



The Mathematical Association of Victoria SPECIALIST MATHEMATICS

Trial written examination 2

2006

Reading time: 15 minutes

Writing time: 2 hours

Student's Name:

QUESTION AND ANSWER BOOK

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	4	4	58
			Total 80

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

These questions have been written and published to assist students in their preparations for the 2006 Specialist Mathematics Examination 2. The questions and associated answers and solutions do not necessarily reflect the views of the Victorian Curriculum and Assessment Authority. The Association gratefully acknowledges the permission of the Authority to reproduce the formula sheet.

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MULTIPLE CHOICE ANSWER SHEET

Student Name:

Circle the letter that corresponds to each correct answer

- 1.
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SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

The asymptotes of the curve $(x + 1)^2 - (y + 2)^2 = 1$ intersect the x -axis at

- A. $x = -3$ and $x = 1$
- B. $x = -2$ and $x = -1$
- C. $x = -1$ and $x = 3$
- D. $x = 1$ and $x = 2$
- E. $x = -1 - \sqrt{5}$ and $x = -1 + \sqrt{5}$

Question 2

If $u = -1 + i$ and $v = 1 - i$ which one of the following represents a real number?

- A. $u + \bar{v}$
- B. $u - v$
- C. uv
- D. $\frac{1}{v}$
- E. $\frac{u}{v}$

Question 3

$16 \operatorname{cis}\left(\frac{\pi}{3}\right)$ is a fourth root of the complex number z .

Another fourth root of z is

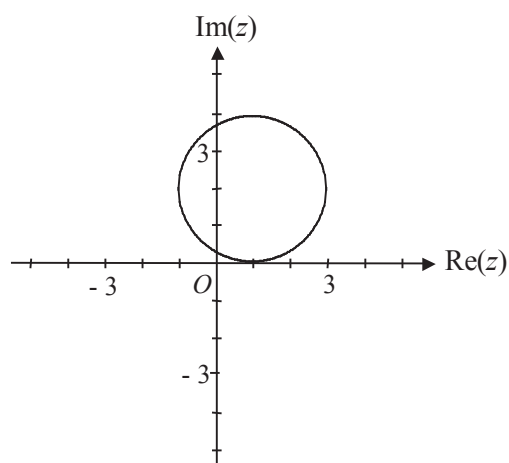
- A. $2 \operatorname{cis}\left(\frac{\pi}{12}\right)$
- B. $2 \operatorname{cis}\left(-\frac{\pi}{6}\right)$
- C. $16 \operatorname{cis}\left(-\frac{\pi}{3}\right)$
- D. $16 \operatorname{cis}\left(\frac{5\pi}{6}\right)$
- E. $16 \operatorname{cis}\left(\frac{4\pi}{3}\right)$

SECTION 1 – continued
TURN OVER

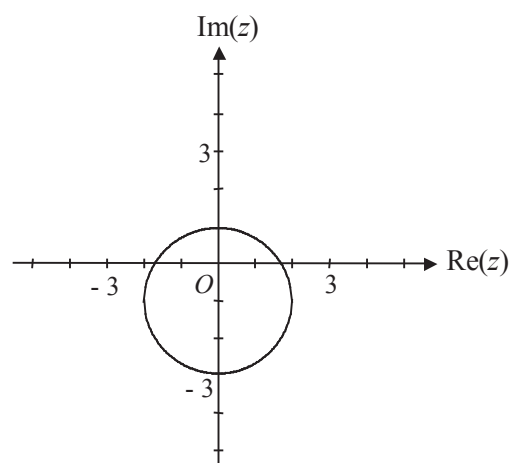
Question 4

The graph of the relation $4 - (z + i)(\bar{z} - i) = 0$, $z \in \mathbb{C}$ could be

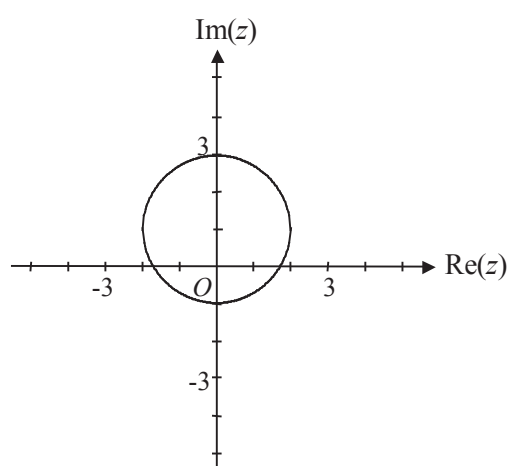
A.



