

Year 2006

VCE

Specialist Mathematics

Solutions

Trial Examination 2



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SECTION 1

ANSWERS

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E
11	A	B	C	D	E
12	A	B	C	D	E
13	A	B	C	D	E
14	A	B	C	D	E
15	A	B	C	D	E
16	A	B	C	D	E
17	A	B	C	D	E
18	A	B	C	D	E
19	A	B	C	D	E
20	A	B	C	D	E
21	A	B	C	D	E
22	A	B	C	D	E

SECTION 1

Question 1

Answer D

The graphs of all alternatives give vertical asymptotes at $x = 0$, $\frac{\pi}{2}$ and π however option D is wrong, it is the reflection in the x-axis

Question 2

Answer D

$$f(g(x)) = \frac{1}{-x^2 - (a-b)x + ab} = \frac{1}{-(x^2 + (a-b)x - ab)} = \frac{1}{-(x-b)(x+a)}$$

So there are vertical asymptotes at $x = -a$ and $x = b$

Question 3

Answer A

$$y = \frac{ax^2 + b}{x} = ax + \frac{b}{x} = ax + bx^{-1}$$

$$\frac{dy}{dx} = a - bx^{-2} = a - \frac{b}{x^2} \quad \text{for turning points } \frac{dy}{dx} = 0$$

$$a = \frac{b}{x^2} \quad x^2 = \frac{b}{a} \quad x = \pm \sqrt{\frac{b}{a}} \quad \text{however there are no turning points, so there is no}$$

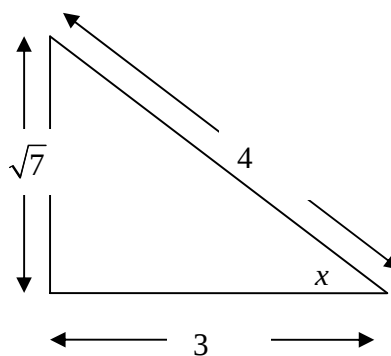
solutions for $x = \pm \sqrt{\frac{b}{a}}$ so a and b must have opposite signs the product $ab < 0$ so $a > 0$ and $b < 0$

Question 4

Answer B

$$\operatorname{cosec}(x) = \frac{1}{\sin(x)} = \frac{4\sqrt{7}}{7}$$

$$\sin(x) = \frac{7}{4\sqrt{7}} = \frac{\sqrt{7}}{4}$$



so from the triangles $\cos(x) = -\frac{3}{4}$ since $\frac{\pi}{2} < x < \pi$ x is obtuse and in the second

quadrant $\sec(x) = \frac{1}{\cos(x)} = -\frac{4}{3}$

Question 5**Answer E**

The ellipse $\frac{(y+1)^2}{18} + \frac{(x-4)^2}{8} = 2$ in standard form is $\frac{(x-4)^2}{16} + \frac{(y+1)^2}{36} = 1$
 it has centre at $(4, -1)$ $a = 4$ $b = 6$ the domain is $4 \pm 4 = [0, 8]$
 and the range $-1 \pm 6 = [-7, 5]$

Question 6**Answer C**

Given $u = 3 \operatorname{cis}\left(\frac{\pi}{4}\right)$, $v = a \operatorname{cis}(b)$ and $\frac{u}{v} = \frac{3 \operatorname{cis}\left(\frac{\pi}{4}\right)}{a \operatorname{cis}(b)} = \frac{3}{a} \operatorname{cis}\left(\frac{\pi}{4} - b\right) = -6 = 6 \operatorname{cis}(\pi)$
 so $\frac{3}{a} = 6$ $a = \frac{1}{2}$ and $\frac{\pi}{4} - b = \pi$ $b = -\frac{3\pi}{4}$

Question 7**Answer E**

$z = (\cos \theta + i \sin \theta)^3 = \operatorname{cis}(3\theta)$ by DeMoivre's Theorem
 $z^2 = (\cos \theta + i \sin \theta)^6 = \operatorname{cis}(6\theta) = \cos(6\theta) + i \sin(6\theta)$ and $z^2 = a + bi$
 Equating real and imaginary parts respectively gives $\cos(6\theta) = a$ and $\sin(6\theta) = b$

Question 8**Answer C**

The circle has radius 2, one of the solutions is $u = 2i$ and $u^3 = 8i^3 = -8i$
 but $P(u) = 0$ so $P(z) = z^3 + 8i = 0$

Question 9**Answer A**

$P(z) = z^3 + bz^2 + cz + d$ and $P(-\alpha i) = 0$ and $P(\beta) = 0$ by the conjugate root theorem $P(\alpha i) = 0$ so $z^2 + \alpha^2$ is one factor, the other is $(z - \beta)$ expanding
 $P(z) = (z^2 + \alpha^2)(z - \beta) = z^3 - \beta z^2 + \alpha^2 z - \alpha^2 \beta = z^3 + bz^2 + cz + d = 0$ equating
 $b = -\beta$ $c = \alpha^2$ $d = -\alpha^2 \beta$

Question 10**Answer C**

The inside of the circle of radius 4, can be represented by $\{z: |z| \leq 4\}$ or $\{z: z\bar{z} \leq 16\}$ or $\{z: \operatorname{Re}^2(z) + \operatorname{Im}^2(z) \leq 16\}$ the upper half plane of the Argand diagram can be represented by $\{z: \operatorname{Im}(z) \geq 0\}$ or $\{z: 0 \leq \operatorname{Arg}(z) \leq \pi\}$ option C is the only correct intersection of all the possible correct representations.

