2 + x - 12y - 10 = 0$$

$$2x - \left(4y + 4x\frac{dy}{dx}\right) + 8y\frac{dy}{dx} + 1 - 12\frac{dy}{dx} = 0$$

Substituting $x = 6$ and $y = 1$ gives:

$$12 - \left(4 + 24\frac{dy}{dx}\right) + 8\frac{dy}{dx} + 1 - 12\frac{dy}{dx} = 0$$

$$9 = 28\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{9}{28}$$

Question 7 Answer D

Let $z = x + iy$ in $|z| = |z + 4|$.

$$|x + iy| = |(x + 4) + iy|$$

$$\sqrt{x^2 + y^2} = \sqrt{(x + 4)^2 + y^2}$$

$$x^2 + y^2 = (x + 4)^2 + y^2$$

$$x^2 + y^2 = x^2 + 8x + 16 + y^2$$

$$8x = -16 \text{ and so } x = -2$$

Now $x = \operatorname{Re}(z)$ and so the answer is D.

Question 8 Answer D

$$\int_0^{\frac{\pi}{6}} \cos(3x)e^{\sin(3x)} dx$$

Let $u = \sin(3x)$ so $\frac{du}{dx} = 3 \cos(3x)$

If $x = 0$ then $u = 0$ and if $x = \frac{\pi}{6}$ then $u = 1$.

$\frac{1}{3} \int_0^1 e^u du$ is the result.

Question 9 Answer A

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = |\overrightarrow{BA}| |\overrightarrow{BC}| \cos \angle ABC$$

$$\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$$

$$= -(\underline{i} - \underline{j} + \underline{k}) + 2\underline{i} + \underline{j} - \underline{k}$$

$$= \underline{i} + 2\underline{j} - 2\underline{k} \text{ and so } |\overrightarrow{BA}| = 3$$

$$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$$

$$= -(\underline{i} - \underline{j} + \underline{k}) + 3\underline{i} + \underline{j}$$

$$= 2\underline{i} + 2\underline{j} - \underline{k} \text{ and so } |\overrightarrow{BC}| = 3$$

$$\overrightarrow{BA} \cdot \overrightarrow{BC} = 2 + 4 + 2 \text{ and therefore } \cos \angle ABC = \frac{8}{9}$$

Question 10 Answer C

$$f(x+h) \approx f(x) + h \log_e(x) \text{ where}$$

$$f(1+0.1) \approx f(1) + 0.1 \times \ln(1)$$

$$= 3$$

$$f(1.1+0.1) \approx f(1.1) + 0.1 \times \ln(1.1)$$

$$= 3 + 0.009531$$

$$= 3.009531$$

$$f(1.2+0.1) \approx f(1.2) + 0.1 \times \ln(1.2)$$

$$= 3.009531 + 0.01823$$

$$= 3.02776$$

Answer: 3.0278

Question 11 Answer B

A volume of revolution around the y -axis is found by evaluating $\int \pi x^2 dy$. The lower limit is $y = 1$ and the upper is e .

$$y = e^{2x}$$

$$2x = \log_e(y)$$

$$x = \frac{1}{2} \log_e(y)$$

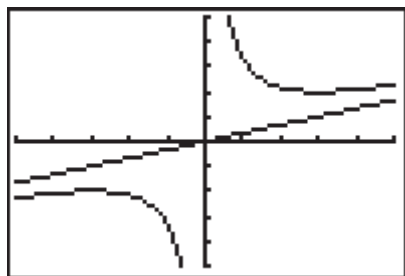
The volume formed by rotating the curve around the y -axis $= \frac{\pi}{4} \int_1^e (\log_e(y))^2 dy$

The volume formed by rotating the line $x = \frac{1}{2}$ around the y -axis $= \pi \int_1^e \left(\frac{1}{2}\right)^2 dy$

Required volume $\frac{\pi}{4} \int_1^e (1 - \log^2(y)) dy$

Question 12 Answer E

The graph of the function $y = \frac{x^2 + 9}{3x}$ has two asymptotes, one vertical ($x = 0$) and the other oblique ($y = \frac{x}{3}$). It also has two turning points as can be seen from the graph of this function. The oblique asymptote with equation $y = \frac{x}{3}$ is also shown.

