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Specialist Mathematics

2006

Trial Examination 2

SECTION 1 Multiple-choice questions

Instructions for Section 1

Answer **all** questions.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1

The graph of $y = \frac{2x^2 + x - 3}{x^2 + 7x - 8}$ has

- A. no straight line asymptotes.
- B. only one straight line asymptote.
- C. only two straight line asymptotes.
- D. only three straight line asymptotes.
- E. $x = 1$ and $x = -8$ as its vertical asymptotes.

Question 2

Which one of the following hyperbolas has straight line asymptotes with equations $y = \frac{x}{2}$ and $y = -\frac{x}{2} - 2$?

- A. $\frac{(x-2)^2}{4} - (y-1)^2 = 1$
- B. $\frac{(x-2)^2}{8} - \frac{(y-1)^2}{2} = 1$
- C. $\frac{(x-2)^2}{12} - \frac{(y-1)^2}{3} = 1$
- D. $\frac{(x+2)^2}{8} - \frac{(y+1)^2}{2} = 1$
- E. $\frac{(x+2)^2}{3} - \frac{(y+1)^2}{12} = 1$

Question 3

The roots of $z^4 + 1$ are

- A. $z^2 - i, z^2 + i$
- B. $-1, 1, -i, i$
- C. $z-1, z+1, z-i, z+i$
- D. $\frac{\sqrt{2}}{2}(1+i), \frac{\sqrt{2}}{2}(1-i), -\frac{\sqrt{2}}{2}(1+i), -\frac{\sqrt{2}}{2}(1-i)$
- E. $z + \frac{\sqrt{2}}{2}(1+i), z + \frac{\sqrt{2}}{2}(1-i), z - \frac{\sqrt{2}}{2}(1+i), z - \frac{\sqrt{2}}{2}(1-i)$

Question 4

Given $z = \sqrt{2} - 3i$ and $w = 3 + i\sqrt{2}$, $(z\bar{w})^{-1}$ is equal to

- A. $\frac{6\sqrt{2} + 7i}{121}$
- B. $\frac{6\sqrt{2} - 7i}{121}$
- C. $\frac{1}{6\sqrt{2} + 7i}$
- D. $\frac{i}{7}$
- E. $\frac{i}{11}$

Question 5

Which one of the following statements is **false** for $z = -1.82 + 0.91i$?

- A. $z \in \left\{ z : \text{Arg}(z) \geq -\frac{5\pi}{6} \right\} \cap \{ z : 2 < |z| < 4 \}$
- B. $z \in \left\{ z : \text{Arg}(z) \geq \frac{5\pi}{6} \right\} \cap \{ z : 2 < |z| < 4 \}$
- C. $z \in \left\{ z : \text{Arg}(z) \geq \frac{5\pi}{6} \right\} \cup \{ z : 3 < |z| < 5 \}$
- D. $z \in \left\{ z : \text{Arg}(z) \leq \frac{5\pi}{6} \right\} \cup \{ z : 2 < |z| < 4 \}$
- E. $z \in \left\{ z : \text{Arg}(z) \leq -\frac{5\pi}{6} \right\} \cup \{ z : 3 < |z| < 5 \}$

Question 6

The simplified form of $\frac{\cos x - \sin x}{\cos x + \sin x}$ is

- A. $\sec(2x) - \tan(2x)$
- B. $\sec(2x) + \tan(2x)$
- C. $\operatorname{cosec}(2x) - \cot(2x)$
- D. $\operatorname{cosec}(2x) + \cot(2x)$
- E. $(\sec(2x) - \tan(2x))^{-1}$

