

INSIGHT

Trial Exam Paper

2006

SPECIALIST MATHEMATICS

Written examination 2

STUDENT NAME:

QUESTION AND ANSWER BOOK

Reading time: 15 minutes

Writing time: 2 hours

Structure of book

<i>Section</i>	<i>Number of questions</i>	<i>Number of questions to be answered</i>	<i>Number of marks</i>
1	22	22	22
2	5	5	58
Total			80

- Students are permitted to bring the following items into the examination: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, once bound reference, one approved graphics calculator or CAS (memory DOES NOT need to be cleared) and, if desired, one scientific calculator.
- Students are NOT permitted to bring sheets of paper or white out liquid/tape into the examination.

Materials provided

- The question and answer book of 27 pages with a separate sheet of miscellaneous formulas.
- Answer sheet for multiple-choice questions

Instructions

- Write your **name** in the box provided and on the multiple-choice answer sheet.
- Remove the formula sheet during reading time.
- You must answer the questions in English.

At the end of the exam

- Place the multiple-choice answer sheet inside the front cover of this book.

Students are NOT permitted to bring mobile phones or any other electronic devices into the examination.

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SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the multiple-choice answer sheet provided.

Choose the response that is **correct** for the question.

One mark will be awarded for a correct answer; no marks will be awarded for an incorrect answer.

Marks **are not** deducted for incorrect answers.

No marks will be awarded if more than one answer is completed for any question.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

Question 1

If $2i$ is a solution of the equation $z^3 - 5z^2 + 4z - mi = 0$, then the value of m will be

- A. $-2i$
- B. $-20i$
- C. -20
- D. 20
- E. $20i$

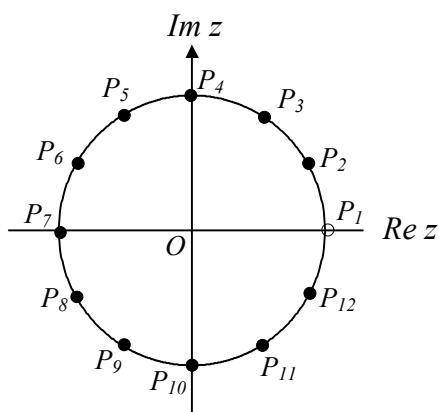
Question 2

If $z = -1 + \sqrt{3}$, then $\text{Arg}(z^2)$ equals

- A. $-\frac{2\pi}{3}$
- B. $-\frac{\pi}{3}$
- C. $\frac{\pi}{3}$
- D. $\frac{2\pi}{3}$
- E. $\frac{4\pi}{3}$

Question 3

Points P_1 to P_{12} are twelve equally spaced points around the circumference of a circle.



Point P_3 represents the complex number $z = a + ib$.