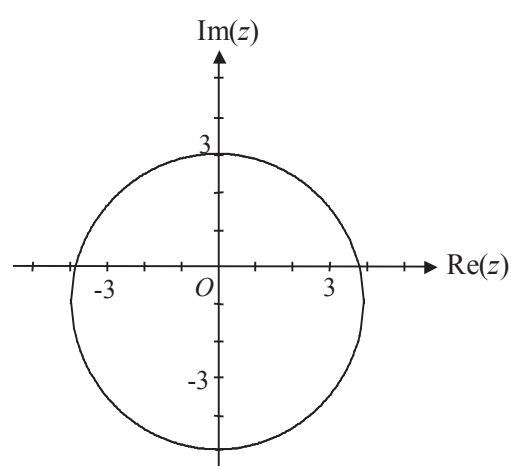
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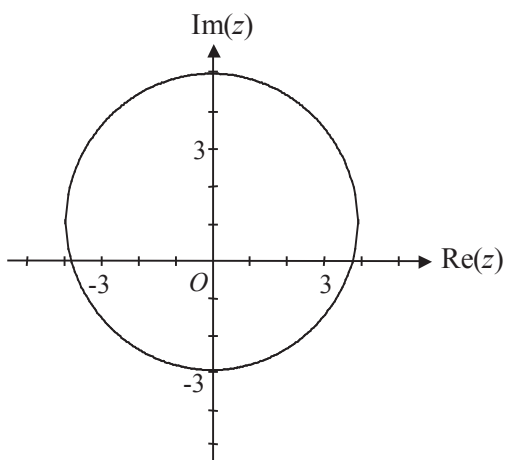
C.



D.



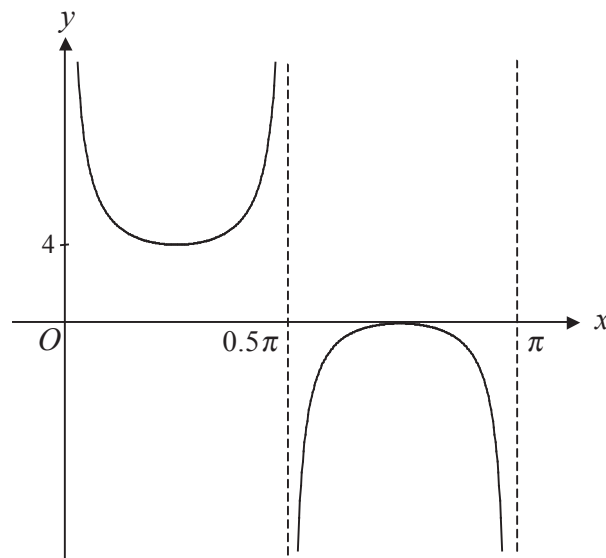
E.



Question 5

The implied domain of the function $f(x) = \arccos(x - m) + n$ is

- A. $[m - 1, m + 1]$
- B. $[-1 - m, 1 - m]$
- C. $[m - n, m + n]$
- D. $[m - n - 1, m - n + 1]$
- E. $[n, \pi + n]$

Question 6

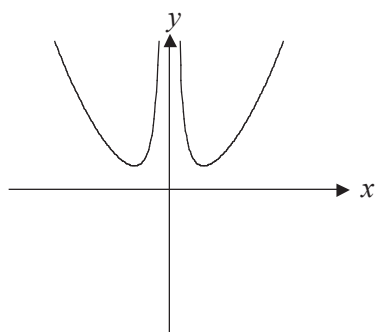
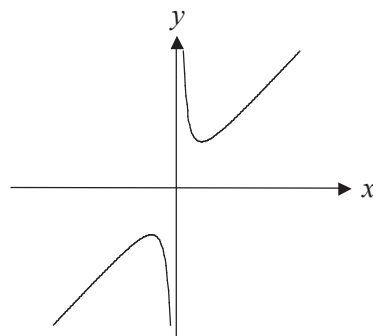
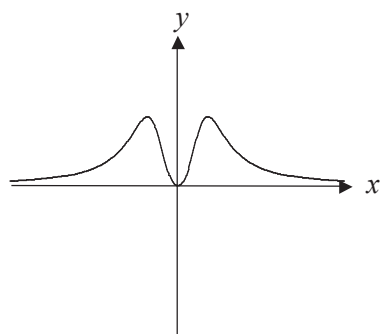
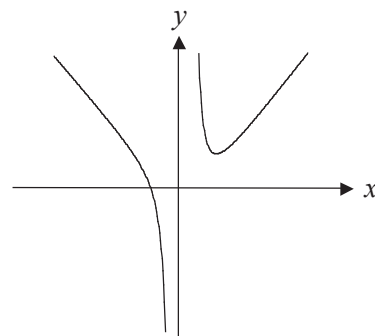
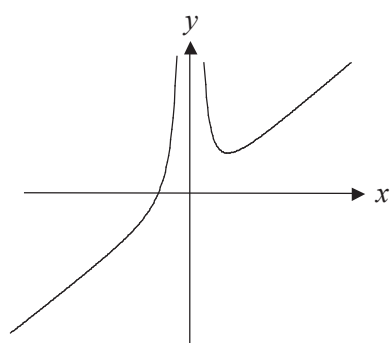
The equation of the graph shown above could be

