

**Q1a**  $\frac{x^2}{3} - \frac{y^2}{2} = 1, x, y \in \mathbb{R}.$

Implicit differentiation,  $\frac{2x}{3} - y \frac{dy}{dx} = 0, \therefore \frac{dy}{dx} = \frac{2x}{3y}.$

**Q1b**  $\frac{dy}{dx} = -1, \therefore \frac{2x}{3y} = -1, \therefore y = -\frac{2x}{3}.$

Since  $\frac{x^2}{3} - \frac{y^2}{2} = 1, \therefore \frac{x^2}{3} - \frac{\left(-\frac{2x}{3}\right)^2}{2} = 1, \frac{x^2}{3} - \frac{2x^2}{9} = 1,$

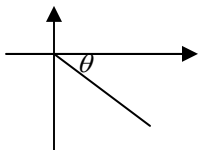
$\frac{x^2}{9} = 1, \therefore x = \pm 3$  and  $y = \mp 2.$

$\therefore (3, -2)$  or  $(-3, 2).$

**Q2a**  $\mathbf{a} = 12\mathbf{i} - 5\mathbf{j}$ , resultant force  $\mathbf{R} = m\mathbf{a} = 24\mathbf{i} - 10\mathbf{j}.$

$|\mathbf{R}| = \sqrt{24^2 + (-10)^2} = 26 \text{ N}.$

Direction: At  $\theta = \tan^{-1}\left(\frac{-10}{24}\right) = \tan^{-1}\left(\frac{-5}{12}\right)$  with  $i$ .



**Q2b** At  $t = 0$ , velocity  $\mathbf{v} = 0. \therefore \mathbf{v} = \int 12\mathbf{i} - 5\mathbf{j} dt = 12t\mathbf{i} - 5t\mathbf{j}.$

At  $t = 2$ , displacement  $\mathbf{s} = \int_0^2 12t\mathbf{i} - 5t\mathbf{j} dt = [6t^2\mathbf{i} - 2.5t^2\mathbf{j}]_0^2$

$= 24\mathbf{i} - 10\mathbf{j}.$

**Q3a**  $\mathbf{r}(t) = 2t\mathbf{i} - (5t + 1)\mathbf{j} + 2\mathbf{k}, \mathbf{v} = \frac{d}{dt}\mathbf{r}(t) = 2\mathbf{i} - 5\mathbf{j}.$

Velocity  $\mathbf{v}$  is constant,  $\therefore$  the particle moves in a straight line.

**Q3b** Speed  $= |\mathbf{v}| = \sqrt{2^2 + (-5)^2} = \sqrt{29}.$

**Q4** Let  $u = \log_e(2x), \frac{du}{dx} = \frac{1}{x}.$

$x \frac{dy}{dx} - \log_e(2x) = 0, \frac{dy}{dx} = \frac{\log_e(2x)}{x},$

$y = \int \frac{\log_e(2x)}{x} dx = \int u \frac{du}{dx} dx = \int u du = \frac{1}{2}u^2 + c = \frac{1}{2}(\log_e(2x))^2 + c$

Since  $f\left(\frac{1}{2}\right) = 0, \therefore c = 0, \therefore y = \frac{1}{2}(\log_e(2x))^2.$

**Q5a**  $T_{CD} = 5g = 5 \times 9.8 = 49 \text{ N}$

**Q5b**  $\frac{T_{BC}}{T_{CD}} = \frac{0.6}{1.0}, T_{BC} = 0.6T_{CD} = 0.6 \times 49 = 29.4 \text{ N}$

**Q6a**  $P(-1, 0, 1), Q(1, -2, 2)$  and  $R(2, 1, 0).$

$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) - (-\mathbf{i} + \mathbf{k}) = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

**Q6b**  $|\overrightarrow{PQ}| = \sqrt{2^2 + (-2)^2 + 1^2} = 3.$

Unit vector in the direction of  $\overrightarrow{PQ} = \frac{1}{3}(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}).$

$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP} = (2\mathbf{i} + \mathbf{j}) - (-\mathbf{i} + \mathbf{k}) = 3\mathbf{i} + \mathbf{j} - \mathbf{k}.$

Scalar resolute of  $\overrightarrow{PR}$  in the direction of  $\overrightarrow{PQ}$

$= (3\mathbf{i} + \mathbf{j} - \mathbf{k}) \cdot \frac{1}{3}(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 1.$

Vector resolute of  $\overrightarrow{PR}$  in the direction of  $\overrightarrow{PQ}$

$= 1 \times \frac{1}{3}(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = \frac{1}{3}(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}).$

Vector resolute of  $\overrightarrow{PR}$  perpendicular to  $\overrightarrow{PQ}$

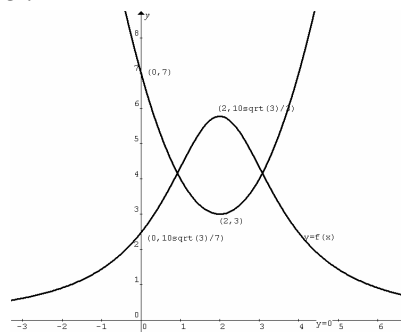
$= (3\mathbf{i} + \mathbf{j} - \mathbf{k}) - \frac{1}{3}(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = \frac{1}{3}(7\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}).$

$\therefore$  shortest distance  $= \frac{1}{3}\sqrt{7^2 + 5^2 + (-4)^2} = \sqrt{10}.$

**Q7a**  $f(x) = \frac{10\sqrt{3}}{x^2 - 4x + 7} = \frac{10\sqrt{3}}{3 + x^2 - 4x + 4} = \frac{10\sqrt{3}}{3 + (x - 2)^2}.$

