

$$Q1 \quad \frac{2\sqrt{3} + 2i}{1 - \sqrt{3}i} = \frac{4cis\left(\frac{\pi}{6}\right)}{2cis\left(-\frac{\pi}{3}\right)} = 2cis\left(\frac{\pi}{6} + \frac{\pi}{3}\right) = 2cis\left(\frac{\pi}{2}\right)$$

$$\begin{aligned} Q2a \quad f(z) &= z^3 - (\sqrt{5} - i)z^2 + 4z - 4\sqrt{5} + 4i \\ &= z^2(z - (\sqrt{5} - i)) + 4(z - (\sqrt{5} - i)) \\ &= (z - (\sqrt{5} - i))(z^2 + 4) \\ \therefore f(\sqrt{5} - i) &= 0. \text{ Hence } \sqrt{5} - i \text{ is a solution of} \\ z^3 - (\sqrt{5} - i)z^2 + 4z - 4\sqrt{5} + 4i &= 0 \end{aligned}$$

Q2b The other two solutions are:

$$z^2 + 4 = 0, \quad z = -2i \text{ or } 2i$$

Q3 By implicit differentiation:

$$3x^2 - 4xy - 2x^2 \frac{dy}{dx} + 4y \frac{dy}{dx} = 0,$$

$$\frac{dy}{dx} = \frac{3x^2 - 4xy}{2x^2 - 4y}.$$

$$\text{At } P(2,3), \quad \frac{dy}{dx} = 3.$$

$$\text{Equation of tangent: } y - 3 = 3(x - 2), \therefore y = 3x - 3$$

$$\begin{aligned} Q4 \quad V &= \int_{-\frac{1}{2}}^0 \pi y^2 dx = \pi \int_{-\frac{1}{2}}^0 \frac{1}{1 - x^2} dx \\ &= \frac{\pi}{2} \int_{-\frac{1}{2}}^0 \left(\frac{1}{1 - x} + \frac{1}{1 + x} \right) dx \quad (\text{partial fractions}) \\ &= \frac{\pi}{2} [-\log_e(1 - x) + \log_e(1 + x)]_{-\frac{1}{2}}^0 \\ &= \frac{\pi}{2} \left[\log_e \frac{1 + x}{1 - x} \right]_{-\frac{1}{2}}^0 = \frac{\pi}{2} \left(-\log_e \frac{1}{3} \right) = \frac{\pi}{2} \log_e 3. \end{aligned}$$

Q5a $u = +4$, $s = +3$, $v = 0$, use $v^2 = u^2 + 2as$ to find a .

$$\therefore a = \frac{v^2 - u^2}{2s} = \frac{0 - 16}{6} = -\frac{8}{3} \text{ ms}^{-2}$$

Q5b Newton's second law: $R = ma$,

$$-\mu N = 6\left(-\frac{8}{3}\right), \quad -\mu(6g) = -6\left(\frac{8}{3}\right), \quad \mu = \frac{8}{3g}.$$

$$Q6a \quad \mathbf{r}(t) = \int (-4 \sin(2t)\mathbf{i} + 6 \cos(2t)\mathbf{j}) dt,$$

$$\mathbf{r}(t) = 2 \cos(2t)\mathbf{i} + 3 \sin(2t)\mathbf{j} + \mathbf{c}.$$

$$\text{Given } \mathbf{r}(0) = 2\mathbf{i}, \mathbf{c} = \mathbf{0}. \therefore \mathbf{r}(t) = 2 \cos(2t)\mathbf{i} + 3 \sin(2t)\mathbf{j}$$

$$Q6b \quad x = 2 \cos(2t), \quad y = 3 \sin(2t).$$

$$\therefore \frac{x^2}{4} = \cos^2(2t) \text{ and } \frac{y^2}{9} = \sin^2(2t)$$

$$\therefore \frac{x^2}{4} + \frac{y^2}{9} = 1.$$

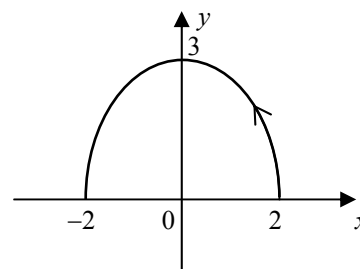
$$\text{Given } 0 \leq t \leq \frac{\pi}{2}, \text{ when } t = 0, \quad x = 2, \quad y = 0;$$