- A. $y = 4\operatorname{cosec}(2x)$
- B. $y = \operatorname{cosec}(2x) + 4$
- C. $y = 2\operatorname{cosec}(2x) + 2$
- D. $y = 4\operatorname{cosec}(0.5x)$
- E. $y = 2\operatorname{cosec}(0.5x) + 2$

SECTION 1 – continued
TURN OVER

Question 7

One of the graphs below does not represent a function of the form $f(x) = x^m + x^{-n}$ where $m, n \in \{1, 2\}$. Which graph is this?

A.**B.****C.****D.****E.**

Question 8

The curve with parametric equations $x = 2 \sin(t)$ and $y = \cos(t)$ is translated 1 unit in the negative x direction and 2 units in the positive y direction.

The Cartesian equation of the translated curve is

- A. $(x + 1)^2 + 4(y - 2)^2 = 4$
- B. $(x - 1)^2 - 4(y + 2)^2 = 4$
- C. $(x + 1)^2 + (y - 2)^2 = 2$
- D. $\frac{(x + 1)^2}{2} + (y - 2)^2 = 1$
- E. $\frac{(x - 1)^2}{4} + (y + 2)^2 = 1$

Question 9

A curve is specified by the parametric equations $x = 2\sqrt{t + 1}$ and $y = t^2 + 1$.

The gradient of the curve at the point where $t = 3$ is

- A. 3
- B. 4
- C. 6
- D. 10
- E. 12

Question 10

The equation of the tangent to the curve $2y - x^2e^y + 1 = 0$ at the point $(1, 0)$ is given by

- A. $y - x + 1 = 0$
- B. $y - 2x + 2 = 0$
- C. $y + 2x - 2 = 0$
- D. $3y - 2x + 2 = 0$
- E. $3y + 2x - 2 = 0$

SECTION 1 – continued
TURN OVER

Question 11

Using a suitable substitution $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^3(2x) dx$ can be expressed as

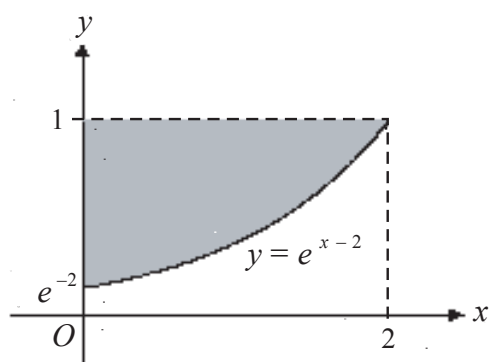
A. $-\int_0^{\frac{\pi}{4}} (1 - u^2) du$

B. $-\int_0^{\frac{\pi}{4}} u^3 du$

C. $-\int_0^1 u^3 du$

D. $2 \int_0^1 (1 - u^2) du$

E. $\frac{1}{2} \int_0^1 (u^2 - 1) du$

Question 12

The graph of $y = e^{x-2}$, $0 \leq x \leq 2$ is shown above. An expression for the volume of the solid of revolution formed when the shaded region is rotated around the x -axis is given by

A. $\pi \int_0^2 (1 - e^{x-2}) dx$

B. $\pi \int_0^2 (1 - e^{x-2})^2 dx$

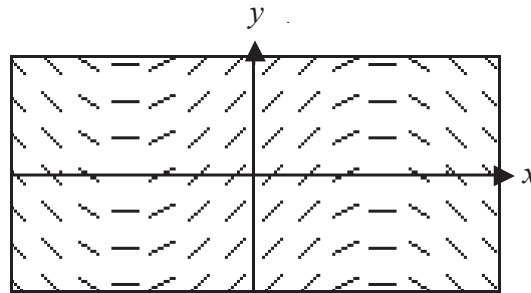
C. $\pi \int_{e^{-2}}^1 (1 - e^{x-2})^2 dx$

D. $2\pi - \pi \int_0^2 e^{(x-2)^2} dx$

E. $2\pi - \pi \int_0^2 e^{2(x-2)} dx$

Question 13

The slope field from a first order differential equation is shown below.



