



Trial Examination 2006

VCE Specialist Mathematics Units 3 & 4

Written Examination 1

Suggested Solutions

Question 1

- a. $2 - 3i$ A1
- b. $(z - 2 - 3i)(z - 2 + 3i)$ M1
 $= (z - 2)^2 - (3i)^2$
 $= z^2 - 4z + 13$ A1
- c. By division or inspection
 $z = 2$ A1

Question 2

- a. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
 $= 3\mathbf{q} - 2\mathbf{p}$ A1
 $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$
 $= 9\mathbf{q} - 6\mathbf{p}$
 $= 3\overrightarrow{AB}$ A1
 So $\overrightarrow{AB}$ is parallel to $\overrightarrow{AC}$ and A is the point in common.
 $\therefore A, B$ and C are collinear. M1
- b. $\overrightarrow{BC} = 6\mathbf{q} - 4\mathbf{p}$
 $= 2\overrightarrow{AB}$
 $\therefore |\overrightarrow{AB}| : |\overrightarrow{BC}| = 1 : 2$ A1

Question 3

Use of $\cos(2x) = 2\cos^2(x) - 1$ M1

LHS $= \cos^2(2x) - \sin^2(x) = (2\cos^2(x) - 1)^2 - \sin^2(x)$ A1
 $= 4\cos^4(x) - 4\cos^2(x) + 1 - (1 - \cos^2(x))$
 $= 4\cos^4(x) - 3\cos^2(x)$ A1
 $= \cos^2(x)(4\cos^2(x) - 3)$
 $= \cos^2(x)(2\cos(x) - \sqrt{3})(2\cos(x) + \sqrt{3}) = \text{RHS}$ A1

Question 4

$f(x) = \arctan(2x)$
 $= \arctan\left(\frac{x}{1/2}\right)$

$f'(x) = \frac{1/2}{1/4 + x^2}$ M1
 $= \frac{2}{4x^2 + 1}$ A1

$f''(x) = -\frac{16x}{(4x^2 + 1)^2}$ A1

Question 5

$$\frac{dT}{dt} = k(T - 20) \quad (k < 0) \quad \text{A1}$$

$$\frac{dt}{dT} = \frac{1}{k(T - 20)}$$

$$t = \frac{1}{k} \log_e(T - 20) + c \quad (\text{since } T - 20 > 0, |T - 20| \text{ is not required}) \quad \text{A1}$$

When $t = 0$, $T = 80$

$$0 = \frac{1}{k} \log_e(60) + c$$

$$c = -\frac{1}{k} \log_e(60) \quad \text{M1}$$

$$t = \frac{1}{k} \log_e\left(\frac{T - 20}{60}\right)$$

When $t = 10$, $T = 65$

$$10 = \frac{1}{k} \log_e\left(\frac{45}{60}\right)$$

$$k = \frac{1}{10} \log_e\left(\frac{3}{4}\right) \quad \text{A1}$$

$$t = \frac{10}{\log_e\left(\frac{3}{4}\right)} \log_e\left(\frac{T - 20}{60}\right)$$

The time taken to cool to 50°C is $\frac{10 \log_e\left(\frac{1}{2}\right)}{\log_e\left(\frac{3}{4}\right)}$ minutes. A1

Question 6

$$\int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{\sqrt{1 - 4x^2}} dx = \frac{1}{2} \int_{\frac{1}{4}\sqrt{\frac{1}{4}}}^{\frac{1}{2}} \frac{1}{\sqrt{\frac{1}{4} - x^2}} dx \quad \text{M1}$$

$$= \frac{1}{2} \left[\sin^{-1}(2x) \right]_{\frac{1}{4}}^{\frac{1}{2}} \quad \text{A1}$$

$$= \frac{1}{2} \left(\sin^{-1}(1) - \sin^{-1}\left(\frac{1}{2}\right) \right)$$

$$= \frac{1}{2} \left(\frac{\pi}{2} - \frac{\pi}{6} \right) \quad \text{A1}$$

$$= \frac{\pi}{6}$$

Hence $k = \frac{1}{6}$ A1

Question 7

$$\frac{dy}{dx} = \sin(x)\cos^2(x)$$

$$y = \int (\sin(x)\cos^2(x))dx$$

Let $u = \cos(x)$ and so $\frac{du}{dx} = -\sin(x)$.

