

**Year 2006**

**VCE**

**Specialist Mathematics**

**Trial Examination 1**



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**Victorian Certificate of Education  
2006**

**STUDENT NUMBER**

Figures  
Words

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**SPECIALIST MATHEMATICS**

**Trial Written Examination 1**

Reading time: 15 minutes

Total writing time: 1 hour

**QUESTION AND ANSWER BOOK**

**Structure of book**

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|-------------------------------------------|------------------------|
| 9                          | 9                                         | 40                     |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers.
- Students are NOT permitted to bring into the examination room: notes of any kind, a calculator, blank sheets of paper and/or white out liquid/tape.

**Materials supplied**

- Question and answer book of 13 pages with a detachable sheet of miscellaneous formulas at the end of this booklet.
- Working space is provided throughout the booklet.

**Instructions**

- Detach the formula sheet from the end of this book during reading time.
- Write your **student number** in the space provided above this page.
- All written responses must be in English.

**Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.**

**Instructions**

Answer **all** questions in the spaces provided.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are not drawn to scale.

Take the acceleration due to gravity to have magnitude  $g \text{ m/s}^2$ , where  $g = 9.8$

**Question 1**

Given the vector  $\underline{b} = -2\underline{i} + y\underline{j} + 4\underline{k}$  find the value of  $y$  if

- a. the length of vector  $\underline{b}$  is 5.

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2 marks

- b. the vector  $\underline{b}$  makes an angle of  $150^\circ$  with the  $y$ -axis.

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2 marks

**Question 2**

A tank has a capacity of 300 litres and contains 200 litres of water in which 8 kilograms of salt has been dissolved. A salt solution of concentration 0.5 kilograms per litre is poured into the tank at a rate of 2 litres per minute, and the well-stirred mixture flows out at a rate of 3 litres per minute. If the amount of salt in the tank at a time  $t$  minutes is  $Q$  kilograms,

- a. set up the differential equation, for the amount of salt  $Q$  kg in the tank at a time  $t$  minutes.

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1 mark

- b. If  $Q = \frac{1}{2}(200 - t) + C(200 - t)^n$  is a solution, where  $C$  is a constant, find the value of  $n$ .

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2 marks

**Question 3**

- a. Show that  $\frac{d}{dx}\left(\cos^{-1}\left(\sqrt{\frac{3}{x}}\right)\right) = \frac{\sqrt{3}}{2x\sqrt{x-3}}$  for  $x > 3$

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2 marks

- b. Hence find the exact value of  $\int_4^{12} \frac{1}{x\sqrt{x-3}} dx$ .

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2 marks

**Question 4**

- a. If  $y = \frac{x}{\sqrt{x-3}}$  then gradient function can be represented as  $\frac{ax+b}{\sqrt{(x-3)^3}}$

find the exact values of  $a$  and  $b$ .

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2 marks

- b. Find using calculus the exact area bounded by the curve  $y = \frac{x}{\sqrt{x-3}}$  the  $x$ -axis and the lines  $x = 3$  and  $x = 4$ .

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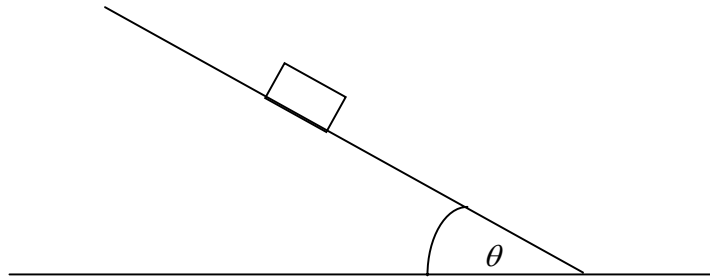
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2 marks

**Question 5**

A block of mass  $m$  is projected with a speed  $U$  so as to move straight up an inclined slide. The slide makes an angle of  $\theta$  with the horizontal and the coefficient of friction between the block and the slide is  $\mu$ .



- a. On the diagram above, mark in all the forces on the block on its movement up the slide.

1 mark

- b. Show that the block travels a distance of  $\frac{U^2}{2g(\sin(\theta) + \mu \cos(\theta))}$  up the slide before coming to rest.

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3 marks

**Question 6**

The area enclosed by the curves with the equations  $y = \cos(2x)$  and  $y = \sin(x)$ ,  $0 \leq x \leq \frac{\pi}{6}$  is rotated about the  $x$ -axis to form a solid of revolution. Find the exact volume of this solid of revolution.

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4 marks

**Question 7**

a. Solve for  $z$  if  $z^4 + z^2 - 12 = 0$

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2 marks

**b.** Given that  $z = a + bi$  where  $a, b \in \mathbb{R}$  find  $z^2$ .

Show that if  $z^2 = -1 - 4\sqrt{3}i$  then  $a^2 - b^2 = -1$  and  $ab = -2\sqrt{3}$ .

Hence find in exact Cartesian form the square roots of  $-1 - 4\sqrt{3}i$

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3 marks

**c.** Use the quadratic formula to solve  $z^2 + \sqrt{3}z + (1 + \sqrt{3}i) = 0$

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3 marks

**Question 8**

$OABC$  is a trapezium. The length of  $OA$  is  $\lambda$  times the length of  $CB$ .

$P$  and  $Q$  are the midpoints of  $OC$  and  $BA$  respectively.