Question 11**Answer E**

When we have the repeated factor $(x+3)^2$ and the non-linear factor (x^2+3) we represent the expression $\frac{3x+1}{(x+3)^2(x^2+3)}$ in partial fractions as

$$\frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{Cx+D}{x^2+3}$$

Question 12**Answer E**

$$\int_1^4 \frac{\cos(\sqrt{x})}{\sqrt{x}} dx = \int_1^4 \cos(\sqrt{x}) \frac{1}{\sqrt{x}} dx$$

$$\text{let } u = \sqrt{x} = x^{\frac{1}{2}} \quad \frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \quad \frac{1}{\sqrt{x}} dx = 2 du$$

change the terminals when $x=4$ $u=2$ and when $x=1$ $u=1$

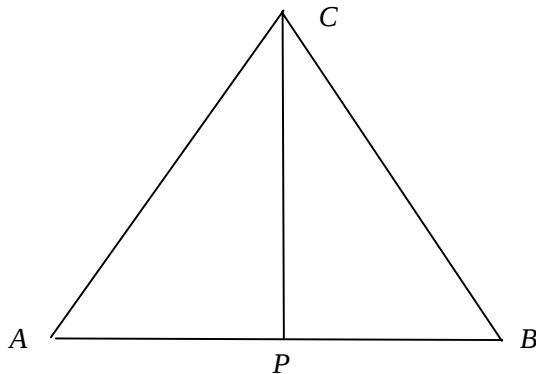
the definite integral becomes $2 \int_1^2 \cos(u) du$

Question 13**Answer D**

At $x=a$ and $x=e$ the gradient changes from negative to positive, this gives a minimum at the two points $x=a$ and $x=e$. At $x=c$ the gradient changes from positive to negative this gives a maximum at the point $x=c$. The graph of $F(x)$ is a quartic and will have two points of inflexions, one somewhere between $x=a$ and $x=c$ and another somewhere between $x=c$ and $x=e$.

Question 14

Answer C



Option A is true $\overrightarrow{AP} = \overrightarrow{PB} = \frac{1}{2} \overrightarrow{AB}$ since P is the mid-point of $\overrightarrow{AB}$

Option B is true since ABC is an equilateral triangle all side lengths are equal

$$|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{AC}|$$

Option C is false $\overrightarrow{AP}$ is perpendicular to $\overrightarrow{PC}$ so $\overrightarrow{AP} \cdot \overrightarrow{PC} = 0$

Option D is true the angle CAB is 60° $\cos(60^\circ) = \frac{1}{2} = \frac{\overrightarrow{AC} \cdot \overrightarrow{AB}}{|\overrightarrow{AC}| |\overrightarrow{AB}|}$

Option E is true the angle PCB is 30° $\cos(30^\circ) = \frac{\sqrt{3}}{2} = \frac{\overrightarrow{CP} \cdot \overrightarrow{CB}}{|\overrightarrow{CP}| |\overrightarrow{CB}|}$

Question 15

Answer A

$$\underline{a} = -2\underline{i} + 2\underline{j} - \underline{k} \text{ and } \underline{b} = 4\underline{i} + y\underline{j} + 2\underline{k} \quad \underline{a} \cdot \underline{b} = -8 + 2y - 2 = 2y - 10$$

$$|\underline{a}| = \sqrt{4 + 4 + 1} = 3 \quad |\underline{b}| = \sqrt{16 + y^2 + 4} = \sqrt{20 + y^2}$$

$$\text{So the angle between } \underline{a} \text{ and } \underline{b} \text{ is } \cos^{-1} \left(\frac{2y - 10}{3\sqrt{(20 + y^2)}} \right)$$

If $y = -4$ then $-2\underline{a} = \underline{b}$ so the vectors $\underline{a}$ and $\underline{b}$ are linearly **dependent**, option A is false

All the other alternatives are true.

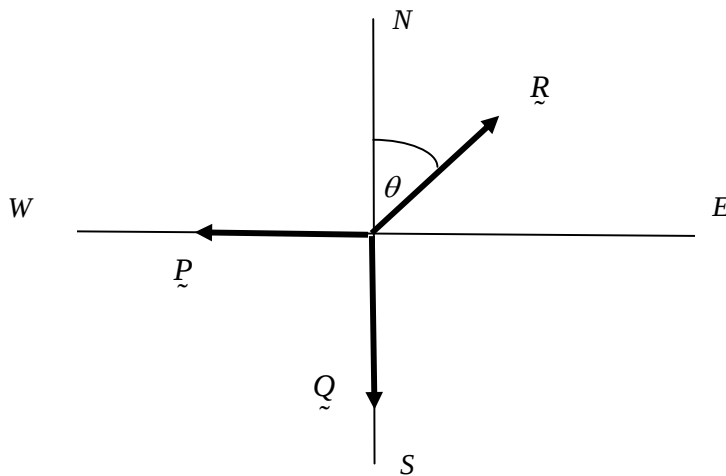
Option C if $\underline{a}$ is perpendicular to $\underline{b}$ then $2y - 10 = 0 \quad y = 5$

Option D if $y > 5$ then the angle between the vectors $\underline{a}$ and $\underline{b}$ is acute.

Option E if $y < 5$ then the angle between the vectors $\underline{a}$ and $\underline{b}$ is obtuse.

