

Year 2007

VCE

Specialist Mathematics

Trial Examination 1



KILBAHA MULTIMEDIA PUBLISHING
PO BOX 2227
KEW VIC 3101
AUSTRALIA

TEL: (03) 9817 5374
FAX: (03) 9817 4334
chemas@chemas.com
www.chemas.com

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**Victorian Certificate of Education
2007**

STUDENT NUMBER

Figures
Words

Letter

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SPECIALIST MATHEMATICS

Trial Written Examination 1

Reading time: 15 minutes

Total writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
9	9	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

- Question and answer book of 14 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

Instructions

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above this page.
- All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Instructions

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

Question 1

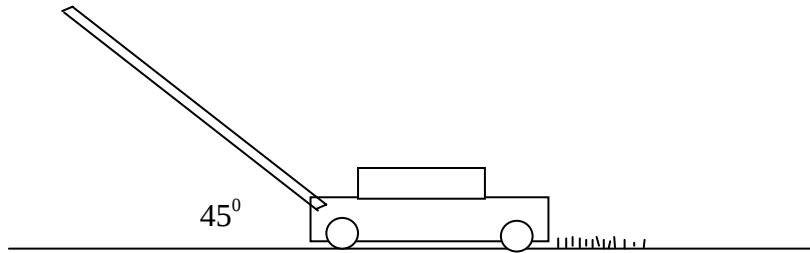
Consider the relation $\frac{x}{e^{2y}} - 5y + 6x^2 + 3 = 0$

Find an expression for $\frac{dy}{dx}$ in terms of x and y .

3 marks

Question 2

A man pushes down along the handle of a motor-mower with a force of T newtons. The handle of the motor-mower is inclined at an angle of 45° to the horizontal lawn. The motor-mower has a mass of 10 kg and the coefficient of friction between the uncut grass and the motor-mower is $\frac{1}{7}$.



- a. On the diagram above, mark in all the forces on the motor-mower.

1 mark

- b. Show that if $T = 12\sqrt{2}$, the motor-mower will not move.

2 marks

- c. Find the acceleration of the motor-mower if $T = \frac{49\sqrt{2}}{2}$.

3 marks

Question 3

Solve the differential equation $\frac{dy}{dx} = \frac{3x-5}{\sqrt{9-4x^2}}$ given that $y = 0$ when $x = 0$.

5 marks

Question 4

Consider the cubic equation $z^3 + pz^2 + qz + 15 = 0$, where p and q are real numbers.
One root of this equation is $1 - 2i$.

- a.** State another root.

1 mark

- b.** Find the values of p and q and all the roots.

3 marks

Question 5

a. Show that $\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$

3 marks

b. Find $\text{Arg}\left(-1 + (\sqrt{3} - 2)i\right)$

1 mark

Question 6

a. Show that $\frac{d}{dx} \left(\tan^{-1} \left(\sqrt{\frac{3}{x}} \right) \right) = \frac{-\sqrt{3}}{2\sqrt{x}(x+3)}$ for $x > 0$.

2 marks

b. Hence, find the exact value of $\int_1^9 \frac{1}{\sqrt{x^3 + 6x^2 + 9x}} dx$.