A solution of this differential equation could be

- A. $y = \sin^{-1}(x)$
- B. $y = \tan(x)$
- C. $y = \sin(x)$
- D. $y = |x|$
- E. $y = x^3$

Question 14

Given $\frac{dy}{dx} = \frac{1}{\log_e(x+1)}$ and $y = -2$ when $x = 1$. The value of y when $x = 2$ is

- A. -1.0897
- B. -0.8816
- C. 0.2958
- D. 0.9102
- E. 1.1184

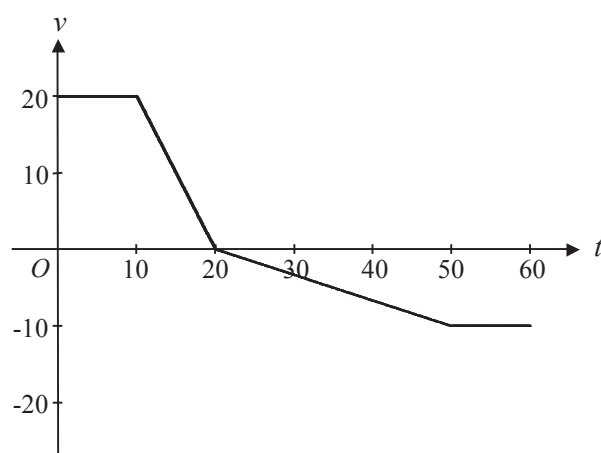
Question 15

A particle moves in a straight line with an acceleration of $\frac{v^2 + 1}{8}$ m/s², where v is its velocity in m/s. When the particle is at the origin, its velocity is 1 m/s.

Correct to one decimal place, how many metres will the particle be from the origin when its velocity is 3 m/s?

- A. 0.6
- B. 1.4
- C. 3.7
- D. 6.4
- E. 9.2

SECTION 1 – continued
TURN OVER

Question 16

The graph above shows the velocity of a particle moving in a straight line at time t seconds, $t \geq 0$. The distance of the particle, in metres, from its starting point after 60 seconds is

- A. 10
- B. 50
- C. 250
- D. 550
- E. 600

Question 17

A child sends a toy skidding across the floor with an initial speed of 4 m/s. The toy travels in a straight line and is subject to constant retardation due to friction. It hits a wall 3 metres away after 1.2 seconds. The speed of the toy, in m/s, when it hits the wall is

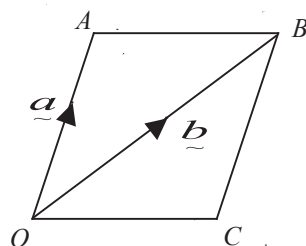
- A. 1.0
- B. 1.3
- C. 1.8
- D. 2.5
- E. 3.0

Question 18

$OABC$ is a rhombus where $\vec{OA} = \vec{a}$ and $\vec{OB} = \vec{b}$.

Which one of the following statements is true?

- A. $\vec{a} \cdot \vec{b} = 0$
- B. $(\vec{b} - \vec{a}) \cdot \vec{b} = 0$
- C. $(\vec{b} - 2\vec{a}) \cdot \vec{b} = 0$
- D. $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$
- E. $|\vec{a}| = |\vec{b}|$



Question 19

If $\vec{p} = \vec{i} + 2\vec{k}$ and $\vec{q} = 5\vec{i} + \vec{j} - \vec{k}$, then the scalar resolute of $\vec{p}$ in the direction of $\vec{q}$ is

- A. $\sqrt{5}$
- B. $\frac{\sqrt{3}}{3}$
- C. $\frac{3}{\sqrt{5}}$
- D. $3\sqrt{3}$
- E. $\frac{7}{3\sqrt{3}}$

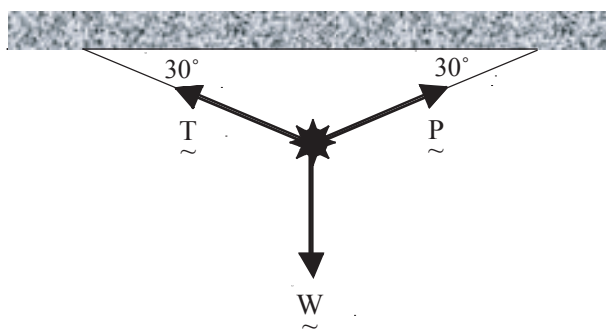
Question 20

When forces of $3\vec{i} - 2\vec{j}$ and $\vec{i} + 5\vec{j}$ newton simultaneously act on a 0.5 kg object, its acceleration, in m/s^2 , will be