**Question 13 Answer D**

A point of inflection occurs where the second derivative is zero and the first derivative does **not** change sign. So, at the point of inflection $\frac{d^2y}{dx^2}$ changes sign and $\frac{dy}{dx}$ does not change sign.

Question 14 Answer C

A stationary point of inflection occurs at $x = -1$ since $f'(x) = 0$ and the derivative of $f'(x)$ is zero as well at that point. $f'(x)$ changes from negative to positive through zero at $x = 2$ and so there is a local minimum at this point for the graph of $y = f(x)$. The rate of change of the gradient of $f'(x)$ is zero at $x = 1$ (but the gradient of $f(x)$ is about -4) and so there is a point of inflection at this point.

Question 15 Answer A

It should be noted that $\frac{dy}{dx}$ is

- positive if $y > 0$
- negative if $y < 0$
- infinite (shown by vertical steepness) as y approaches zero.

Only alternative A displays these properties.

Question 16 Answer D

The direction of motion at $t = 2$ is given by $\dot{\mathbf{r}}(2)$.

$$\dot{\mathbf{r}}(t) = (3 - 4t)\mathbf{i} + 6\mathbf{j} + \frac{1}{2\sqrt{t}}\mathbf{k}$$

Substituting $t = 2$: $-5\mathbf{i} + 6\mathbf{j} + \frac{1}{2\sqrt{2}}\mathbf{k}$ which is equivalent to $-5\mathbf{i} + 6\mathbf{j} + \frac{\sqrt{2}}{4}\mathbf{k}$.

Question 17 Answer B

Let the coordinates of P be $(-a, -b)$ where a and b are both positive. Hence $z = -a - bi$.
 Q (by inspection) is $-b + ai$.

Testing each of the alternatives given:

A. $iz = i(-a - bi) = b - ai$ which is not Q . However, it is the negative of what is required and so the answer is B.

Alternatively, rotation clockwise through a right angle is equivalent to multiplication by $-i$.

Question 18 Answer E

$$\begin{aligned} v &= \int (5 \sin(2t) - 1) dt \\ &= -\frac{5}{2} \cos(2t) - t + c \end{aligned}$$

At $t = 0$, $v = 0$ and so $c = 2.5$

$$v = -\frac{5}{2} \cos(2t) - t + 2.5$$

At $t = 1$, $v = -2.5 \cos(2) - 1 + 2.5$

Answer: $v = 2.54$

Question 19 Answer B

Acceleration down slope = $g \sin(\theta)$ where

$$\theta = \arctan\left(\frac{1}{5}\right).$$

$\therefore a = \frac{g}{\sqrt{26}}$ is Jimmy's uniform acceleration.

$$u = 0, s = 250, a = \frac{g}{\sqrt{26}} \text{ and } v^2 = u^2 + 2as$$

$$\therefore v^2 = 0 + 2 \times \frac{g}{\sqrt{26}} \times 250 \text{ and so } v = 30.9995$$

$$\text{Momentum} = mv$$

$$= 75 \times 30.9995$$

$$= 2325 \text{ kgms}^{-1}$$

Question 20 Answer A

Let $\underline{i}$ and $\underline{j}$ be unit vectors acting east and north respectively.

The sum of the two forces are:

$$8\underline{j} + (-6\cos 60^\circ \underline{j} - 6\sin 60^\circ \underline{i})$$