2 marks

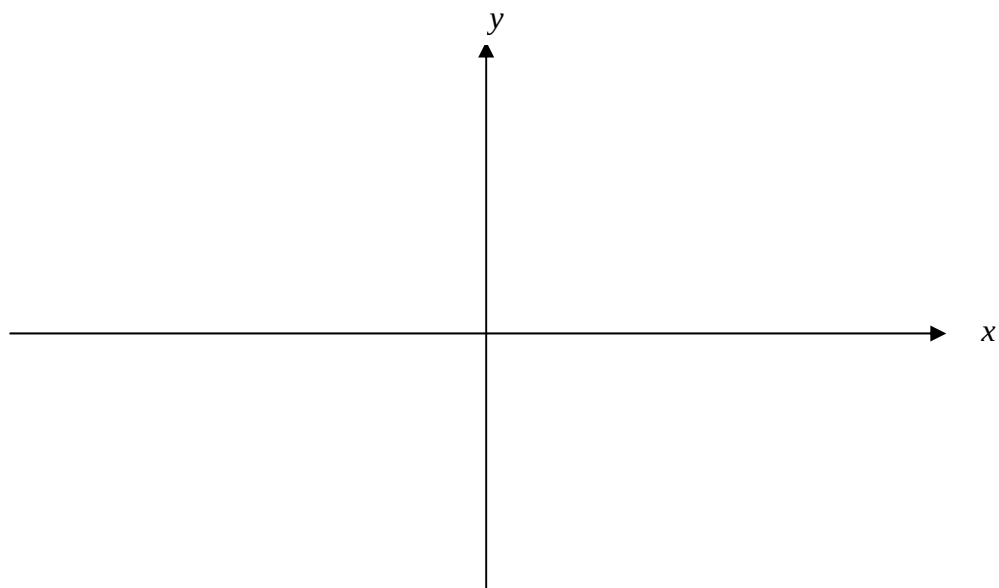
Question 7

The position vector of a moving particle is given by $\underline{r}(t) = \frac{3}{2}(e^{2t} + e^{-2t})\underline{i} + \frac{5}{2}(e^{2t} - e^{-2t})\underline{j}$ for $t \geq 0$.

- a.** If the Cartesian equation of the path can be expressed in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ find the values of a and b , where a and b are positive real constants.

3 marks

- b.** Sketch the path of the particle on the axes provided.



1 mark

Question 8

The area enclosed by the curve with the equation $y = 6\sin\left(\frac{\pi x}{2}\right)$ is rotated about the x -axis, to form a solid of revolution between $0 \leq x \leq b$, where b is a real constant and $0 < b < 4$.

- a.** Find the volume of the solid of revolution, in terms of b .

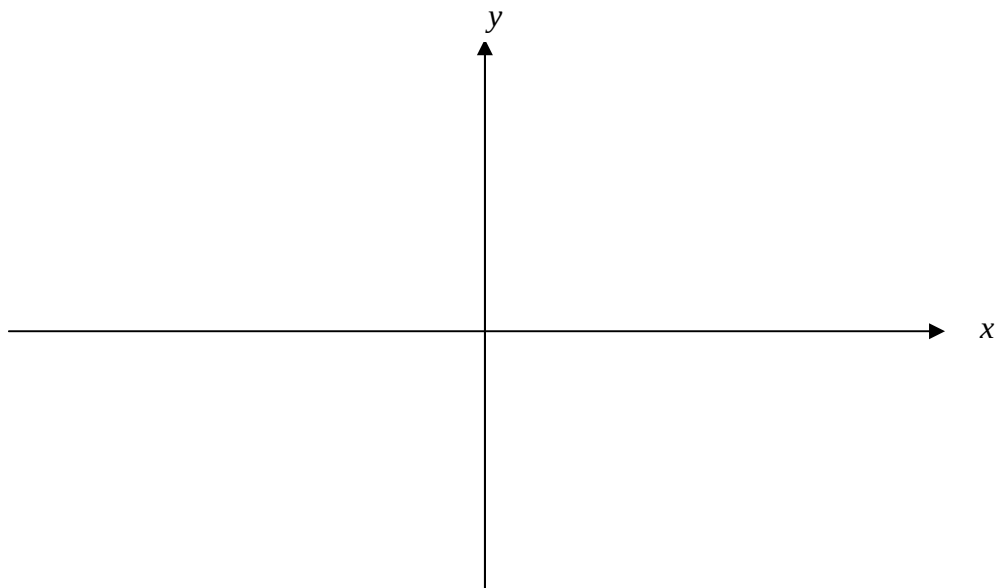
3 marks

- b.** Find the value of b if the volume is $9(7\pi + 2)$.

1 mark

Question 9

- a. Sketch the graph with the equation $y = \frac{4}{x^2 - 4x}$, clearly indicating the location of all asymptotes and any turning points.



2 marks

- b. Find the exact area bounded by $y = \frac{4}{x^2 - 4x}$, the x-axis and the lines $x = 1$ and $x = 2$.

4 marks

EXTRA WORKING SPACE

[illegible]

END OF EXAMINATION

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of triangle: $\frac{1}{2}bc \sin(A)$

sine rule: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex Numbers)

$$z = x + yi = r(\cos(\theta) + i \sin(\theta)) = r \operatorname{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$-\pi < \operatorname{Arg}(z) \leq \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

Vectors in two and three dimensions

$$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

Mechanics

momentum: $\underline{p} = m\underline{v}$

equation of motion: $\underline{R} = m\underline{a}$

sliding friction: $F \leq \mu N$

constant (uniform) acceleration:

$$v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u + v)t$$

acceleration: $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \quad \text{for } x > 0$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:
$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:
$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:
$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x)$

END OF FORMULA SHEET