- A. 2.5
- B. 5
- C. 10
- D. $2\vec{i} + 1.5\vec{j}$
- E. $8\vec{i} + 6\vec{j}$

Question 21

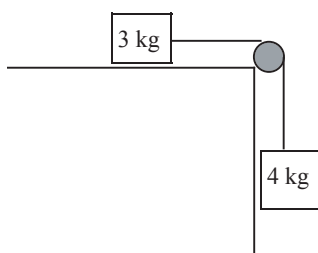
A decoration is suspended in equilibrium from a ceiling by two strings, each making an angle of 30° with the horizontal. The forces acting on the decoration are shown by vectors $\vec{T}$ and $\vec{P}$ which represent the tension in the strings, and by vector $\vec{W}$ which represents the weight force.



Which one of the following statements is true?

- A. $\vec{T} = \vec{P}$
- B. $\vec{T} + \vec{P} = \vec{W}$
- C. $\vec{T} + \vec{P} = 2\vec{W}$
- D. $\vec{T} + \vec{P} = -\vec{W}$
- E. $\vec{T} + \vec{P} = -2\vec{W}$

SECTION 1 – continued
TURN OVER

Question 22

The diagram above shows a 3 kg mass on a smooth horizontal surface connected to a 4 kg mass by a light string which passes over a smooth pulley. The tension in the string, in newtons, would be

- A. $\frac{g}{4}$
- B. $\frac{3g}{4}$
- C. $\frac{4g}{7}$
- D. $\frac{12g}{7}$
- E. $7g$

END OF SECTION 1
TURN OVER

Working Space

TURN OVER

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

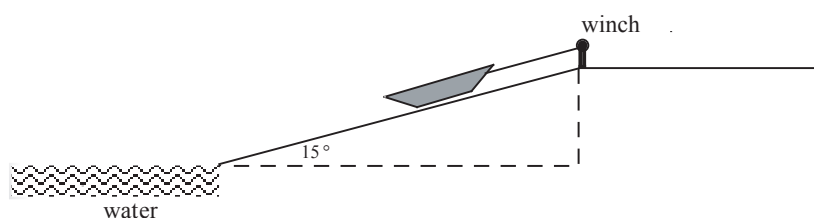
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$.

Question 1

A winch is used to pull a 1000 kg boat up a ramp that is inclined at an angle of 15° to the horizontal. The winch applies a pulling force of P newton along a rope and the boat moves up the ramp with an acceleration 0.1 m/s^2 . The coefficient of friction between the boat and the ramp is 0.25.

- a. On the diagram below show all forces acting on the boat as it is pulled up the ramp.



1 mark

- b. Find the force, P , correct to the nearest newton.

3 marks

SECTION 2 – continued

When the boat reaches the top of the ramp, the rope breaks and the boat slides back down the ramp into the water.

- c. Find the acceleration of the boat as it slides down the ramp.

Write your answer in m/s^2 correct to two decimal places.

3 marks

- d. If the ramp is 6 metres long, find the speed of the boat when it enters the water.

Write your answer in m/s correct to two decimal places.

2 marks

- e. How many seconds after the rope breaks will the boat be back in the water?

Write your answer correct to one decimal place.

1 mark

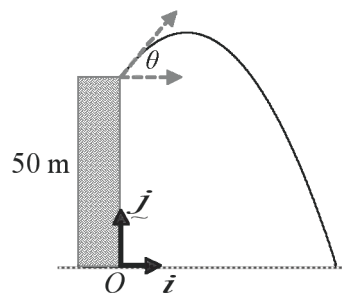
Total 10 marks

SECTION 2 – continued
TURN OVER

Question 2

A stone is projected upwards from the top of a 50 m building at an angle of θ° to the horizontal with a velocity of $\mathbf{v} = 20 \cos(\theta) \mathbf{i} + (20 \sin(\theta) - 9.8t) \mathbf{j}$ m/s at time t , $t \geq 0$.

The diagram below shows the path of the stone. Point O is the origin at ground level.



- a. Determine the initial speed of the stone.

1 mark

- b. Write a vector equation for the position of the stone at any time t seconds.

2 marks

- c. If $\theta = 60^\circ$

- i. Determine the greatest height above the ground the stone will reach and the time it takes to reach this height. Write answers correct to one decimal place.