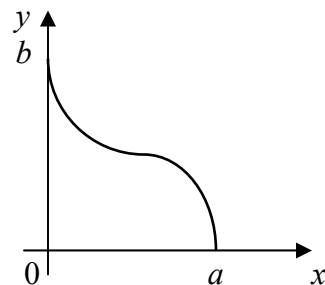
Question 7

Given $f(x) = 3 \sec\left(\frac{x - \pi}{2}\right) + 1$, $0 < x \leq \pi$, then

- A. $f^{-1}(x) = 2 \cos^{-1}\left(\frac{3}{x-1}\right) + \pi$, $0 < x \leq \pi$
- B. $f^{-1}(x) = 3 \cos^{-1}\left(\frac{2}{x-1}\right) + \pi$, $0 < x \leq \pi$
- C. $f^{-1}(x) = 3 \cos^{-1}\left(\frac{2}{x-1}\right) + \pi$, $x \geq 4$
- D. $f^{-1}(x) = 2 \cos^{-1}\left(\frac{3}{x-1}\right) + \pi$, $x \geq 4$
- E. $f^{-1}(x) = \cos^{-1}\left(\frac{6}{x-1} + \pi\right)$, $x \geq 4$

Question 8

The graph of $y = \frac{b}{\pi} \cos^{-1}\left(\frac{2x-a}{a}\right)$ is shown below,



the value of $\int_0^b x dy$ is

- A. $\frac{ab\pi}{6}$
- B. $\frac{3ab}{2\pi}$
- C. $\frac{ab}{2}$
- D. $\frac{a^2 + b^2}{2\pi}$
- E. $\frac{(a+b)^2}{2\pi}$

Question 9

The points on the curve $x^2 - y^2 = \frac{3}{4}$ where the gradient is 2 are

- A. $\left(-\frac{1}{2}, -1\right), \left(\frac{1}{2}, 1\right)$
- B. $(-2, -1), (2, 1)$
- C. $\left(-1, \frac{1}{2}\right), \left(1, \frac{1}{2}\right)$
- D. $(-2, 1), (2, 1)$
- E. $\left(-1, -\frac{1}{2}\right), \left(1, \frac{1}{2}\right)$

Question 10

The gradient(s) of the curve $y = \log_e |x+1|$ where $y=1$ is/are

- A. $-e, e$
- B. $-e-1, e-1$
- C. $e+1, e-1$
- D. $-e^{-1}, e^{-1}$
- E. $1-e^{-1}, 1+e^{-1}$

Question 11

If $a < b < -1$, then $\int_a^b \log_e |x| dx$ is equal to

- A. $\int_{-b}^{-a} \log_e |x| dx$
- B. $\int_b^a \log_e |x| dx$
- C. $-\int_{-b}^{-a} \log_e |x| dx$
- D. $\int_b^a \log_e (x) dx$
- E. $\int_{-b}^{-a} \log_e (-x) dx$

Question 12

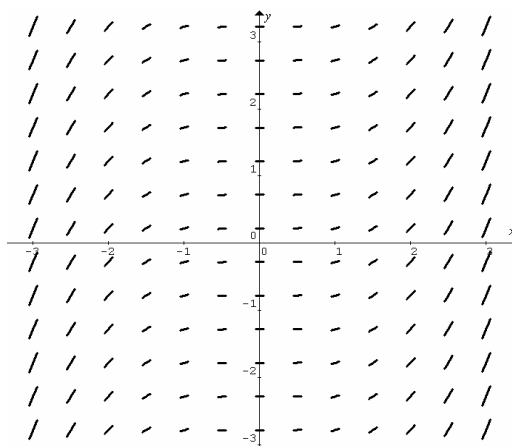
The graph of $f : [3, 6] \rightarrow \mathbb{R}, f(x) = \sqrt{9 - (x - 3)^2}$ is shown below. Length measure is in cm.