$$= -6\sin 60^\circ \underline{i} + (8 - 6\cos 60^\circ) \underline{j}$$

$$= -3\sqrt{3} \underline{i} + 5 \underline{j}$$

$$\text{Magnitude} = \sqrt{(27 + 25)}$$

$$= \sqrt{52}$$

$$= 2\sqrt{13}$$

Question 21 Answer B

$$-mg = \frac{mv^2}{1000} = ma$$

$$a = -\frac{1000g + v^2}{1000}$$

$$v \frac{dv}{dx} = -\frac{1000g + v^2}{1000}$$

$$\frac{dv}{dx} = -\frac{1000g + v^2}{1000v}$$

$$\frac{dx}{dv} = -\frac{1000v}{1000g + v^2}$$

$$x = \int -\frac{1000v}{1000g + v^2} dv$$

$$x = -500 \ln(1000g + v^2) + c$$

When $x = 0$, $v = 400$

$$x = 500 \ln \frac{9800 + 400^2}{1000g + v^2}$$

When $v = 0$, $x = 1426$ m

Question 22 Answer C

$$s = -60, a = -9.8, u = 20, t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$-60 = 20t - 4.9t^2$$

$$4.9t^2 - 20t - 60 = 0$$

Solving this quadratic using TI-83 QUAD PRGM: $t = -2.01, 6.09$

Ignoring the negative solution, $t = 6.09$ seconds.

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Section 2

Question 1

a. $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i = 1 \operatorname{cis}\left(\frac{3\pi}{4}\right)$

Apply de Moivre's theorem:

$$z^2 = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

$$r^2 \operatorname{cis}(2\theta) = 1 \operatorname{cis}\left(\frac{3\pi}{4}\right)$$

M1

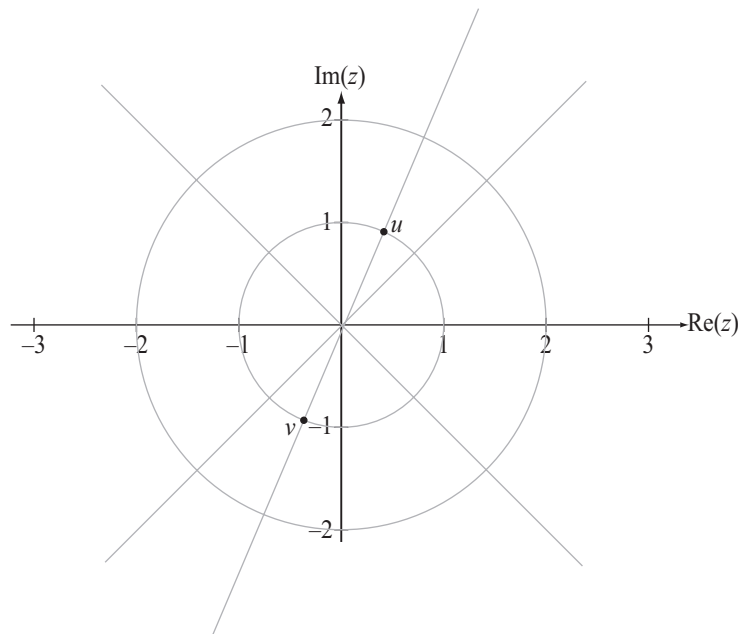
$$r^2 = 1 \therefore r = 1, u = 1 \operatorname{cis}\left(\frac{3\pi}{4}\right)$$

$$2\theta = \frac{3\pi}{4} + 2k\pi, k = 0, 1$$

$$u = 1 \operatorname{cis}\left(\frac{3\pi}{8}\right) \text{ and } v = 1 \operatorname{cis}\left(-\frac{5\pi}{8}\right)$$

A2

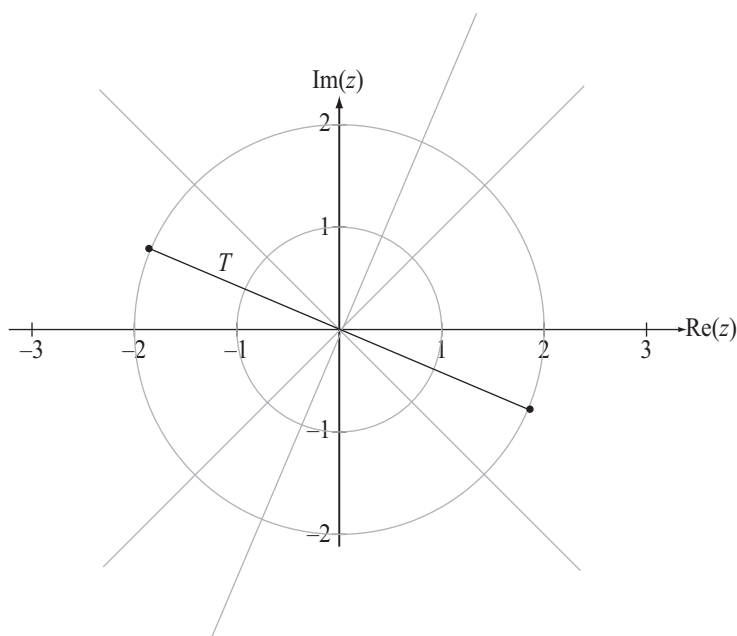
b.