M1

$$y = -\int u^2 du$$

A1

$$y = -\frac{u^3}{3} + c$$

$$y = -\frac{\cos^3(x)}{3} + c$$

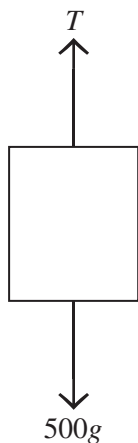
A1

Apply the condition to find the value of c .

$$-\frac{4}{3} = -\frac{1}{3} + c \text{ and so } c = -1$$

$$\text{Hence } y = -\left(\frac{\cos^3(x)}{3} + 1\right).$$

A1

Question 8**a.**

where T = tension in the cable

A1

$$\mathbf{b.} \quad \Sigma F = ma \Rightarrow 500 \times g - T = 500 \times 1.8$$

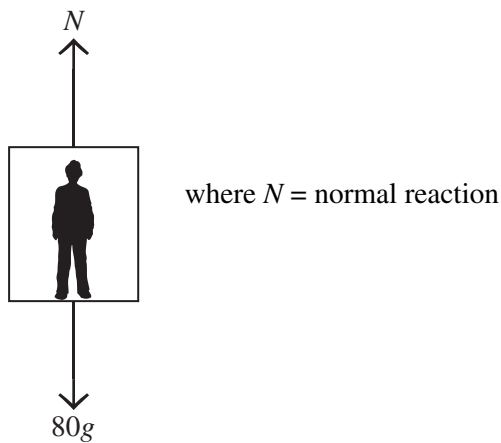
$$T = 500(g - 1.8)$$

$$= 500 \times 8$$

$$= 4000 \text{ N}$$

A1

c.



A1

d. $\Sigma F = ma \Rightarrow 80 \times g - N = 80 \times 1.8$

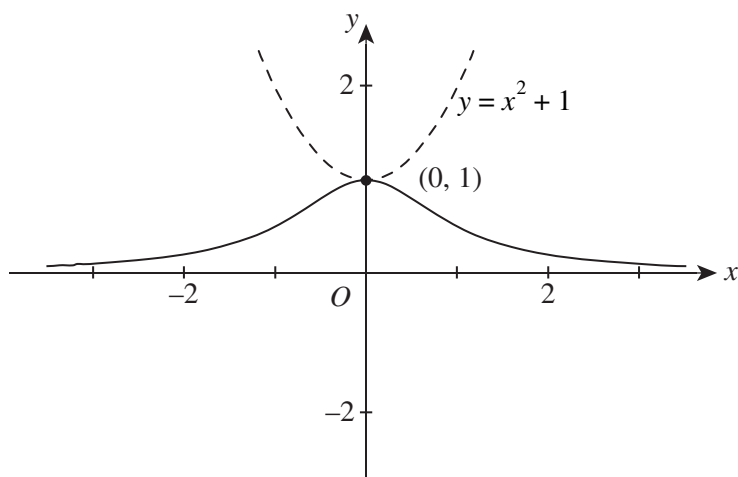
$$N = 80 \times 8$$

$$= 640 \text{ N}$$

A1

Question 9

a.



Maximum turning point labelled correctly and horizontal asymptote at $y = 0$.

A1

b. $b^2 - 4c < 0$ or $b^2 < 4c$

A1

c. $f(x) = (x^2 + bx + c)^{-1}$

$$f'(x) = \frac{-(2x + b)}{(x^2 + bx + c)^2}$$

M1

$$f'(x) = 0 \text{ when } 2x + b = 0$$

Since $x = -3$, $-6 + b = 0$ and so $b = 6$.

A1

d. There is an asymptote at $x = 0$,
so $x^2 + 6x + c$ has x as a factor.

$$\therefore c = 0$$

A1

e. $f(x) = \frac{1}{x^2 + 6x}$

$$= \frac{1}{x(x+6)}$$
$$= \frac{1}{6} \left(\frac{1}{x} - \frac{1}{x+6} \right) \quad \text{A1}$$

So $A = -\frac{1}{6} \int_{-2}^{-1} \left(\frac{1}{x} - \frac{1}{x+6} \right) dx$

$$= -\frac{1}{6} \left[\log_e \left| \frac{x}{x+6} \right| \right]_{-2}^{-1} \quad \text{M1}$$

$$= -\frac{1}{6} \left(\log_e \left(\frac{1}{5} \right) - \log_e \left(\frac{1}{2} \right) \right)$$
$$= -\frac{1}{6} \log_e \left(\frac{2}{5} \right) \quad \left(= \frac{1}{6} \log_e \left(\frac{5}{2} \right) \right) \quad \text{A1}$$