The shaded region is rotated about the x -axis to form a solid of revolution. The volume of this solid is given by

- A. 18 cm^3 B. 36 cm^3 C. $18\pi \text{ cm}^3$ D. $36\pi \text{ cm}^3$ E. $\frac{36}{\pi} \text{ cm}^3$

Question 13



The direction field for a certain first order differential equation is shown above. The first order differential equation could be

- A. $\frac{dy}{dx} = \frac{k}{x}$, where k is a real constant
 B. $\frac{dy}{dx} = k \log_e x$, where k is a real constant
 C. $\frac{dy}{dx} = e^{kx}$, where k is a real constant
 D. $\frac{dy}{dx} = kx^3$, where k is a real constant
 E. $\frac{dy}{dx} = kx^2$, where k is a real constant

Question 14

The definite integral $\int_0^{\frac{\pi}{3}} \cot\left(\frac{\pi}{2} - x\right) dx$ can be expressed as

- A. $\int_{\frac{1}{2}}^1 \frac{dx}{x}$
- B. $\int_{\frac{1}{2}}^1 \left(-\frac{1}{x}\right) dx$
- C. $\left[-\log_e \left|\frac{\pi}{2} - x\right|\right]_0^{\frac{\pi}{3}}$
- D. $\left[\sec^2\left(\frac{x}{2}\right)\right]_0^{\frac{\pi}{3}}$
- E. $\left[\cos ec^2\left(\frac{\pi}{2} - x\right)\right]_0^{\frac{\pi}{3}}$

Question 15

Euler's method, with a step size of 0.2, is used to solve the differential equation $\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$, with initial condition $y = -2$ when $x = 1$. When $x = 1.4$ the approximate value for y is

- A. $-2 + \frac{0.2}{\sqrt{2.44}}$
- B. $-2 + \frac{0.2}{\sqrt{2}} + \frac{0.2}{\sqrt{2.44}}$
- C. $-2 + \frac{0.2}{\sqrt{2}} + \frac{0.4}{\sqrt{2.44}}$
- D. $-2 + \frac{0.2}{\sqrt{2.44}} + \frac{0.2}{\sqrt{2.96}}$
- E. $-2 + \frac{0.2}{\sqrt{2.44}} + \frac{0.4}{\sqrt{2.96}}$

Question 16

The position vector of a particle at time $t \geq 0$ is given by $\mathbf{r}(t) = 3\sin(t)\mathbf{i} + \sqrt{3}\cos(t)\mathbf{j}$, where $\mathbf{i}$ points to east and $\mathbf{j}$ points to north. At $t = \frac{\pi}{3}$, the particle is moving in the direction

- A. SE B. SW C. NE D. NW E. N30°W

Question 17

$\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are the position vectors of three collinear points P, Q and R respectively. Point Q divides the line segment PR into the ratio 1 : 4.

$\mathbf{q}$ is equal to

- A. $\frac{1}{3}(\mathbf{p} + 4\mathbf{r})$
- B. $\frac{1}{4}(\mathbf{p} + 4\mathbf{r})$
- C. $\frac{1}{5}(\mathbf{p} + 4\mathbf{r})$
- D. $\frac{1}{5}(4\mathbf{p} + \mathbf{r})$
- E. $\frac{1}{4}(4\mathbf{p} + \mathbf{r})$

Question 18

A particle of mass m kg slides along a rough horizontal surface. The reaction force of the surface on the particle makes an angle θ° with the direction of motion of the particle. Which one of the following statements is true?

- A. $\theta = 0$
- B. $0 < \theta < 90$
- C. $\theta = 90$
- D. $90 < \theta < 180$
- E. $\theta = 180$

Question 19

A body of mass 2 kg moves with velocity $\mathbf{v}(t) = \cos(2t)\mathbf{i} - 5\mathbf{j}$ ms⁻¹ at time t (in seconds). The magnitude of the rate of change of its momentum with respect to time (in kgms⁻²) at $t = \frac{\pi}{4}$ is

- A. -4
- B. -2
- C. 2
- D. 4
- E. 0

Question 20

A body of mass 5 kg falls vertically from rest. Assuming that air resistance is negligible, the distance fallen by the body in the third second is