A2 for u and v in correct positions

A1 for u and v swapped

c.



Region T is the intersection of the inside of the circle centre $(0, 0)$ and radius 2 with the perpendicular bisector of the line segment joining u and v.

A1 perpendicular bisector

A1 inside of the circle

A1 intersection of the two clearly labelled.

d. For what values of n is u^n purely imaginary?

$$\begin{aligned} u^n &= \left[1 \operatorname{cis} \left(\frac{3\pi}{8} \right) \right]^n \\ &= 1 \operatorname{cis} \left(\frac{3n\pi}{8} \right) \end{aligned}$$

If this is purely imaginary then $\cos \left(\frac{3n\pi}{8} \right) = 0$

M1

$$\frac{3n\pi}{8} = \frac{(2k-1)\pi}{2} \text{ where } k \in \mathbb{Z} \text{ or alternatively}$$

$$\frac{3n\pi}{8} = \frac{(2k+1)\pi}{2}$$

M1 for equating angle to $\frac{\text{odd } \pi}{2}$

Hence $n = \frac{4}{3}(2k-1)$ where $k \in \mathbb{Z}$ **or** $n = \frac{4}{3}(2k+1)$ for any multiple of $\frac{\pi}{2}$

A1

Question 2

a. $1 + \log_e x = 0$

$$\log_e x = -1 \text{ so } x = e^{-1}$$

$$\text{Coordinates of } A = (e^{-1}, 0)$$

A1 for $(e^{-1}, 0)$

b. i.
$$f'(x) = \frac{x\left(\frac{1}{x}\right) - 1(1 + \log_e(x))}{x^2}$$

$$= \frac{1 - 1 - \log_e(x)}{x^2}$$

$$= \frac{-\log_e(x)}{x^2}$$

A1

ii. $f'(x) = 0$

$$\therefore \frac{-\log_e(x)}{x^2} = 0$$

$$\log_e(x) = 0$$

$$\text{When } x = 1, y = 1$$

$$\text{Coordinates of } B = (1, 1)$$

A1 for $(1, 1)$

c.
$$f''(x) = \frac{x^2\left(-\frac{1}{x}\right) + 2x\log_e(x)}{x^4}$$

$$= \frac{-x + 2x\log_e(x)}{x^4}$$

$$= \frac{-1 + 2\log_e(x)}{x^3}$$

$$= 0 \text{ if } x = e^{0.5}$$

A1

M1 for equating second derivative to zero
A1 for both coordinates correct

$$\text{Coordinates of } C = (e^{0.5}, 1.5e^{-0.5})$$

d. $\sqrt{e} \approx 1.65$

$$f''(1.6) = -0.0146$$

$$f''(1.7) = 0.0125$$

$f''(x)$ changes sign so the concavity changes, hence the point of inflection.

A1

It should be noted that the first derivative does **not** change sign at $x = 1.65$.