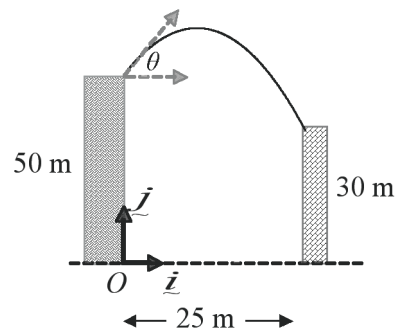
2 marks

SECTION 2 – continued

- ii. How far from the base of the building will the stone strike the ground? Write your answer in metres correct to one decimal place.

2 marks

A second stone is projected upwards from the top of the building with a velocity of $\mathbf{v} = 20\cos(\theta)\mathbf{i} + (20\sin(\theta) - 9.8t)\mathbf{j}$ m/s, $t > 0$. It strikes the top of another building that is 30 m high at a horizontal distance of 25 m away.



- d. Determine the angle of projection of the stone.
Write your answer in degrees correct to one decimal place.

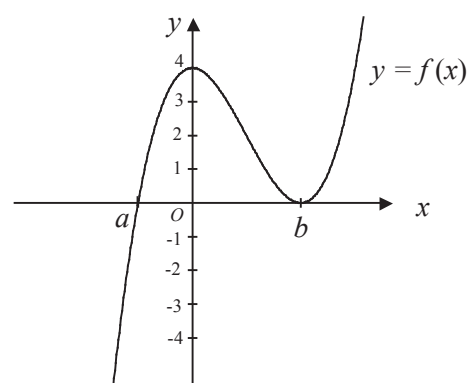
3 marks

Total 10 marks

SECTION 2 – continued
TURN OVER

Question 3

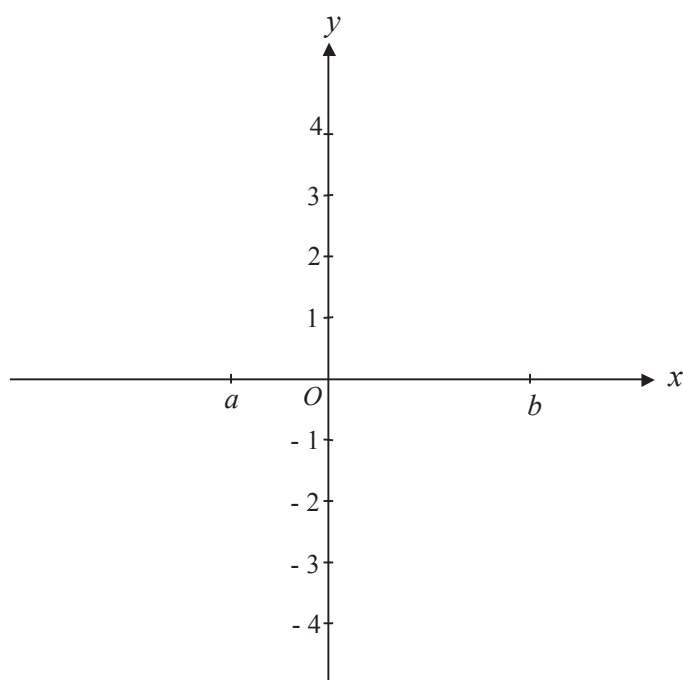
On the axes below, a graph of $y = f(x)$ is shown.



- a. Determine the equation of $f(x)$ in terms of a and b .

2 marks

- b. i. Sketch a graph of $\frac{1}{f(x)}$ showing all key features clearly.