**Q7b** Sketch the graph of the quadratic function

$y = x^2 - 4x + 7$ , then the graph of the reciprocal with dilation factor of  $10\sqrt{3}.$



**Q7c** Area

$= \int_{-1}^3 \frac{10\sqrt{3}}{x^2 - 4x + 7} dx = 10 \int_{-1}^3 \frac{\sqrt{3}}{3 + (x - 2)^2} dx = 10 \left[ \tan^{-1}\left(\frac{x - 2}{\sqrt{3}}\right) \right]_{-1}^3$

$= 10 \left[ \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \tan^{-1}\left(-\sqrt{3}\right) \right] = 10 \left( \frac{\pi}{6} + \frac{\pi}{3} \right) = 5\pi.$

**Q8a** Use the quadratic formula to find the zeros of

$$x^2 + i2\sqrt{3}x - 4.$$

$$x = \frac{-i2\sqrt{3} \pm \sqrt{(i2\sqrt{3})^2 - 4(1)(-4)}}{2(1)} = \frac{-i2\sqrt{3} \pm 2}{2} = \pm 1 - i\sqrt{3}.$$

$$\therefore x^2 + i2\sqrt{3}x - 4 = (x - 1 + i\sqrt{3})(x + 1 + i\sqrt{3}).$$

**Q8b** Express  $1 - i\sqrt{3}$  in polar form.  $1 - i\sqrt{3} = 2\text{cis}\left(-\frac{\pi}{3}\right).$

$$\sqrt{1 - i\sqrt{3}} = \left[2\text{cis}\left(-\frac{\pi}{3} + 2k\pi\right)\right]^{\frac{1}{2}} = \sqrt{2}\text{cis}\left(-\frac{\pi}{6} + k\pi\right)$$

$$= \sqrt{2}\text{cis}\left(-\frac{\pi}{6}\right) = \sqrt{2}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \frac{\sqrt{2}}{2}(\sqrt{3} - i) \text{ when } k = 0,$$

$$\text{or } = \sqrt{2}\text{cis}\left(\frac{5\pi}{6}\right) = \sqrt{2}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = -\frac{\sqrt{2}}{2}(\sqrt{3} - i) \text{ when } k = 1.$$

$$\text{Similarly, } -1 - i\sqrt{3} = 2\text{cis}\left(-\frac{2\pi}{3}\right).$$

$$\sqrt{-1 - i\sqrt{3}} = \left[2\text{cis}\left(-\frac{2\pi}{3} + 2k\pi\right)\right]^{\frac{1}{2}} = \sqrt{2}\text{cis}\left(-\frac{\pi}{3} + k\pi\right)$$

$$= \sqrt{2}\text{cis}\left(-\frac{\pi}{3}\right) = \sqrt{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = \frac{\sqrt{2}}{2}(1 - i\sqrt{3}) \text{ when } k = 0,$$

$$\text{or } = \sqrt{2}\text{cis}\left(\frac{2\pi}{3}\right) = \sqrt{2}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -\frac{\sqrt{2}}{2}(1 - i\sqrt{3}) \text{ when }$$

$$k = 1.$$

$$\begin{aligned} \text{Q8c } x^4 + i2\sqrt{3}x^2 - 4 &= (x^2 - 1 + i\sqrt{3})(x^2 + 1 + i\sqrt{3}) \\ &= \left(x - \frac{\sqrt{2}}{2}(\sqrt{3} - i)\right)\left(x + \frac{\sqrt{2}}{2}(\sqrt{3} - i)\right)\left(x - \frac{\sqrt{2}}{2}(1 - i\sqrt{3})\right)\left(x + \frac{\sqrt{2}}{2}(1 - i\sqrt{3})\right) \end{aligned}$$

**Q9a**  $v(t) = \frac{5(1-2t)}{1+2t}, t \geq 0.$

When  $v = 0$ ,  $\frac{5(1-2t)}{1+2t} = 0$ ,  $1-2t = 0$ ,  $t = \frac{1}{2}.$

**Q9b** At  $0 \leq t < \frac{1}{2}$ ,  $v > 0$ , the particle is moving *forwards*.

$$\text{Displacement} = \int_0^{\frac{1}{2}} \frac{5(1-2t)}{1+2t} dt = \int_0^{\frac{1}{2}} \left(\frac{10}{2t+1} - 5\right) dt$$

$$= \left[5 \log_e |2t+1| - 5t\right]_0^{\frac{1}{2}} = 5\left(\log_e 2 - \frac{1}{2}\right).$$

$$\text{Distance} = |\text{displacement}| = 5\left(\log_e 2 - \frac{1}{2}\right)$$

At  $\frac{1}{2} < t \leq 1$ ,  $v < 0$ , the particle is moving *backwards*.

$$\text{Displacement} = \int_{\frac{1}{2}}^1 \left(\frac{10}{2t+1} - 5\right) dt$$

$$= \left[5 \log_e |2t+1| - 5t\right]_{\frac{1}{2}}^1 = 5(\log_e 3 - 1) - 5\left(\log_e 2 - \frac{1}{2}\right) = 5\left(\log_e \frac{3}{2} - \frac{1}{2}\right)$$

$$\text{Distance} = |\text{displacement}| = -5\left(\log_e \frac{3}{2} - \frac{1}{2}\right).$$

$$\text{Total distance} = 5\left(\log_e 2 - \frac{1}{2}\right) - 5\left(\log_e \frac{3}{2} - \frac{1}{2}\right) = 5 \log_e \frac{4}{3}.$$

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