- A. 24.5 m B. 44.1 m C. 19.6 m D. 34.3 m E. 78.4 m

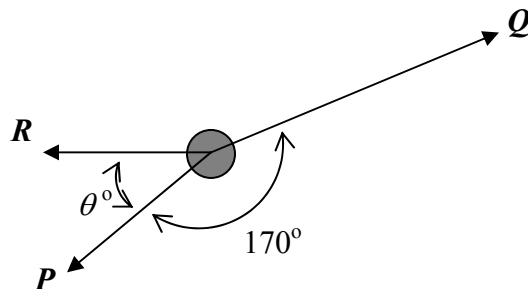
Question 21

Vectors $\mathbf{a} = 3\mathbf{i} + p\mathbf{j}$, $\mathbf{b} = 2\mathbf{i} - 5\mathbf{j}$ and $\mathbf{c} = 5\mathbf{i} + 2\mathbf{j}$ are coplanar. They are linearly dependent when

- A. $p = \frac{3}{2}$ only
- B. $p = \frac{3}{2}$ or $-\frac{7}{2}$ only
- C. $p \in \mathbb{R} \setminus \left\{-\frac{15}{2}\right\}$
- D. $p \in \mathbb{R} \setminus \left\{\frac{6}{5}\right\}$
- E. $p \in \mathbb{R} \setminus \left\{-\frac{15}{2}, \frac{6}{5}\right\}$

Question 22

A body of mass 5 kg is in equilibrium when it is acted upon by three concurrent coplanar forces $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ as shown in the diagram below. $\mathbf{P} = 10$ newtons and $\mathbf{Q} = 20$ newtons. The angle between $\mathbf{P}$ and $\mathbf{Q}$ is 170° and the angle between $\mathbf{P}$ and $\mathbf{R}$ is θ° .



The value of θ is closest to

- A. 15 B. 20 C. 25 D. 30 E. 35

SECTION 2 Extended-answer questions

Instructions for Section 2

Answer **all** questions.

A decimal approximation will not be accepted if an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this exam are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ ms}^{-2}$, where $g = 9.8$.

Question 1

Consider the set of complex numbers $S = \{z : |z| = \arg z\}$. Let $r = |z|$ and $\theta = \arg z$.

a. Complete the following table.

1 mark

θ	0		2		4		6
r		1		3		5	

b. Find the complex number $z = x + yi$ in S such that $\arg z = \frac{\pi}{3}$.

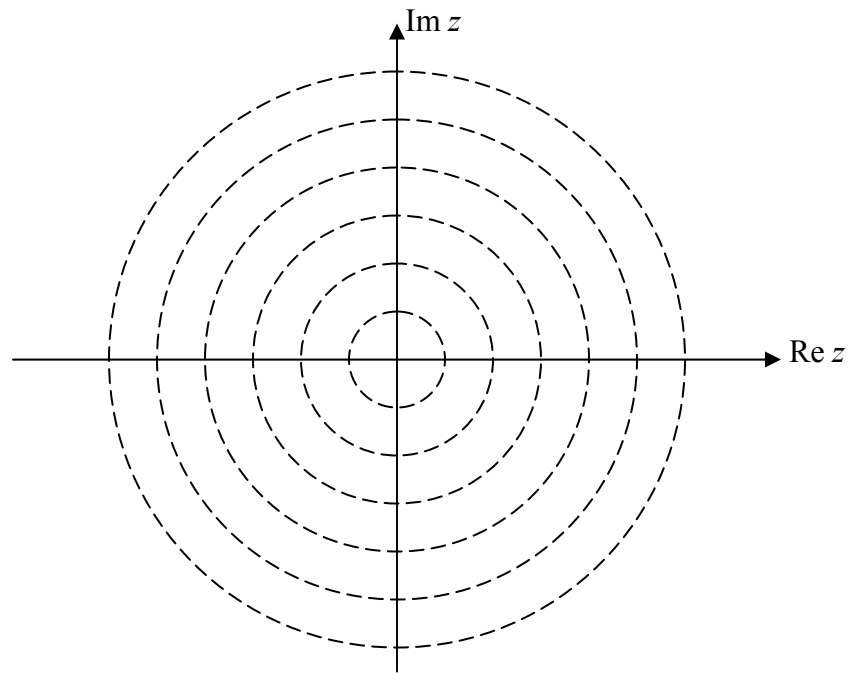
2 marks

c. Given $w = \frac{\pi}{2}i$, and $0 < \arg w < \pi$, determine whether or not $w \in S$. Explain.