$$\text{when } t = \frac{\pi}{4}, \quad x = 0, \quad y = 3;$$

$$\text{when } t = \frac{\pi}{2}, \quad x = -2, \quad y = 0.$$

$$\therefore -2 \leq x \leq 2 \text{ and } 0 \leq y \leq 3.$$

Q6c



Q7a

$$x_0 = 1, \quad y_0 = 1$$

$$x_1 = 1.1, \quad y_1 \approx 1 + 0.1 \times \frac{1}{1} = 1 + 0.1 = 1.1$$

$$x_2 = 1.2, \quad y_2 \approx 1.1 + 0.1 \times \frac{1}{1.1} = 1.1 + \frac{0.1}{1.1} = \frac{1.31}{1.1} = \frac{131}{110}.$$

$$Q7b \quad \frac{dy}{dx} = \frac{1}{x}, \quad y = \int \frac{1}{x} dx = \log_e x + c, \text{ where } x > 0.$$

$$\text{Since } y(1) = 1, \therefore c = 1 \text{ and } y = \log_e x + 1.$$

$$\text{When } x = 1.2, \quad y = \log_e(1.2) + 1.$$

Q8a

y	-2	-1	0	1	2
$\frac{dy}{dx}$	2.5	1	0.5	1	2.5

$$\text{Note: } \frac{dy}{dx} = \frac{1 + y^2}{2} \text{ is independent of } x.$$

Graph in part c.

Q8b $\frac{dx}{dy} = \frac{2}{1+y^2}$, $x = 2 \int \frac{1}{1+y^2} dy$, $x = 2 \tan^{-1} y + c$.

Given $y = -1$ when $x = 0$,

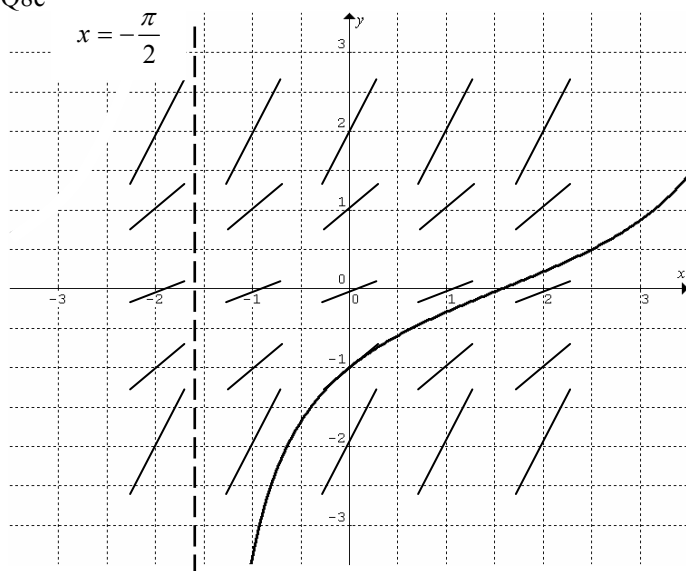
$$0 = 2 \tan^{-1}(-1) + c, \quad 0 = 2 \left(-\frac{\pi}{4} \right) + c.$$

$$\therefore c = \frac{\pi}{2} \text{ and } x = 2 \tan^{-1} y + \frac{\pi}{2}.$$

$$\text{Hence } y = \tan \frac{1}{2} \left(x - \frac{\pi}{2} \right).$$

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Q8c



Q9 $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$, $\mathbf{v} = \frac{d}{dt}\mathbf{r} = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j}$.

Given $\mathbf{v} = -y\mathbf{i} + x\mathbf{j}$, $\therefore \frac{dx}{dt} = -y$ and $\frac{dy}{dt} = x$.

$$\mathbf{a} = \frac{d}{dt}\mathbf{v} = -\frac{dy}{dt}\mathbf{i} + \frac{dx}{dt}\mathbf{j}.$$

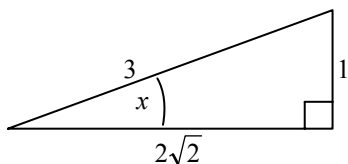
$$\therefore \mathbf{a} = -x\mathbf{i} - y\mathbf{j} = -(\mathbf{r}).$$

Hence $\mathbf{a} = -\mathbf{r}$.

Q10 $\tan(2x) = \frac{4\sqrt{2}}{7}$, $\frac{2 \tan x}{1 - \tan^2 x} = \frac{4\sqrt{2}}{7}$, where $0 \leq x < \frac{\pi}{4}$.

$$\therefore \tan^2 x + \frac{7}{2\sqrt{2}} \tan x - 1 = 0, \text{ where } 0 \leq x < \frac{\pi}{4}.$$

$$\tan x = \frac{-\frac{7}{2\sqrt{2}} + \sqrt{\frac{49}{8} + 4}}{2} = \frac{1}{2\sqrt{2}}. \quad (\text{Quadratic formula})$$



From diagram, $\sin x = \frac{1}{3}$.