Question 16**Answer B**

The velocity vector is $\dot{r}(t) = 4e^{\frac{t}{2}}\hat{i} - 2\sin\left(\frac{t}{2}\right)\hat{j}$ integrating with respect to t to get the position vector $r(t) = \int 4e^{\frac{t}{2}} dt \hat{i} - \int 2\sin\left(\frac{t}{2}\right) dt \hat{j} = 8e^{\frac{t}{2}}\hat{i} + 4\cos\left(\frac{t}{2}\right)\hat{j} + \underline{C}$ now using the initial conditions $r(0) = 3\hat{i} - 3\hat{j} = 8\hat{i} + 4\hat{j} + \underline{C}$ so $\underline{C} = -5\hat{i} - 7\hat{j}$ and

$$r(t) = \left(8e^{\frac{t}{2}} - 5\right)\hat{i} + \left(4\cos\left(\frac{t}{2}\right) - 7\right)\hat{j}$$
Question 17**Answer A**

Option A is false if $\theta = 45$ then $P = Q = \frac{\sqrt{2}R}{2}$

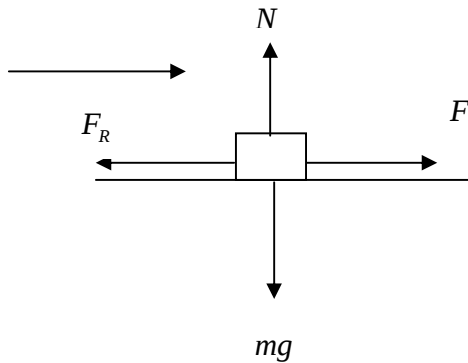
All the other alternatives are true

Option B $R^2 = P^2 + Q^2$

Option C $P = R \sin(\theta)$ and $Q = R \cos(\theta)$

Option D $\tan(\theta) = \frac{P}{Q}$

Option E as vectors $\underline{P} + \underline{Q} + \underline{R} = \underline{0}$

Question 18**Answer E**

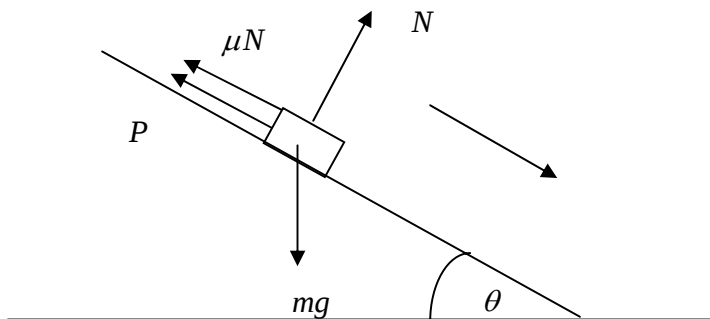
$$m = 60 \text{ kg} \quad g = 9.8 \text{ m/s}^2 \quad \mu = 0.75$$

$$N = mg = 588 \text{ newtons}$$

$$\mu N = 441 \text{ newtons}$$

movement with constant acceleration

is only possible if $F > \mu N$

Question 19**Answer B**

resolving the forces, perpendicular to the plane

$$N - mg \cos(\theta) = 0 \Rightarrow N = mg \cos(\theta)$$

resolving the forces up and parallel to the plane.

$$P + \mu N - mg \sin(\theta) = 0$$

$$P + \mu mg \cos(\theta) - mg \sin(\theta) = 0$$

$$P = mg \sin(\theta) - \mu mg \cos(\theta)$$

$$P = mg (\sin \theta - \mu \cos \theta)$$

Question 20**Answer A**

$$\tan^{-1}\left(\frac{1}{2t}\right) \frac{dt}{dy} = 1 \quad \frac{dy}{dt} = \tan^{-1}\left(\frac{1}{2t}\right) \text{ using a calculator program gives}$$

t	y
1.00	2.0000
1.25	2.1159
1.50	2.2110

Question 21**Answer B**

The solution curves are of the form $x = \sin(2t) + c$ so differentiating,

the differential equation is $\frac{dx}{dt} = 2 \cos(2t)$

Question 22**Answer B**

The volume in the tank is $V = A h$ $\frac{dV}{dh} = A$

and $\frac{dV}{dt} = \text{inflow} - \text{outflow} = Q - c\sqrt{h}$

$\frac{dh}{dt} = \frac{dh}{dV} \frac{dV}{dt}$ so that $\frac{dh}{dt} = \frac{Q - c\sqrt{h}}{A}$ $h(0) = h_0$