The complex number $i^{11}\bar{z}$ is represented by point

- A. P_2
- B. P_5
- C. P_8
- D. P_9
- E. P_{11}

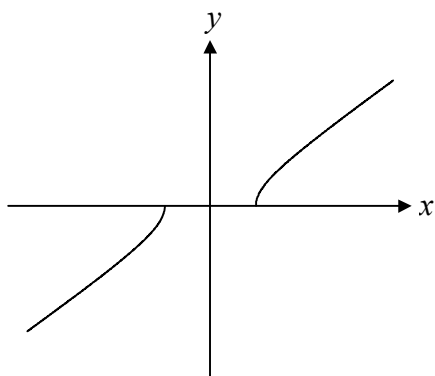
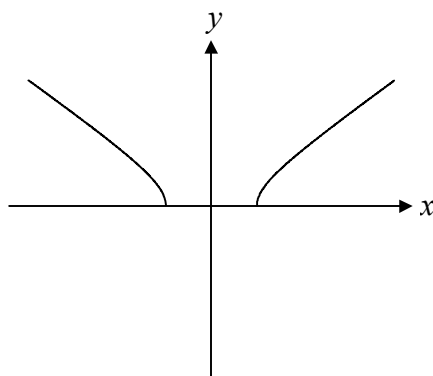
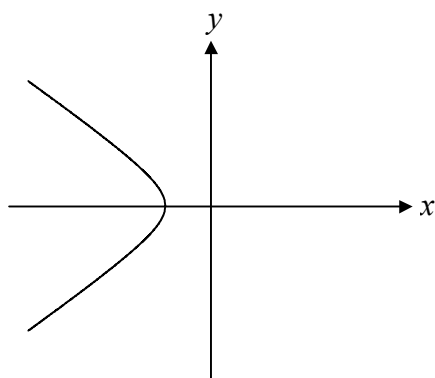
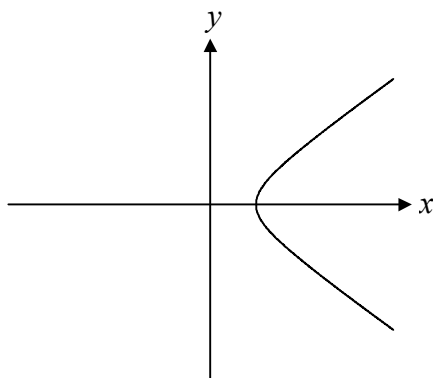
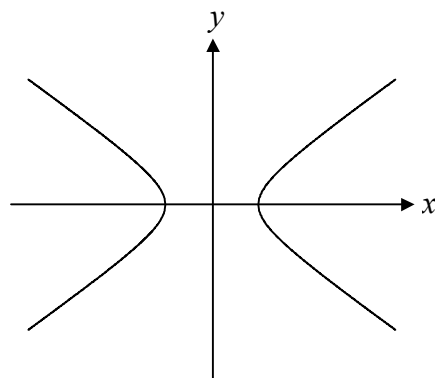
Question 4

The range of the function $f(x) = \cos^{-1}(x - \pi) - 1$ is

- A. $[\pi - 1, \pi + 1]$
- B. $[-1, \pi - 1]$
- C. $[0, \pi]$
- D. $[-2, 0]$
- E. $[-1, 1]$

Question 5

A graph of the curve specified by the parametric equations $x = \sec(t)$, $y = \tan(t)$ where $t \in [0, \pi]$ could be

A.**B.****C.****D.****E.**

Question 6

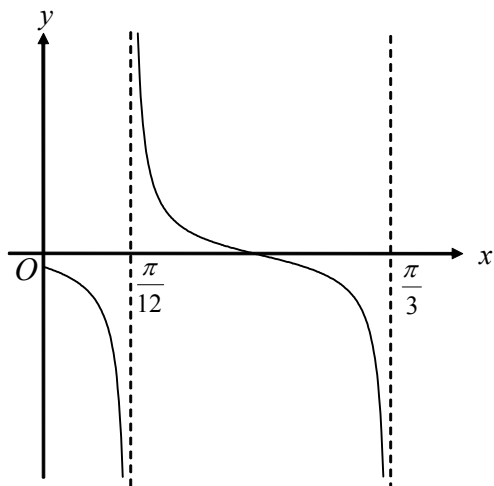
Consider the function $f : R \rightarrow R$ where $f(x) = 4x^3 - 3x^4$

Which one of the following statements is not true?

- A. f has two stationary points
- B. f has two points of inflexion
- C. f' is maximum when $x = \frac{2}{3}$
- D. $\frac{1}{f}$ has three asymptotes
- E. $f = \frac{1}{f}$ has three solutions

Question 7

A graph of $f : \left[0, \frac{\pi}{3}\right)$ where $f(x) = \cot\left(nx - \frac{\pi}{3}\right)$ is sketched below.