- e. Equate the first derivative to the gradient between the points $(0, 0)$ and $\left(\frac{1 + \log_e(x)}{x}\right)x$.

$$-\frac{\log_e(x)}{x^2} = \frac{\frac{1 + \log_e(x)}{x} - 0}{x - 0}$$

$$-\frac{\log_e(x)}{x^2} = \frac{1 + \log_e(x)}{x^2}$$

$$-\log_e(x) = 1 + \log_e(x)$$

$$-2\log_e(x) = 1 \text{ and so } \log_e(x) = -\frac{1}{2}$$

$$\therefore x = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

f. $\int_0^{\frac{1}{\sqrt{e}}} \left(\frac{e}{2}x\right)dx - \int_{\frac{1}{e}}^{\frac{1}{\sqrt{e}}} \left(\frac{1 + \log_e(x)}{x}\right)dx$

(This first integral represents the area under the tangent which passes through the origin.)

M1 the difference of two integrals

A1 correct limits

$$= \left[\frac{ex^2}{4} \right]_0^{\frac{1}{\sqrt{e}}} - \frac{1}{2} \left[(1 + \log_e(x))^2 \right]_{\frac{1}{e}}^{\frac{1}{\sqrt{e}}}$$

A1 (correct anti-derivative of the $\frac{1 + \log_e(x)}{x}$ function)

$$= 0.25 - \frac{1}{2}[(1 - 0.5)^2 - (1 - 1)^2]$$

$$= \frac{1}{8}$$

Question 3

- a. $\overrightarrow{OA} = -\underline{i} + 3\underline{j} + 2\underline{k}$, $\overrightarrow{OB} = 3\underline{i} + 6\underline{j} + \underline{k}$ and $\overrightarrow{OC} = -4\underline{i} + 4\underline{j} + 3\underline{k}$

$$\overrightarrow{BC} \cdot \overrightarrow{BA} = (\overrightarrow{BO} + \overrightarrow{OC}) \cdot (\overrightarrow{BO} + \overrightarrow{OA})$$

M1 dot product

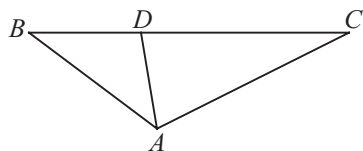
$$= (-3\underline{i} - 6\underline{j} - \underline{k} - 4\underline{i} + 4\underline{j} + 3\underline{k}) \cdot (-3\underline{i} - 6\underline{j} - \underline{k} - \underline{i} + 3\underline{j} + 2\underline{k})$$

$$= (-7\underline{i} - 2\underline{j} + 2\underline{k}) \cdot (-4\underline{i} - 3\underline{j} + \underline{k})$$

A1 for these vectors

$$= 28 + 6 + 2$$

$$= 36$$



b. $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

A1

$$= -\underline{q} + k\underline{p}$$

Let D be a point on the side BC and let $\overrightarrow{BC} = \underline{p}$, $\overrightarrow{BA} = \underline{q}$ and $\overrightarrow{BD} = k\underline{p}$, $k \in R$.

SOLUTIONS – continued

- c. If $\overrightarrow{AD} \perp \overrightarrow{BC}$ then $\overrightarrow{AD} \cdot \overrightarrow{BC} = 0$.

Find k such that $(-q + kp) \cdot p = 0$

$$\text{So } -q \cdot p + kp \cdot p = 0$$

$$k = \frac{q \cdot p}{p \cdot p}$$

$$q \cdot p = 36 \text{ (from part (a))}$$

$$p \cdot p = (-7\hat{i} - 2\hat{j} + 2\hat{k}) \cdot (-7\hat{i} - 2\hat{j} + 2\hat{k}) \quad \text{M1}$$

$$= 57$$

$$\text{So } k = \frac{36}{57} = \frac{12}{19} \quad \text{A1}$$

- d. Find the magnitude of $\overrightarrow{AD}$ with $k = \frac{12}{19}$ for the shortest distance because we need $\overrightarrow{AD} \perp \overrightarrow{BC}$.