Let  $\overrightarrow{OA} = \underline{a}$

- a.** Express  $\overrightarrow{PQ}$  in terms of  $\lambda$  and  $\underline{a}$

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3 marks

- b.** What is the ratio of the lengths of  $PQ$  to  $OA$ ?

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1 mark

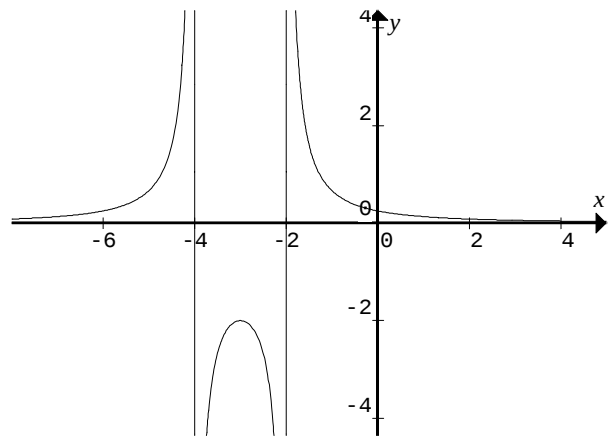
**Question 9**

- a. The graph shown has the form

$$y = \frac{a}{x^2 + bx + c}$$

The graph has vertical asymptotes at  $x = -4$  and  $x = -2$  and has a range of  $(-\infty, -2] \cup (0, \infty)$ .

Prove that  $a = 2$   $b = 6$  and  $c = 8$



2 marks

- b. If the area bounded by the curve  $y = \frac{2}{x^2 + 6x + 8}$  the coordinates axes and  $x = 2$  can be expressed in the form  $\log_e(p)$  find using calculus the exact value of  $p$ .

3 marks

**EXTRA WORKING SPACE**

[illegible]

**END OF EXAMINATION**

# **SPECIALIST MATHEMATICS**

## **Written examination 1**

### **FORMULA SHEET**

#### **Directions to students**

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

## Specialist Mathematics Formulas

### Mensuration

area of a trapezium:  $\frac{1}{2}(a+b)h$

curved surface area of a cylinder:  $2\pi rh$

volume of a cylinder:  $\pi r^2 h$

volume of a cone:  $\frac{1}{3}\pi r^2 h$

volume of a pyramid:  $\frac{1}{3}Ah$

volume of a sphere:  $\frac{4}{3}\pi r^3$

area of triangle:  $\frac{1}{2}bc \sin(A)$

sine rule:  $\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$

cosine rule:  $c^2 = a^2 + b^2 - 2ab \cos(C)$

### Coordinate geometry

ellipse:  $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$       hyperbola:  $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

### Circular ( trigonometric ) functions

$$\cos^2(x) + \sin^2(x) = 1$$

$$1 + \tan^2(x) = \sec^2(x)$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$$

$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

| function | $\sin^{-1}$                                  | $\cos^{-1}$ | $\tan^{-1}$                                  |
|----------|----------------------------------------------|-------------|----------------------------------------------|
| domain   | $[-1, 1]$                                    | $[-1, 1]$   | $R$                                          |
| range    | $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ | $[0, \pi]$  | $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ |

## Algebra ( Complex Numbers )

$$z = x + yi = r(\cos \theta + i \sin \theta) = r \operatorname{cis} \theta$$

$$|z| = \sqrt{x^2 + y^2} = r \qquad -\pi < \operatorname{Arg} z \leq \pi$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2) \qquad \frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$$

$$z^n = r^n \operatorname{cis}(n\theta) \text{ (de Moivre's theorem )}$$

## Vectors in two and three dimensions

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$|\underline{r}| = \sqrt{x^2 + y^2 + z^2} = r \qquad \underline{r}_1 \cdot \underline{r}_2 = r_1 r_2 \cos \theta = x_1 x_2 + y_1 y_2 + z_1 z_2$$

$$\dot{\underline{r}} = \frac{d\underline{r}}{dt} = \frac{dx}{dt} \underline{i} + \frac{dy}{dt} \underline{j} + \frac{dz}{dt} \underline{k}$$

## Mechanics

momentum:  $\underline{p} = m\underline{v}$

equation of motion:  $\underline{R} = m\underline{a}$

sliding friction:  $F \leq \mu N$

constant ( uniform ) acceleration:

$$v = u + at \qquad s = ut + \frac{1}{2}at^2 \qquad v^2 = u^2 + 2as \qquad s = \frac{1}{2}(u + v)t$$

acceleration:  $a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left( \frac{1}{2}v^2 \right)$

## Calculus

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

$$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax)$$

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

$$\frac{d}{dx}(\sin^{-1}(x)) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1}(x)) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^2}$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + c, \quad n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e(x) + c, \quad \text{for } x > 0$$

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$$

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$$

$$\int \sec^2(ax) dx = \frac{1}{a} \tan(ax) + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \quad a > 0$$

$$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$$

product rule:  $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

quotient rule:  $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

chain rule:  $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$

Euler's method    If  $\frac{dy}{dx} = f(x)$ ,  $x_0 = a$  and  $y_0 = b$ , then  $x_{n+1} = x_n + h$  and  $y_{n+1} = y_n + hf(x)$

**END OF FORMULAE SHEET**