END OF SECTION 1 SUGGESTED ANSWERS

SECTION 2

Question 1

a.i. $3x^2 + 18x - y^2 + 4y + 11 = 0$

$$3(x^2 + 6x) - (y^2 - 4y) = -11$$

$$3(x^2 + 6x + 9) - (y^2 - 4y + 4) = -11 + 27 - 4 \quad \text{M1}$$

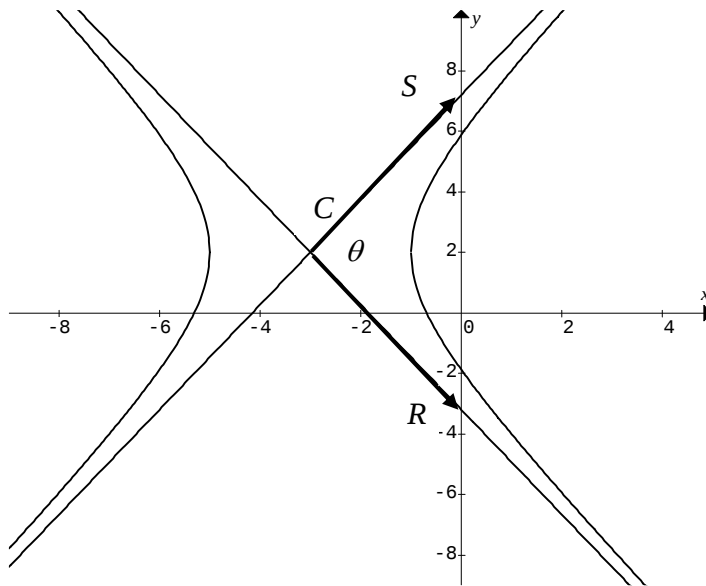
$$3(x+3)^2 - (y-2)^2 = 12 \quad \frac{(x+3)^2}{4} - \frac{(y-2)^2}{12} = 1 \quad \text{A1}$$

ii. centre (h, k) where $h = -3$ $k = 2$ $a = 2$ $b = 2\sqrt{3}$

hyperbola, centre $(-3, 2)$ vertices $(-5, 2)$ $(-1, 2)$ A1

asymptotes $\frac{y-2}{2\sqrt{3}} = \pm \frac{x+3}{2}$ $y = \pm\sqrt{3}(x+3) + 2$

$$y = \sqrt{3}x + 2 + 3\sqrt{3} \quad \text{and} \quad y = -\sqrt{3}x + 2 - 3\sqrt{3} \quad \text{A1}$$



b.i $3x^2 + 18x - y^2 + 4y + 11 = 0$

differentiating implicitly

$$\frac{d}{dx}(3x^2) + \frac{d}{dx}(18x) - \frac{d}{dx}(y^2) + \frac{d}{dx}(4y) + \frac{d}{dx}(11) = 0$$

$$6x + 18 - 2y \frac{dy}{dx} + 4 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(2y - 4) = 6x + 18$$

$$\frac{dy}{dx} = \frac{3x+9}{y-2} = \frac{3(x+3)}{y-2}$$

A1

- ii. When the tangent is parallel to the y-axis. $\frac{dy}{dx} \rightarrow \infty \Rightarrow y - 2 = 0 \quad y = 2$

These are obviously the vertices $(-5, 2) \quad (-1, 2)$ A1

- c. $R(0, 2 - 3\sqrt{3})$, $S(0, 3\sqrt{3} + 2)$ and $C(-3, 2)$
 $\overrightarrow{CS} = \overrightarrow{OS} - \overrightarrow{OC} = 3\mathbf{i} + 3\sqrt{3}\mathbf{j}$ $\overrightarrow{CR} = \overrightarrow{OR} - \overrightarrow{OC} = 3\mathbf{i} - 3\sqrt{3}\mathbf{j}$ M1
 $|\overrightarrow{CS}| = \sqrt{9 + 27} = 6$ $|\overrightarrow{CR}| = \sqrt{9 + 27} = 6$
 $\overrightarrow{CS} \cdot \overrightarrow{CR} = 9 - 9 \times 3 = -18$

The angle between $\overrightarrow{CS}$ and $\overrightarrow{CR}$ is θ where $\cos(\theta) = \frac{\overrightarrow{CS} \cdot \overrightarrow{CR}}{|\overrightarrow{CS}| |\overrightarrow{CR}|} = \frac{-18}{36} = -\frac{1}{2}$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$$
 A1

this angle represents the angle between the asymptotes at the centre of the hyperbola. A1

- d. $m = -4 + yi$, $f = -7 + 2i$ and $z = x + yi$
 $|z - f| = 2|z - m|$
 $|(x + yi) - (-7 + 2i)| = 2|(x + yi) - (-4 + yi)|$
 $|(x + 7) + (y - 2)i| = 2|(x + 4) + (y - y)i|$ M1
 $\sqrt{(x + 7)^2 + (y - 2)^2} = 2\sqrt{(x + 4)^2}$ squaring both sides and expanding M1
 $x^2 + 14x + 49 + y^2 - 4y + 4 = 4(x^2 + 8x + 16) = 4x^2 + 32x + 64$
 $3x^2 + 18x - y^2 + 4y + 11 = 0$
 $\frac{(x + 3)^2}{4} - \frac{(y - 2)^2}{12} = 1$ the hyperbola again A1