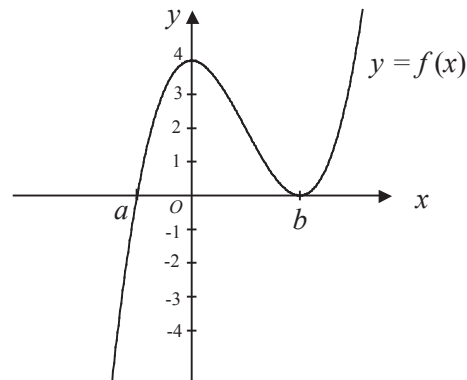
2 marks

- ii. Write down the domain and range of $\frac{1}{f(x)}$

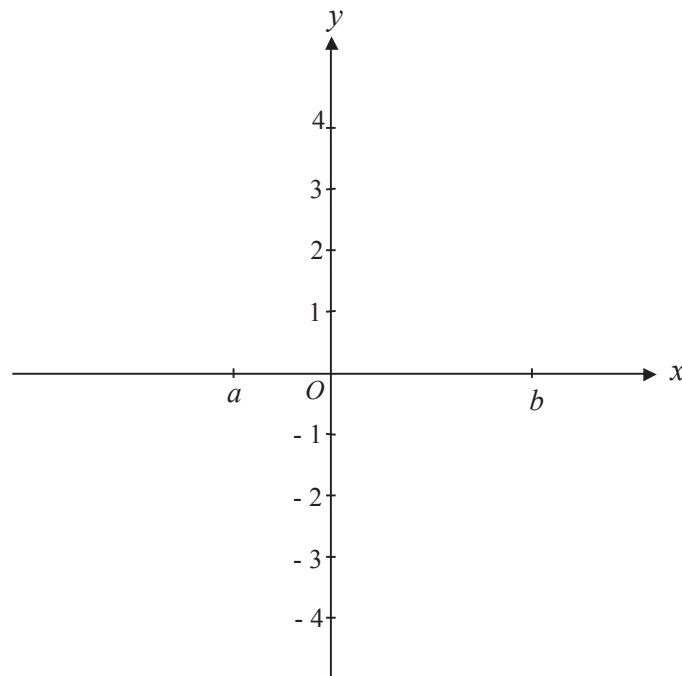
1 mark

SECTION 2 – continued

- c. Given the graph of $y = f(x)$.



Sketch a graph of $g(x)$ where $g(x) = \int f(x) dx$ and $g(0) = -1$.
Show all key features of $g(x)$ clearly on the axes below.



3 marks

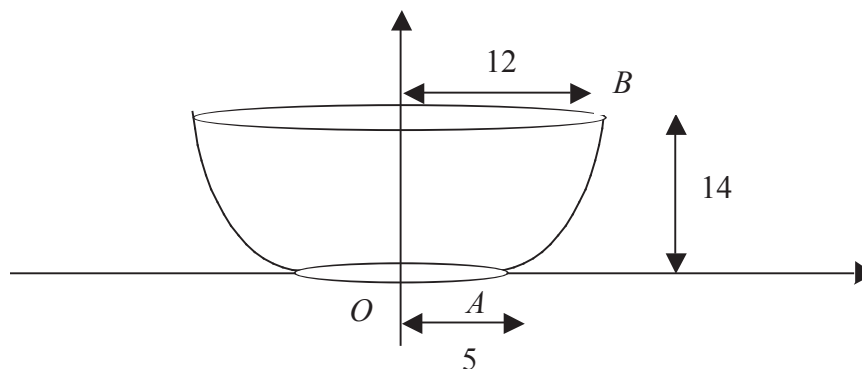
Total 8 marks

SECTION 2 – continued
TURN OVER

Question 4

The diagram below shows a bowl, the base and the top are circular with radii of 5 and 12 cm respectively and the height of the bowl is 14 cm.

The origin O is at the centre of the base of the bowl, with the co-ordinates axes as shown.



- a. If the arc AB can be modelled as a parabola $y = ax^2 + bx + c$ show that

$$a = \frac{2}{17}, \quad b = 0 \quad \text{and} \quad c = -\frac{50}{17}$$

2 marks

- [illegible]

[illegible]

SECTION 2 – continued
TURN OVER

- e. The bowl is filled with water to a height of 9 cm (no more water is coming into the bowl). Unfortunately at this time, the base of the bowl has developed a small crack and the water is leaking from the bowl at a rate of $28\sqrt{h}$ cm³/min. Set up a definite integral for the time for the height of the water to fall from 9 cm to 1 cm.

2 marks

- f. Hence find the time for the height of the water to fall from 9 cm to 1 cm. Write your answer correct to the nearest minute.

1 mark

Total 13 marks

SECTION 2 – continued

Question 5

a. Given points $P(-2, -1)$, $Q(-2 - 2\sqrt{3}, 5)$, $R(-2 + 2\sqrt{3}, 5)$ and $C(-2, 3)$

i. Find using a vector method the angle between $\overrightarrow{CP}$ and $\overrightarrow{CQ}$

3 marks

ii. Show that PQR forms an equilateral triangle.

