

Trial Examination 2006

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Question and Answer Booklet

Reading time: 15 minutes

Writing time: 1 hour

Student's Name: _____

Teacher's Name: _____

Structure of Booklet

Number of questions	Number of questions to be answered	Number of marks
9	9	40

Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.

Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

Materials supplied

Question and answer booklet of 8 pages with a detachable sheet of miscellaneous formulas in the centrefold.

Working space is provided throughout the booklet.

Instructions

Write **your name** and your **teacher's name** in the space provided above on this page.

All written responses must be in English.

Students are NOT permitted to bring mobile phones and/or any other electronic communication devices into the examination room.

Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2006 VCE Specialist Mathematics Units 3 & 4 Written Examination 1.

Instructions

Answer all questions in the spaces provided.

A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, appropriate working must be shown.

Unless otherwise indicated, the diagrams in this booklet are **not** drawn to scale.

Question 1

Given that $z = 2 + 3i$ is a solution of the equation $z^3 - 6z^2 + 21z - 26 = 0$,

- a. write down another complex solution of the equation.

1 mark

- b. Hence find a quadratic factor of the left-hand side of the equation.

2 marks

- c. Find the third solution.

1 mark

Total 4 marks

Question 2

A , B and C are three points on a plane such that $\overrightarrow{OA} = \underline{p}$, $\overrightarrow{OB} = 3\underline{q} - \underline{p}$ and $\overrightarrow{OC} = 9\underline{q} - 5\underline{p}$.

- a. Show that A , B and C are collinear.

3 marks

- b. State the ratio $AB : BC$.

1 mark
Total 4 marks

Question 3

Prove that $\cos^2(2x) - \sin^2(x) = \cos^2(x)(2\cos(x) - \sqrt{3})(2\cos(x) + \sqrt{3})$.

4 marks

Question 4

If $f(x) = \arctan(2x)$, find $f''(x)$.

3 marks

Question 5

A casserole is removed from the oven and placed on a bench in the kitchen. The casserole's rate of cooling is proportional to the difference between its temperature and the temperature of its surroundings. When removed from the oven, the casserole is at a temperature of 80°C , and it cools to 65°C in 10 minutes. If the temperature of the kitchen is kept at a constant 20°C , find the exact time for the casserole to cool to 50°C .

5 marks

Question 6

Find the exact value of k if $\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{1-4x^2}} dx = k\pi$.

4 marks

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VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Formula Sheet

Directions to students

Detach this formula sheet during reading time.
This formula sheet is provided for your reference.

SPECIALIST MATHEMATICS FORMULAS**Mensuration**

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc \sin(A)$

sine rule: $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos(C)$

Coordinate geometry

ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

hyperbola: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Circular (trigonometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r\operatorname{cis}(\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg}(z) \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, \quad n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \quad \text{for } x > 0$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), \quad x_0 = a \text{ and } (y_0 = b), \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos(\theta) = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt}\underline{i} + \frac{dy}{dt}\underline{j} + \frac{dz}{dt}\underline{k}$$

Mechanics

momentum: $\underline{p} = m\underline{v}$

equation of motion: $\underline{R} = m\underline{a}$

sliding friction: $F \leq \mu N$

END OF FORMULA SHEET

Question 7

Solve the differential equation $\frac{dy}{dx} - \sin(x)\cos^2(x) = 0$ given that $y(0) = -\frac{4}{3}$.

4 marks

Question 8

A lift of mass 420 kg, containing a passenger of mass 80 kg, is descending with an acceleration of 1.8 m/s^2 .

- a. Draw a diagram showing the forces acting on the lift.

1 mark

- b. Find the tension in the lift cable.

1 mark

- c. Draw a diagram showing the forces acting on the passenger.

1 mark

- d. Find the force exerted by the floor of the lift on the passenger.