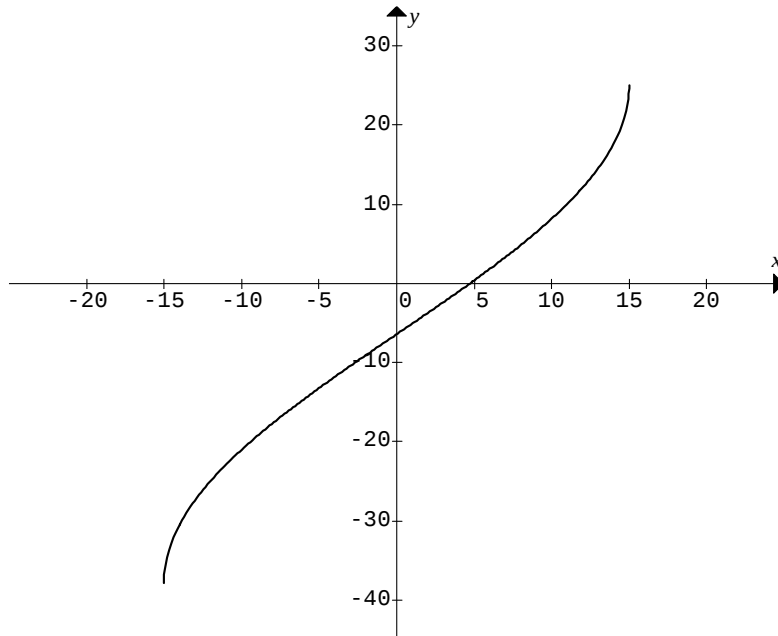
- e. $\mathbf{r}(t) = (-3 + 2\sec(2t))\mathbf{i} + (2 + 2\sqrt{3}\tan(2t))\mathbf{j}$
 $x = -3 + 2\sec(2t)$ $y = 2 + 2\sqrt{3}\tan(2t)$
 $\sec(2t) = \frac{x + 3}{2}$ $\tan(2t) = \frac{y - 2}{2\sqrt{3}}$ M1
 $\sec^2(2t) - \tan^2(2t) = 1$
 $\frac{(x + 3)^2}{4} - \frac{(y - 2)^2}{12} = 1$ the hyperbola again A1

Question 2

a. domain $|x| \leq 15 = [-15, 15]$

A1

b.



A1

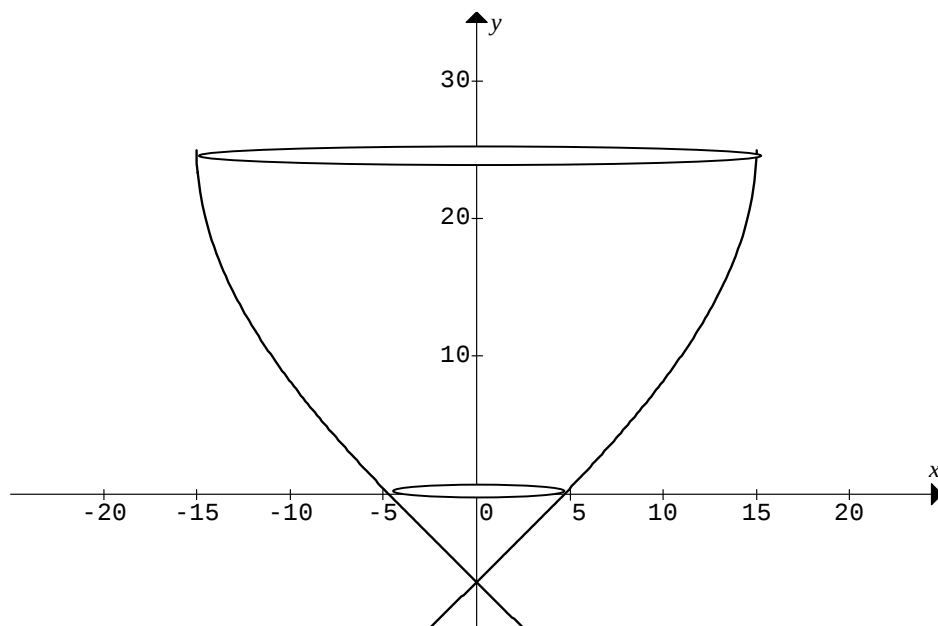
c. $f(x) = 25 - 20 \cos^{-1}\left(\frac{x}{15}\right) \quad x \in [-15, 15]$

end points $(-15, 25 - 20\pi)$ $(15, 25)$

$$f'(x) = \frac{20}{\sqrt{225 - x^2}} \quad x \in (-15, 15)$$

the function is defined at the end-points
but the gradient is not defined at the end-points.

A1



d. when $y = 0$ $25 - 20 \cos^{-1}\left(\frac{x}{15}\right) = 0 \Rightarrow x = 4.72984$

the diameter of the base of the vase is $2 \times 4.72984 = 9.460$ cm A1

e. $f'(10) = \frac{20}{\sqrt{225-100}} = \frac{4\sqrt{5}}{5} \approx 1.78885 = \tan(\theta)$ M1

$\theta = \tan^{-1}(1.789) = 60.794^\circ = 60^\circ 48'$ A1

f. i. $y = 25 - 20 \cos^{-1}\left(\frac{x}{15}\right)$

$20 \cos^{-1}\left(\frac{x}{15}\right) = 25 - y$

$\cos^{-1}\left(\frac{x}{15}\right) = \frac{25-y}{20}$ M1

$\frac{x}{15} = \cos\left(\frac{25-y}{20}\right)$

$x = 15 \cos\left(\frac{25-y}{20}\right)$ M1

$V_y = \pi \int_a^b x^2 dy$

$V = 225\pi \int_0^{25} \cos^2\left(\frac{25-y}{20}\right) dy$ A1

ii. $V = 225\pi \int_0^{25} \cos^2\left(\frac{25-y}{20}\right) dy = 10950.90 \text{ cm}^3$ A1