$$\overrightarrow{AD} = -q + kp \text{ with } k = \frac{12}{19} \quad \text{M1}$$

$$\overrightarrow{AD} = 4\hat{i} + 3\hat{j} - \hat{k} + \frac{12}{19}(-7\hat{i} - 2\hat{j} + 2\hat{k}) \quad \text{M1}$$

$$= -\frac{8}{19}\hat{i} + \frac{33}{19}\hat{j} + \frac{5}{19}\hat{k} \quad \text{A1}$$

$$|\overrightarrow{AD}| = \sqrt{\left(-\frac{8}{19}\right)^2 + \left(\frac{33}{19}\right)^2 + \left(\frac{5}{19}\right)^2}$$

$$= \sqrt{\frac{62}{19}}$$

$$|\overrightarrow{AD}| = 1.81 \text{ to 2 decimal places} \quad \text{A1}$$

Question 4

- a. i.** Rate of mass going in = $0.1 \text{ kg/litre} \times 5 \text{ litre/min}$ **M1**
 $= 0.5 \text{ kg/min}$ **A1**

$$\begin{aligned} \text{Rate of mass going out} &= \frac{x}{800} \text{ kg/litre} \times 5 \text{ litre/min} \\ &= \frac{5x}{800} \text{ kg/min} \end{aligned} \quad \textbf{A1}$$

$$\frac{dx}{dt} = 0.5 - \frac{x}{160} = \frac{80 - x}{160}$$

- ii.** $a = 80$ and $b = 160$ **A1**

iii. $\frac{dt}{dx} = \frac{160}{80 - x}$ **M1** for inversion

$$t = \int \frac{160}{80 - x} dx = -160 \log_e |80 - x| + c \quad \textbf{M1 log recognition}$$

When $t = 0$, $x = 10$ so $c = 160 \log_e 70$

$$t = 160 \log_e \left(\frac{70}{80 - x} \right) \quad \textbf{A1}$$

$$\frac{80 - x}{70} = e^{-\frac{t}{160}}$$

$$80 - 70e^{-\frac{t}{160}} = x \quad \textbf{A1}$$

- iv.** When $t = 12$, $x = 15.058 \text{ kg}$ **A1**

- b.** Rate of mass going in = $0.1 \text{ kg/litre} \times 5 \text{ litre/min}$
 $= 0.5 \text{ kg/min}$

Number of litres in the tank after t minutes $800 + 4t$ **M1** for volume $800 + At$
 where A is a constant

$$\begin{aligned} \text{Rate of mass going out} &= \frac{x}{800 + 4t} \text{ kg/litre} \times 1 \text{ litre/min} \\ &= \frac{x}{800 + 4t} \text{ kg/min} \end{aligned}$$

$$\frac{dx}{dt} = 0.5 - \frac{x}{800 + 4t} \quad \textbf{A1}$$

Question 5

a. $\underline{a} = 0\underline{i} - 9.8\underline{j}$

$$\begin{aligned}\underline{v} &= \int (0\underline{i} - 9.8\underline{j}) dt \\ &= c\underline{i} - (9.8t + d)\underline{j} \text{ where } c \text{ and } d \text{ are constants.}\end{aligned}$$

M1

When $t = 0$, $\underline{v} = 5\underline{i}$ and so $c = 5$ and $d = 0$.

$$\begin{aligned}\underline{r} &= \int (5\underline{i} - 9.8t\underline{j}) dt \\ &= (5t + e)\underline{i} - (4.9t^2 + f)\underline{j} \text{ where } e \text{ and } f \text{ are constants.}\end{aligned}$$

When $t = 0$, $\underline{a} = 0\underline{i} + 40\underline{j}$ and so $e = 0$ and $f = 40$.

$$\underline{r} = 5t\underline{i} + (40 - 4.9t^2)\underline{j} \text{ as required.}$$

A1

b. The $\underline{j}$ component of $\underline{r} = 5t\underline{i} + (40 - 4.9t^2)\underline{j}$ is zero when the toy touches the ground.

$$40 - 4.9t^2 = 0 \text{ so } t = \sqrt{\frac{40}{4.9}}$$

It reaches the ground after 2.86 seconds.

A1

The horizontal distance reached is the $\underline{i}$ component: $5t$

It travels 14.29 metres before reaching the ground.