The value of n could be

- A. $\frac{1}{4}$
- B. $\frac{1}{3}$
- C. 3
- D. 4
- E. 8

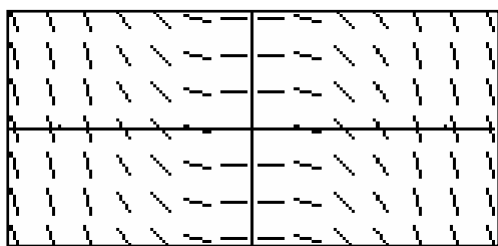
Question 8

The gradient of the curve $y^2 = 4x + 6y - 5$ is $-\frac{2}{3}$ at the point where y equals

- A. 0
- B. 0.15
- C. 1.25
- D. 5
- E. 6

Question 9

The slope field from a first order differential equation is shown below.



If $a \in R$, a solution of this differential equation could be

- A. $y = a \log_e(x)$
- B. $y = a \cos(x)$
- C. $y = a \tan^{-1}(x)$
- D. $y = \frac{a}{x^2}$
- E. $y = ax^3$

Question 10

Given $\frac{dy}{dx} = \sqrt{\sin(2x)}$ and $y = \sqrt{2}$ when $x = \frac{\pi}{12}$.

The value of y when $x = \frac{\pi}{3}$ is

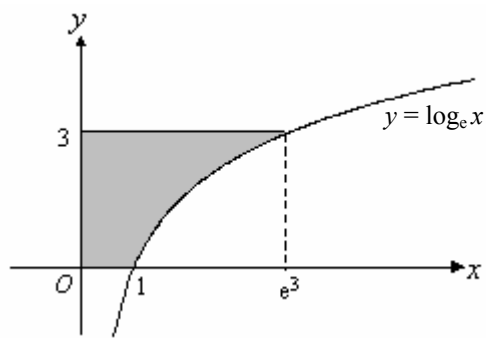
- A. 0.2500
- B. 0.7298
- C. 0.9306
- D. 1.4369
- E. 2.1440

Question 11

Using a suitable substitution, $\int_5^{10} \frac{1}{x^2} e^{\frac{10}{x}} dx$ can be expressed as

- A. $\int_1^2 \frac{100}{u^2} e^u du$
- B. $100 \int_1^2 u^2 e^u du$
- C. $-10 \int_2^1 e^u du$
- D. $\frac{1}{10} \int_1^2 e^u du$
- E. $-\frac{1}{10} \int_5^{10} e^u du$

Question 12



The graph of $y = \log_e x$ is shown above. The volume of the solid of revolution formed when the shaded region is rotated around the y -axis is given by

- A. $\pi \int_0^3 (3 - \log_e x)^2 dx$
- B. $\pi \int_1^{e^3} (\log_e x)^2 dx$
- C. $\pi \int_1^{e^3} (3 - e^y)^2 dy$
- D. $\pi \int_0^3 e^y dy$
- E. $\pi \int_0^3 e^{2y} dy$

Question 13

A spherical ice ball initially has radius 0.9 cm. It is placed in a drink and melts at a constant rate of $1.5 \text{ cm}^3/\text{minute}$. When the radius is 0.6 cm, the rate, in cm/minute, at which the radius is decreasing is

- A. $\frac{5}{24\pi}$
- B. $\frac{25}{72\pi}$
- C. $\frac{25}{24\pi}$
- D. $\frac{54\pi}{25}$
- E. $\frac{36\pi}{25}$

Question 14

A tank initially contains 200 litres of pure water. A salt solution with a concentration of 0.2 kg/litre is poured into the tank at a rate of 5 litres/minute. The mixture is kept uniform by stirring and flows out of the tank at a rate of 3 litres/minute.

Let Q be the amount of salt in the tank after t minutes.

$\frac{dQ}{dt}$ is equal to

- A. $5 - \frac{3Q}{200 + 2t}$
- B. $5 - \frac{3Q}{200}$
- C. $(5 - 3t) \frac{Q}{200}$
- D. $1 - \frac{3Q}{200 - 2t}$
- E. $1 - \frac{3Q}{200 + 2t}$

Question 15

Let $\underline{u} = 6\underline{i} + 2\underline{j} - 3\underline{k}$ and $\underline{v} = 2\underline{i} - \underline{j} + 3\underline{k}$.