Question 3

a. $45a = -260\sqrt{v}$
 $9a = -52\sqrt{v}$

A1

b. $a = \frac{dv}{dt} = -\frac{52\sqrt{v}}{9}$ inverting both sides

$$\frac{dt}{dv} = -\frac{9}{52\sqrt{v}}$$

$$\int \frac{dv}{\sqrt{v}} = -\frac{52}{9} \int 1 dt$$

M1

$$\int v^{-\frac{1}{2}} dv = -\frac{52t}{9} + C_1$$

$$2\sqrt{v} = -\frac{52t}{9} + C_1 \quad \text{now when } t=0 \quad v=9$$

$$2\sqrt{9} = C_1 = 6$$

$$2\sqrt{v} = 6 - \frac{52t}{9} \quad \text{now when } v=1$$

$$2 = 6 - \frac{52t}{9}$$

$$\frac{52t}{9} = 4$$

$$t = \frac{36}{52} = \frac{9}{13}$$

The head-wind blows for $\frac{9}{13}$ secs

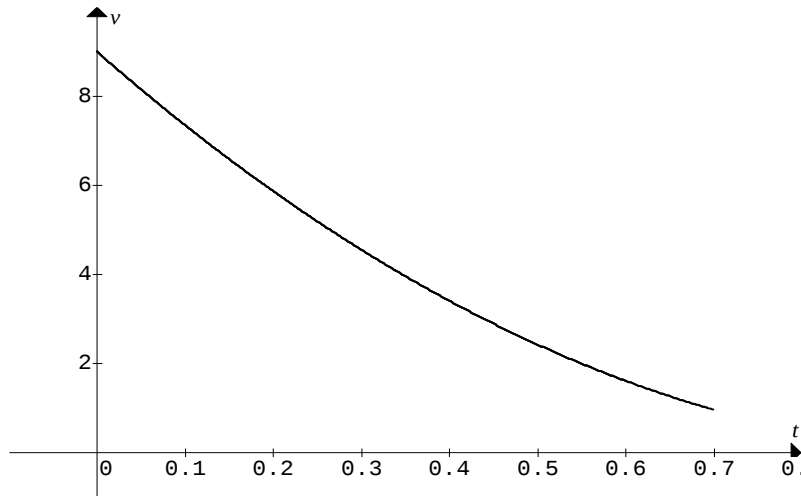
A1

c.i. $\sqrt{v} = 3 - \frac{26t}{9}$

$$v = v(t) = \left(3 - \frac{26t}{9}\right)^2 = \frac{(27 - 26t)^2}{81}$$

A1

ii.



Note the restricted domain for t as $\left[0, \frac{9}{13}\right]$, graph passes through the points

$$(0, 9) \text{ and } \left(\frac{9}{13}, 1\right) \quad \text{A1}$$

d. Since the area under the velocity time graph represents the displacement

$$D = \int_0^{\frac{9}{13}} \left(3 - \frac{26t}{9}\right)^2 dt = 3 \quad \text{A1}$$

Pete rode his bike exactly 3 metres during the head-wind. A1

e. $45a = -260\sqrt{v}$ use $a = v \frac{dv}{dx}$

$$45v \frac{dv}{dx} = -260\sqrt{v}$$

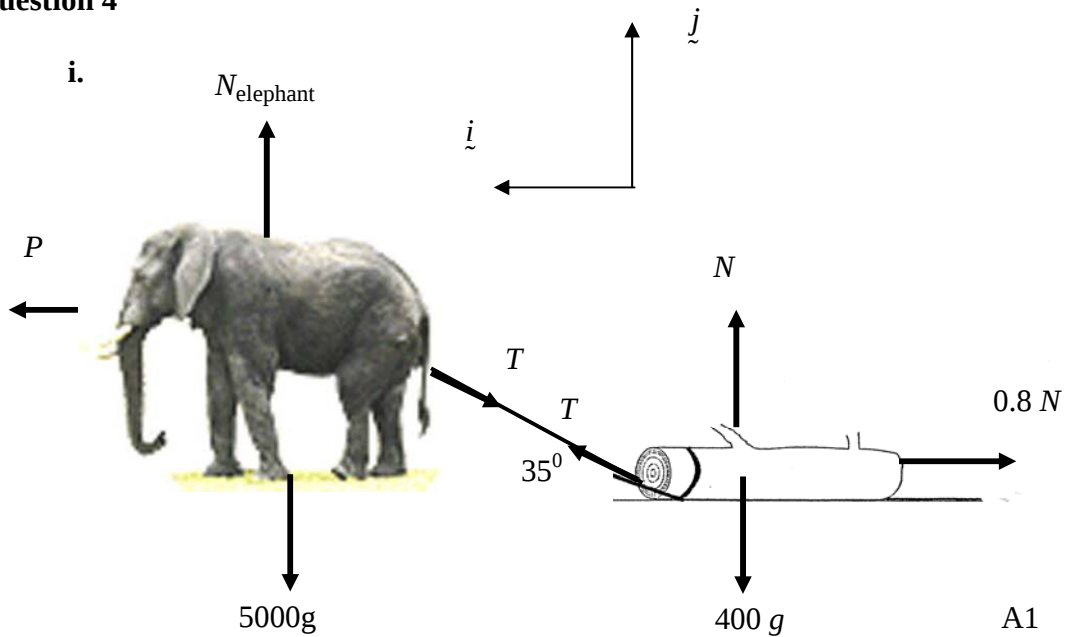
$$\frac{dv}{dx} = -\frac{52}{9\sqrt{v}} \quad \text{A1}$$

f. $\frac{dv}{dx} = -\frac{52}{9\sqrt{v}}$ inverting both sides

$$\frac{dx}{dv} = -\frac{9\sqrt{v}}{52} \quad \text{integrating with respect to } v \quad \text{M1}$$

$$x = -\frac{9}{52} \int_9^1 \sqrt{v} dv = \frac{9}{52} \int_1^9 \sqrt{v} dv \quad \text{A1}$$

$$D = \frac{9}{52} \left[\frac{2}{3} v^{\frac{3}{2}} \right]_1^9 = \frac{3}{26} (27 - 1) = 3 \quad \text{A1}$$

Question 4**a. i.**

ii Using Newtons 2nd Law of Motion
resolving horizontally around the log in the $\hat{i}$ direction.

$$(1) \quad T \cos(35^\circ) - 0.8N = 400 \times 0.1$$

resolving vertically around the log in the $\hat{j}$ direction.