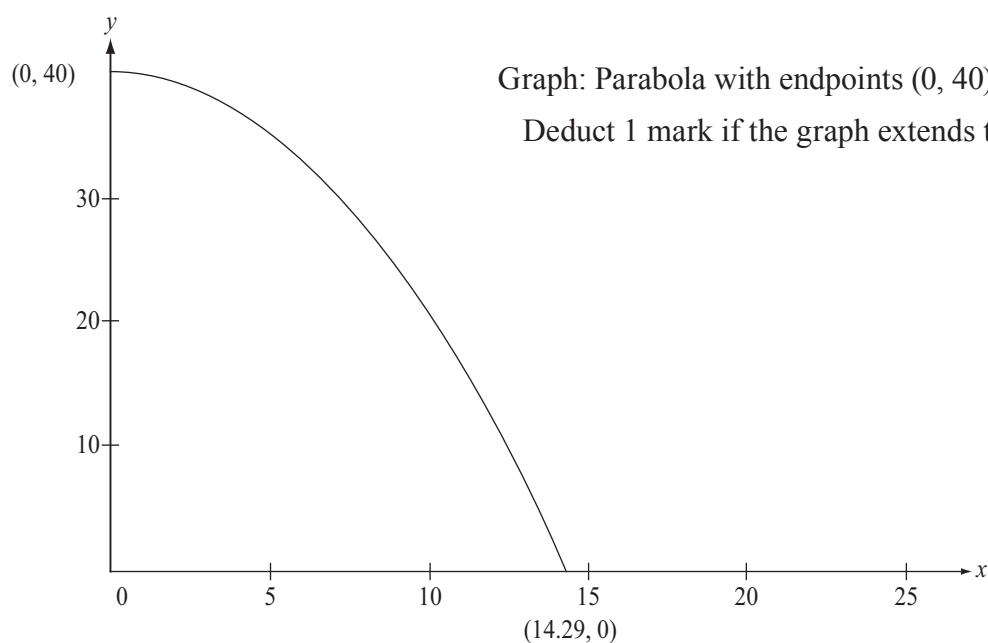
A1

c. $x = 5t$ so $t = \frac{x}{5}$ and $y = 40 - 4.9t^2$

A1

Substituting for t into y gives: $y = 40 - 4.9\left(\frac{x}{5}\right)^2$

$$\therefore y = 40 - \frac{49x^2}{250}$$

A1

- d. $\underline{v} = 5\underline{i} - 9.8\underline{tj}$ and $t = \sqrt{\frac{40}{4.9}}$ when the plane hits the ground. **M1**

The magnitude of the velocity at the point of impact with the ground is

$$\sqrt{\left(25 + 9.8^2 \times \frac{40}{4.9}\right)} \text{ which is } \sqrt{809}. \quad \text{A1}$$

$$\text{Magnitude of momentum} = |mv|$$

$$= 1.5 \times \sqrt{809}$$

$$= 42.7 \text{ kg m/s to 3 significant figures.} \quad \text{A1}$$

e. $\tan(\theta) = \frac{40}{x}$

$$\frac{d}{dt}(\tan(\theta)) = \frac{d}{dt}\left(\frac{40}{x}\right)$$

$$\text{So } \frac{d}{d\theta}(\tan(\theta)) \frac{d\theta}{dt} = \frac{d}{dx}\left(\frac{40}{x}\right) \frac{dx}{dt} \quad \text{M1}$$

$$\therefore \frac{d\theta}{dt} = -\frac{40}{x^2 \sec^2(\theta)} \frac{dx}{dt} \quad \text{A1}$$

f. $\frac{dx}{dt} = 5$ as it is moving with a constant speed of 5 m/s.

$$\frac{d\theta}{dt} = -\frac{200 \cos^2(\theta)}{x^2}$$

$$\text{Now } \cos(\theta) = \frac{x}{\sqrt{x^2 + 1600}} \quad \text{M1}$$

$$\text{When } x = 10, \cos(\theta) = \frac{1}{\sqrt{17}}, \sec^2(\theta) = 17 \text{ and so } \frac{d\theta}{dt} = -\frac{2}{17} \quad \text{A1}$$