The vector resolute of $\underline{u}$ in the direction of $\underline{v}$ is

- A. $\frac{1}{49}(2\underline{i} - \underline{j} + 3\underline{k})$
- B. $\frac{1}{7}(2\underline{i} - \underline{j} + 3\underline{k})$
- C. $\frac{1}{14}(2\underline{i} - \underline{j} + 3\underline{k})$
- D. $\frac{1}{\sqrt{14}}(2\underline{i} - \underline{j} + 3\underline{k})$
- E. $\frac{1}{7\sqrt{14}}(2\underline{i} - \underline{j} + 3\underline{k})$

Question 16

Points A , B and C are collinear such that $AB : BC = 1 : 3$

If $\vec{OA} = \underline{a}$ and $\vec{OC} = \underline{c}$ then $\vec{OB}$ equals

- A. $\frac{1}{4}(3\underline{a} + \underline{c})$
- B. $\frac{1}{4}(\underline{a} + 3\underline{c})$
- C. $\frac{1}{4}(5\underline{a} - \underline{c})$
- D. $\frac{1}{3}(2\underline{a} + \underline{c})$
- E. $\frac{1}{3}(\underline{a} - 3\underline{c})$

Question 17

The position of a particle at time t is given by $\underline{r}(t) = (t^3 + 2t)\underline{i} + 5t\underline{j} - t^2\underline{k}$.

The magnitude of its acceleration when $t = 1$ is

- A. $3\underline{i} + 5\underline{j} - \underline{k}$
- B. $6\underline{i} - 2\underline{k}$
- C. $2\sqrt{10}$
- D. $3\sqrt{6}$
- E. $\sqrt{35}$

Question 18

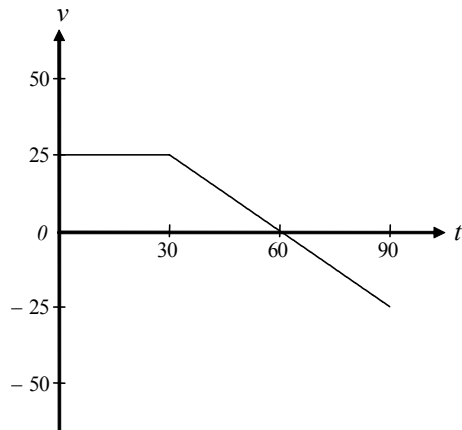
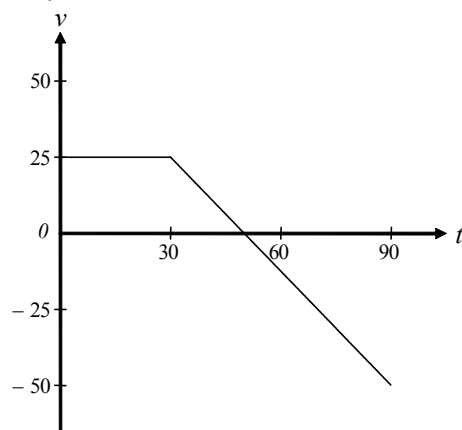
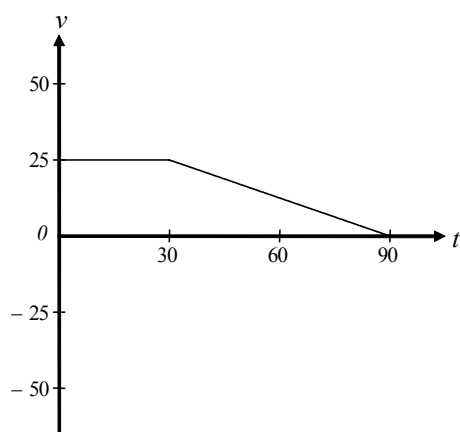
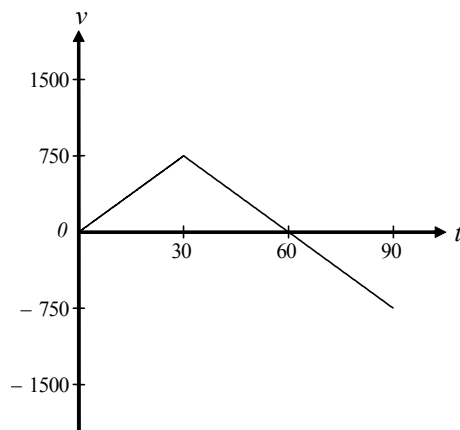
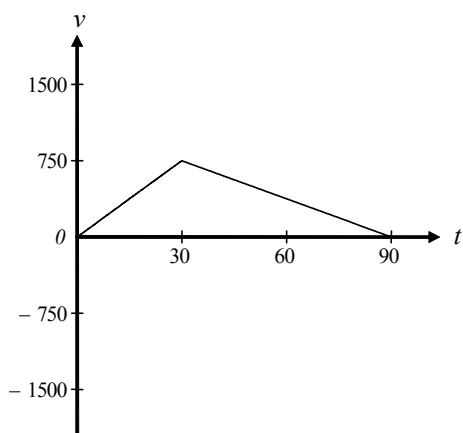
A particle is moving in a straight line with an acceleration of $-20x + 20 \text{ m/s}^2$, where x is its displacement, in metres, from a fixed point O . If the particle is travelling with a velocity of 6 m/s when it is 3 metres to the right of O , its maximum speed, in m/s, is