M1

$$(2) \quad T \sin(35^\circ) + N - 400g = 0$$

$$(2) \Rightarrow N = 400g - T \sin(35^\circ) \text{ into (1)}$$

M1

$$T \cos(35^\circ) - 0.8(400g - T \sin(35^\circ)) = 40$$

$$T(\cos(35^\circ) + 0.8 \sin(35^\circ)) - 0.8 \times 400g = 40$$

$$T = \frac{40(1 + 10 \times 0.8 \times 9.8)}{\cos(35^\circ) + 0.8 \sin(35^\circ)} = 2485.11 = 2485 \text{ newtons}$$

A1

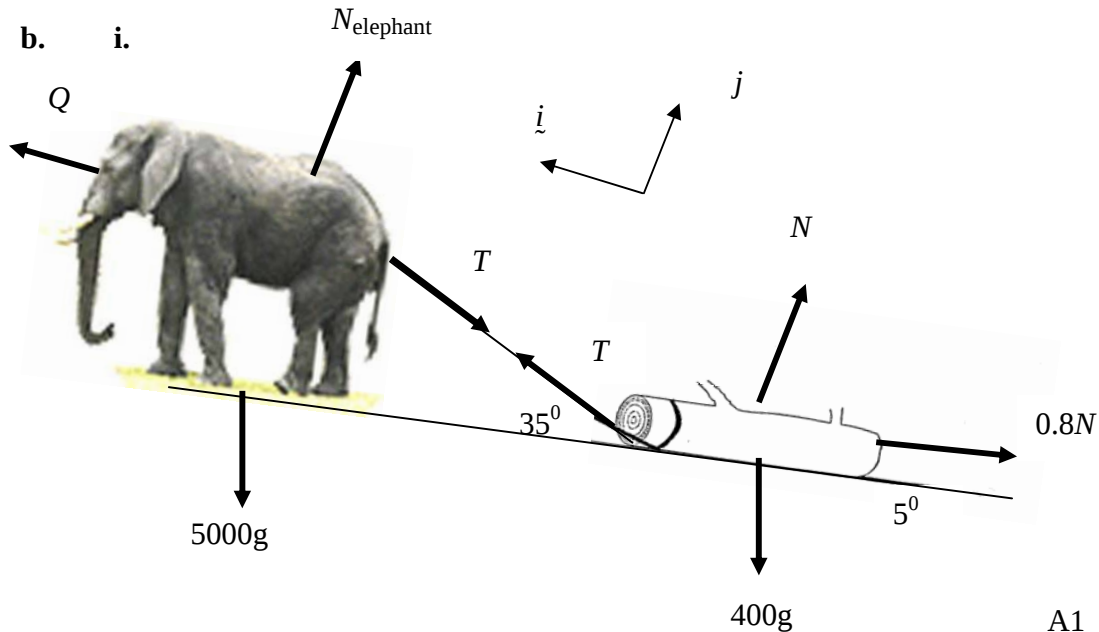
iii. resolving horizontally around the elephant in the $\hat{i}$ direction.

$$(3) \quad P - T \cos(35^\circ) = 5000 \times 0.1$$

$$P = T \cos(35^\circ) + 500 = 2535.68$$

$$P = 2536 \text{ newtons}$$

A1



- ii.** constant speed is zero acceleration. Note the 0.1 m/s is not used.
resolving up and parallel to the hill around the log in the $\hat{i}$ direction.

$$(1) T \cos(35^\circ) - 0.8N - 400g \sin(5^\circ) = 0$$

resolving perpendicular to the hill around the log in the $\hat{j}$ direction

$$(2) N + T \sin(35^\circ) - 400g \cos(5^\circ) = 0 \quad \text{M1}$$

$$(2) \Rightarrow N = 400g \cos(5^\circ) - T \sin(35^\circ) \text{ into (1)}$$

$$T \cos(35^\circ) - 0.8(400g \cos(5^\circ) - T \sin(35^\circ)) - 400g \sin(5^\circ) = 0$$

$$T(\cos(35^\circ) + 0.8 \sin(35^\circ)) = 400g(\sin(5^\circ) + 0.8 \cos(5^\circ)) \quad \text{M1}$$

$$T = \frac{400g(\sin(5^\circ) + 0.8 \cos(5^\circ))}{\cos(35^\circ) + 0.8 \sin(35^\circ)} = 2711.801$$

$$T = 2712 \text{ newtons} \quad \text{A1}$$

- iii.** resolving horizontally around the elephant in the $\hat{i}$ direction.