2 marks

d. Plot the graph of the seven complex numbers shown in the table in part a.

3 marks



e. Each member of $S = \{z : |z| = \arg z\}$ has a complex conjugate.

Find T , the set of complex conjugates of $z \in S$. Specify the values of $\arg z$ for $z \in T$.

3 marks

Total 11 marks

Question 2

Two particles P and Q are in motion. Their position vectors at time t , $\mathbf{p}$ and $\mathbf{q}$, are given by

$$\mathbf{p} = (\log_e(t + 0.5))\mathbf{i} + (t + 0.5)\mathbf{j} \text{ and } \mathbf{q} = (e^{t-0.5})\mathbf{i} + (t - 0.5)\mathbf{j}, \text{ where } 0 \leq t \leq 1.$$

- a. Find the distance between the two particles at time t . 3 marks

- b. Hence find
- i. the closest approach of the two particles and the time that it occurs, 2 marks

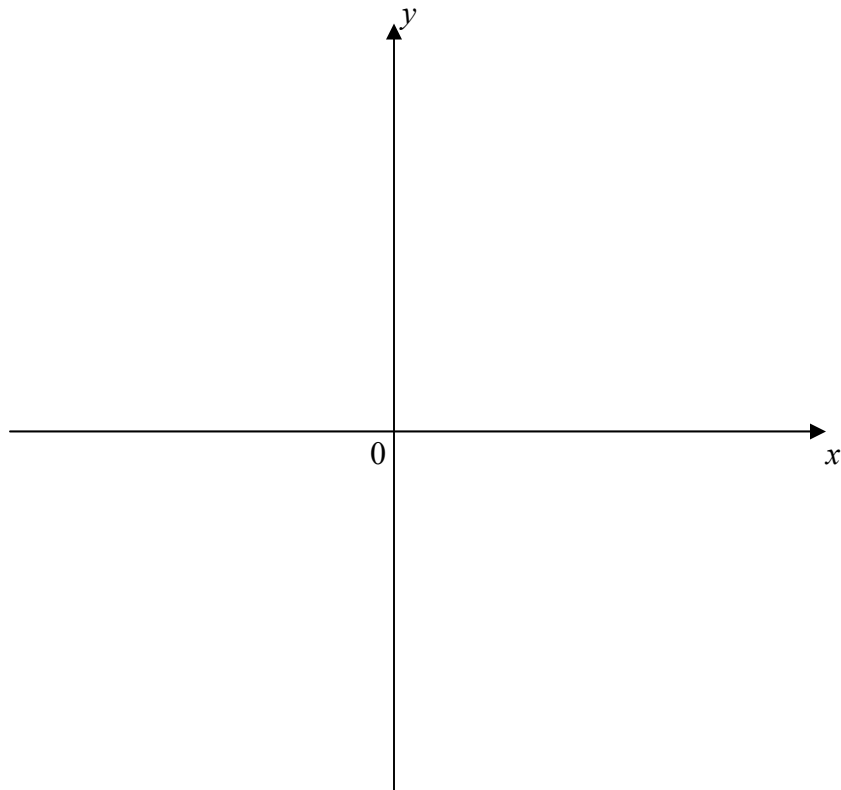
- ii. the greatest distance between the two particles and the time it occurs. 2 marks

- c. Find the time when the two particles move in the same direction. 3 marks

- d.** Find the cartesian equation, in terms of x and y , of the path of particle P, and the same of particle Q. 2 marks

- e.** Hence sketch the two paths on the same set of axes (use the same scale for both axes), and explain your answers for t to parts **bi** and **c**.