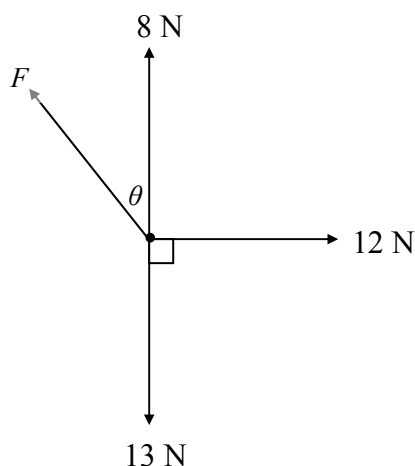
- A. 6.0
- B. 9.8
- C. 10.0
- D. 10.8
- E. 12.0

Question 19

A particle travels in a straight line with a constant velocity of 25 m/s for 30 seconds. It then decelerates for 60 seconds and returns to its original position.

The velocity-time graph that best represents the motion of the particle is

A.**B.****C.****D.****E.**

Question 20

Four forces are acting on a particle as shown in the diagram above.

The particle will be in equilibrium when F , measured in newtons, is equal to

- A. $5 \cos \theta$
- B. $12 \sin \theta$
- C. $\frac{\cos \theta}{12}$
- D. 5
- E. 13

Question 21

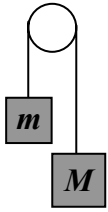
A motorbike is travelling at a speed of 60 km/hr on a straight road. A school zone is observed in the distance and over the next 10 seconds it reduces speed to 40 km/hr.

If the mass of the motorbike is 900 kg, the change in momentum, measured in kg m/s, in the direction of motion is

- A. -6480
- B. -5000
- C. -1800
- D. -500
- E. -180

Question 22

A mass of m kg is attached to a second mass of M kg, $m < M$, by a light string passing over a smooth pulley as shown below. The tension in the string is T newtons.



The acceleration, in m/s^2 , of the M kg mass is

- A. g
- B. Mg
- C. $\frac{Mg - T}{m}$
- D. $\frac{g(M - m)}{(M + m)}$
- E. $\frac{g(M + m)}{(M - m)}$

SECTION 2

Instructions for Section 2

Answer all the questions in the spaces provided.