$$(3) Q - T \cos(35^\circ) - 5000g \sin(5^\circ) = 0$$

$$Q = T \cos(35^\circ) + 5000g \sin(5^\circ) = 6492.01$$

$$Q = 6492 \text{ newtons} \quad \text{A1}$$

Question 5 $\vec{r}(t) = (-25t^2 + 52.5t) \vec{i} + \left(2e^{-\frac{7t}{2}} \left| \cos\left(\frac{17\pi t}{10}\right) \right| \right) \vec{j} \quad t \geq 0$

a. $\vec{r}(0) = 0\vec{i} + 2\vec{j} = 2\vec{j}$

The ball was 2 metres above the ground when it left the bowlers hand. A1

b. differentiating using the product in the $\vec{j}$ component M1

$$\dot{\vec{r}}(t) = (-50t + 52.5) \vec{i} + \left(-\frac{7}{2} \times 2e^{-\frac{7t}{2}} \left| \cos\left(\frac{17\pi t}{10}\right) \right| + 2e^{-\frac{7t}{2}} \times -\frac{17\pi}{10} \left| \sin\left(\frac{17\pi t}{10}\right) \right| \right) \vec{j}$$

$$\dot{\vec{r}}(t) = (-50t + 52.5) \vec{i} - \left(e^{-\frac{7t}{2}} \left(7 \left| \cos\left(\frac{17\pi t}{10}\right) \right| + \frac{17\pi}{5} \left| \sin\left(\frac{17\pi t}{10}\right) \right| \right) \right) \vec{j} \quad \text{A1}$$

c. $\dot{\vec{r}}(0) = 52.5\vec{i} - 7\vec{j} \quad |\dot{\vec{r}}(0)| = \sqrt{52.5^2 + (-7)^2} = 53.0 \text{ m/s}$ M1

$$m = 155 \text{ gm} = 0.155 \text{ kg}$$

magnitude of the momentum of the ball when it left the bowlers hand

$$|\vec{p}| = m|\dot{\vec{r}}(0)| = 0.155 \times 53.0 = 8.21 \text{ kg m/s} \quad \text{A1}$$

d. The ball hits the ground when $2e^{-\frac{7t}{2}} \left| \cos\left(\frac{17\pi t}{10}\right) \right| = 0$

$$\cos\left(\frac{17\pi t}{10}\right) = 0 \quad \text{M1}$$

$$\frac{17\pi t}{10} = \frac{\pi}{2}$$

$$t = \frac{5}{17} \text{ sec} \quad \text{hits the ground after } 0.294 \text{ seconds} \quad \text{A1}$$

$$\vec{r}\left(\frac{5}{17}\right) = \left(-25\left(\frac{5}{17}\right)^2 + 52.5 \times \frac{5}{17} \right) \vec{i} + \left(2e^{-\frac{7 \times \frac{5}{17}}{2}} \left| \cos\left(\frac{17\pi}{10} \times \frac{5}{17}\right) \right| \right) \vec{j}$$

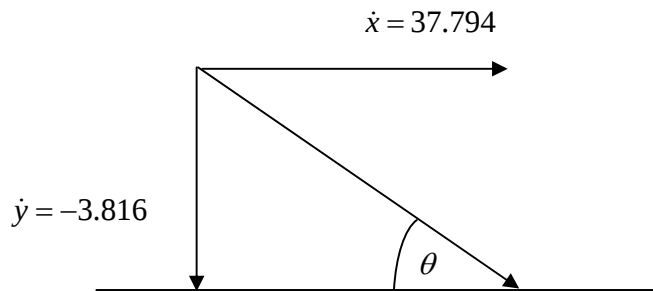
$$\vec{r}\left(\frac{5}{17}\right) = 13.278\vec{i} + 0\vec{j}$$

$$\vec{r}\left(\frac{5}{17}\right) = 13.278\vec{i} \quad (20 - 13.278 = 6.72)$$

The ball strikes the ground 6.72 metres from the batting stumps A1

e.
$$\vec{r}\left(\frac{5}{17}\right) = \left(-50 \times \frac{5}{17} + 52.5\right) \vec{i} + \left(e^{-\frac{7.5}{17}} \left(-7 \left|\cos\left(\frac{\pi}{2}\right)\right| - \frac{17\pi}{5} \left|\sin\left(\frac{\pi}{2}\right)\right|\right)\right) \vec{j}$$

$$\vec{r}\left(\frac{5}{17}\right) = 37.794 \vec{i} - 3.8156 \vec{j}$$



M1

$$\tan(\theta) = \frac{\dot{y}}{\dot{x}} = \frac{3.8156}{37.794}$$

$$\theta = \tan^{-1}(0.101) = 5.765^\circ = 5^\circ 46'$$

The ball strikes the ground at an angle of $5^\circ 46'$

A1

f. when $x = 20$ $20 = -25t^2 + 52t$

$$25t^2 - 52.5t + 20 = 0$$

$$t = \frac{1}{2} = 0.5 \text{ seconds}$$

Hits the batting stumps after 0.5 seconds

A1

$$\vec{r}\left(\frac{1}{2}\right) = 20\vec{i} + \left(2e^{-\frac{7 \times 0.5}{2}} \left|\cos\left(\frac{17\pi}{20}\right)\right|\right) \vec{j}$$

$$\vec{r}\left(\frac{1}{2}\right) = 20\vec{i} + 0.3097 \vec{j}$$

hits the stumps 31 cm above the ground.

A1

g. Total distance of 20 metres in 0.5 seconds, gives the average velocity of the ball as 40 m/s or $40 \times \frac{60 \times 60}{1000} = 144 \text{ km/hr}$

A1

END OF SECTION 2 SUGGESTED ANSWERS