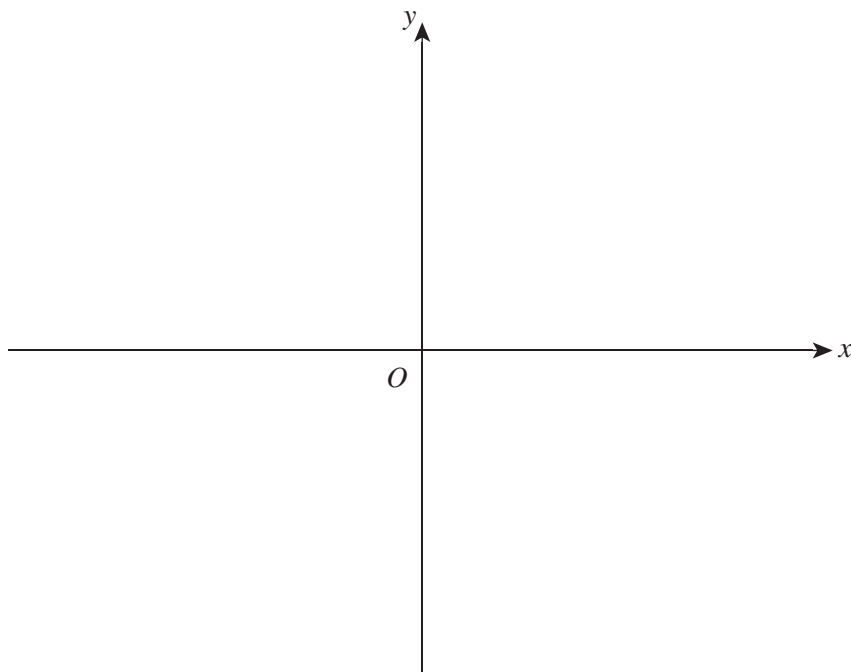
1 mark

Total 4 marks

Question 9

Consider the function with the rule $y = \frac{1}{x^2 + bx + c}$ where b and c are real valued constants.

- a. Sketch the graph when $b = 0$ and $c = 1$ on the axes below, labelling any axis intercepts, stationary points and asymptotes.

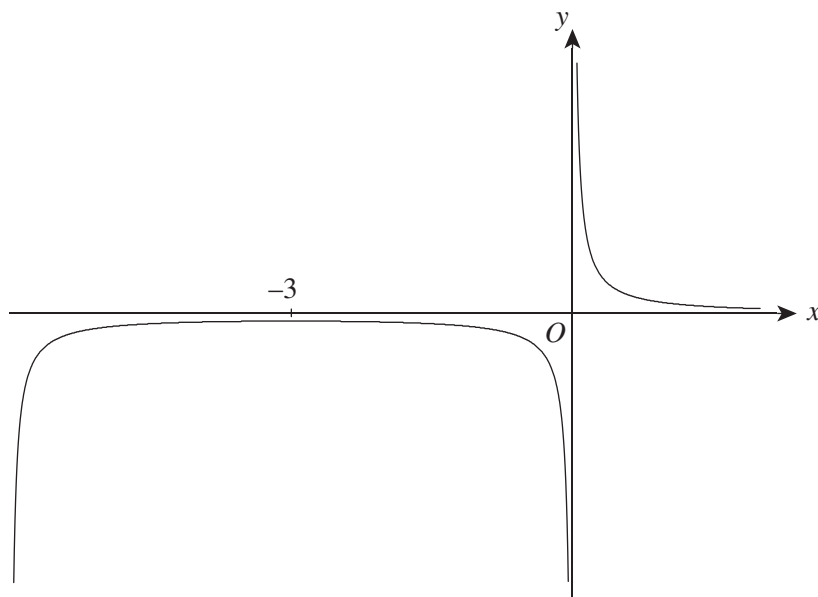


1 mark

- b. What is the relationship between b and c so that the graph will have no vertical asymptotes?

1 mark

Part of the graph of the function f , with rule $f(x) = \frac{1}{x^2 + bx + c}$ is shown below. The function has a turning point when $x = -3$ and a vertical asymptote with equation $x = 0$.



- c. Find $f'(x)$ and use it to show that $b = 6$.

2 marks

- d. Find the value of c .

1 mark

- e. **Hence** find the exact area of the region enclosed by the graph of f , the x -axis and the lines $x = -2$ and $x = -1$.

3 marks
Total 8 marks

END OF QUESTION AND ANSWER BOOKLET