4 marks
Total 16 marks

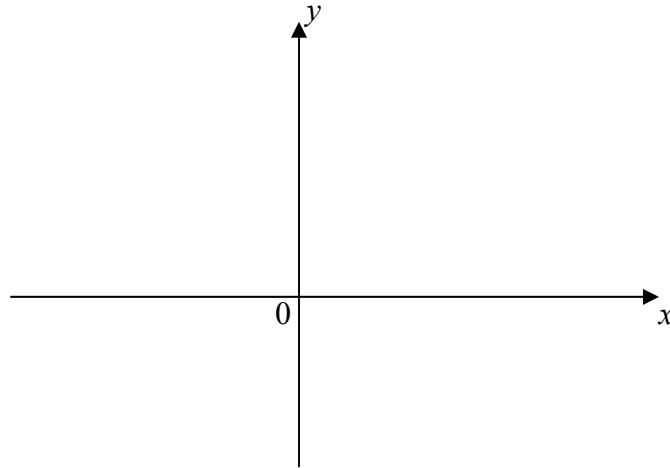


Question 3

Consider the function $f : [0, 2] \rightarrow \mathbb{R}, f(x) = \frac{1}{3}(x-1)^2(x+1)^2$. Lengths are measured in metres.

a. Find the range of the function. 1 mark

b. Sketch the graph of f showing the coordinates of intercepts, stationary point and end points. 2 marks



c. Use calculus to find the exact area of the region enclosed by the graph of the function, the y -axis and the line $y = 3$. 3 marks

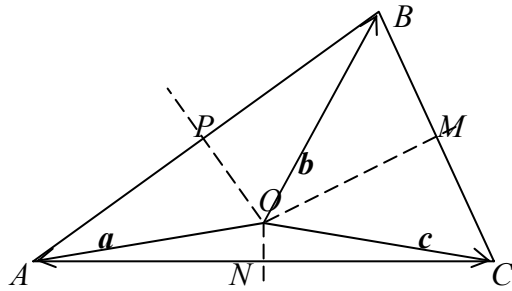
d. The region described in part **c** is rotated about the y -axis. Find the exact volume of the solid of revolution.

5 marks
Total 11 marks

Question 4

Consider $\triangle ABC$ in the following diagram.

$\overline{OM}$ and $\overline{ON}$ are perpendicular bisectors of $\overline{BC}$ and $\overline{AC}$ respectively, and $\overline{OP}$ bisects $\overline{AB}$.



Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$.

a. Express $\overrightarrow{AC}$, $\overrightarrow{BC}$ and $\overrightarrow{BA}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. 1 mark

b. Express $\overrightarrow{OM}$, $\overrightarrow{ON}$ and $\overrightarrow{OP}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. 2 marks

c. Hence show that
 i. $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}|$, 3 marks

ii. $\overline{OP}$ is perpendicular to $\overline{BA}$. 1 mark

d. Show that $|\overrightarrow{AC}|^2 + |\overrightarrow{BC}|^2 + |\overrightarrow{BA}|^2 = 2d^2[3 - (\cos \alpha + \cos \beta + \cos \gamma)]$, where $d = |\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}|$ and α , β and γ are angles between $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{b}$ and $\mathbf{c}$, and $\mathbf{c}$ and $\mathbf{a}$ respectively.

3 marks
Total 10 marks

Question 5

A 5-kg particle, moving at 10 ms^{-1} , experiences a force of magnitude $\frac{500}{25 - t^2}$ newtons at time $t \geq 0$ s until it comes to a stop.

a. Show that a differential equation relating v and t is $\frac{dv}{dt} = -\frac{100}{25 - t^2}$. 1 mark

b. Show that the velocity of the particle at time t seconds is $10 \left(1 - \log_e \left| \frac{5+t}{5-t} \right| \right) \text{ ms}^{-1}$. 4 marks

c. Find the exact time when the particle comes to a stop.

2 marks

d. i. Write down a definite integral for the stopping distance.

1 mark

ii. Hence determine the stopping distance to the nearest metre.

2 marks
Total 10 marks

End of Exam 2