2 marks

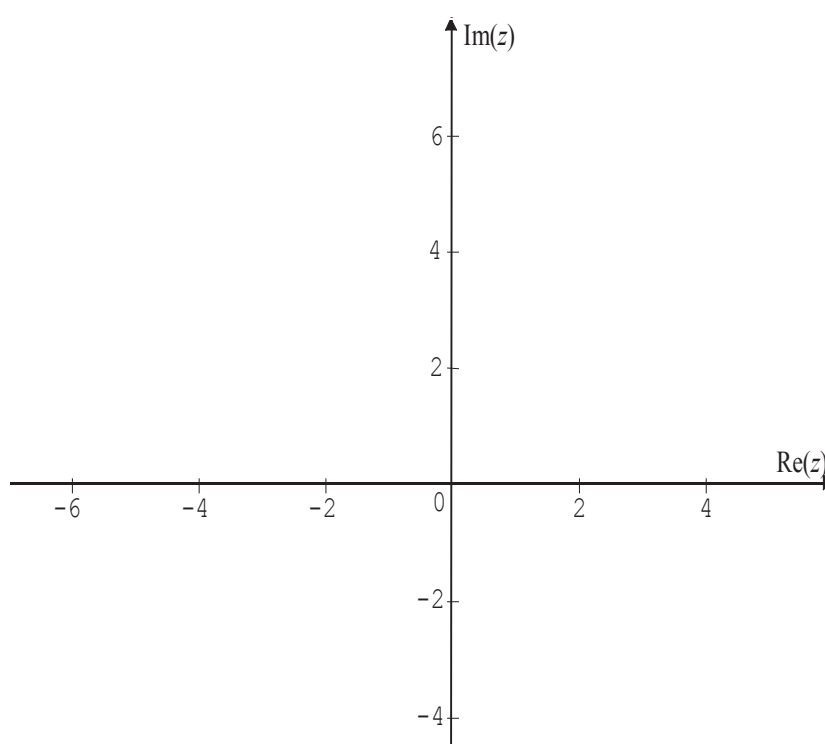
SECTION 2 – continued
TURN OVER

- b.** Let S be defined by $\{z : |z - c| = 4\}$ where $c = -2 + 3i$ and $p = -2 - i$, $q = -2 - 2\sqrt{3} + 5i$ and $r = -2 + 2\sqrt{3} + 5i$

- i.** Verify that p , q and r all belong to S

3 marks

- ii.** Sketch S on the Argand diagram below and plot the points p , q and r labelling them as P , Q and R respectively.



2 marks

SECTION 2 – continued

- c. i. Show that the equation $(z - c)^3 = 64i$ can be expressed in the form
 $z^3 + (6 - 9i)z^2 - (15 + 36i)z - (46 + 73i) = 0$

2 marks

SECTION 2 – continued
TURN OVER

- ii. Hence find in exact Cartesian form all of the roots of the equation

$$z^3 + (6 - 9i)z^2 - (15 + 36i)z - (46 + 73i) = 0$$

Explain your results geometrically.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

5 marks

Total 17 marks

END OF SECTION 2

SPECIALIST MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

Specialist Mathematics Formulas

Mensuration

area of a trapezium:	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder:	$2\pi rh$
volume of a cylinder:	$\pi r^2 h$
volume of a cone:	$\frac{1}{3}\pi r^2 h$
volume of a pyramid:	$\frac{1}{3}Ah$
volume of a sphere:	$\frac{4}{3}\pi r^3$
area of a triangle:	$\frac{1}{2}bc \sin A$
sine rule:	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
cosine rule:	$c^2 = a^2 + b^2 - 2ab \cos C$

Coordinate geometry

ellipse:	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	hyperbola:	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
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Circular (trigometric) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x) \cos(y) + \cos(x) \sin(y)$$

$$\cos(x+y) = \cos(x) \cos(y) - \sin(x) \sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x) \tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2 \cos^2(x) - 1 = 1 - 2 \sin^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x) \cos(y) - \cos(x) \sin(y)$$

$$\cos(x-y) = \cos(x) \cos(y) + \sin(x) \sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)}$$

function	$\sin^{-1}$	$\cos^{-1}$	$\tan^{-1}$
domain	$[-1, 1]$	$[-1, 1]$	R
range	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[0, \pi]$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Algebra (Complex numbers)

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem)}$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, n \neq -1$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\int \frac{-1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, a > 0$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int \frac{a}{a^2+x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Euler's method:

$$\text{If } \frac{dy}{dx} = f(x), x_0 = a \text{ and } y_0 = b, \text{ then } x_{n+1} = x_n + h \text{ and } y_{n+1} = y_n + hf(x_n)$$

acceleration:

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$\text{constant (uniform) acceleration: } v = u + at \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as \quad s = \frac{1}{2}(u+v)t$$

Vectors in two and three dimensions

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = r$$

$$\vec{r}_1 \cdot \vec{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

Mechanics

momentum:

$$\vec{p} = m\vec{v}$$

equation of motion:

$$\vec{R} = m\vec{a}$$

friction:

$$F \leq \mu N$$

END OF FORMULA SHEET