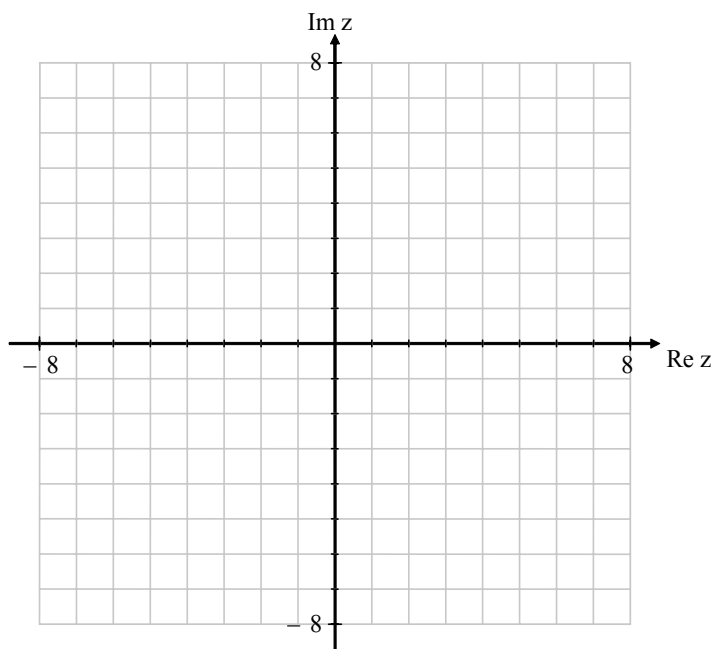
A decimal approximation will not be accepted if an exact answer is required to a question.

In questions where more than one mark is available, approximate working must be shown.

Unless otherwise indicated, the diagrams in this book have not been drawn to scale.

Take the **acceleration due to gravity** to have magnitude $g \text{ m/s}^2$, where $g = 9.8$

Question 1



a. Let $P = 4\sqrt{2}\text{cis}\left(\frac{\pi}{4}\right)$.

Express P in Cartesian form and plot and label this point in the Argand plane above.

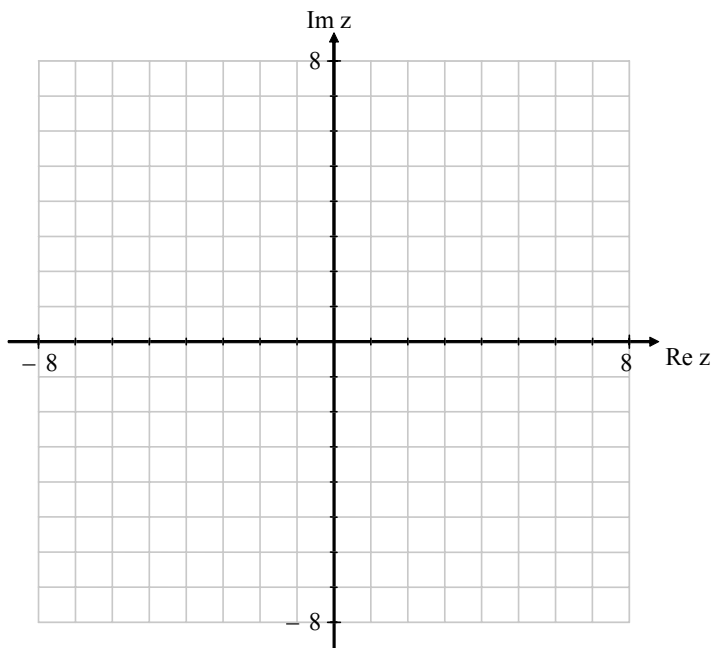
1 mark

- b. i.** Find an equivalent Cartesian equation for

$$\{z : |z + 2 - 4i| = |z - 2|, z \in C\}$$

2 marks

- ii.** Hence sketch $\{z : |z + 2 - 4i| = |z - 2|, z \in C\}$ on the Argand plane below.



1 mark

- c.** Describe the key features of the relation defined by $\{z : |z - i| = 5\}$

2 marks

- d.** M and N are the points of intersection of the relations $\{z : |z - i| = 5\}$ and $\{z : |z + 2 - 4i| = |z - 2|\}$. Determine points M and N in Cartesian form using your graphics calculator.

2 marks

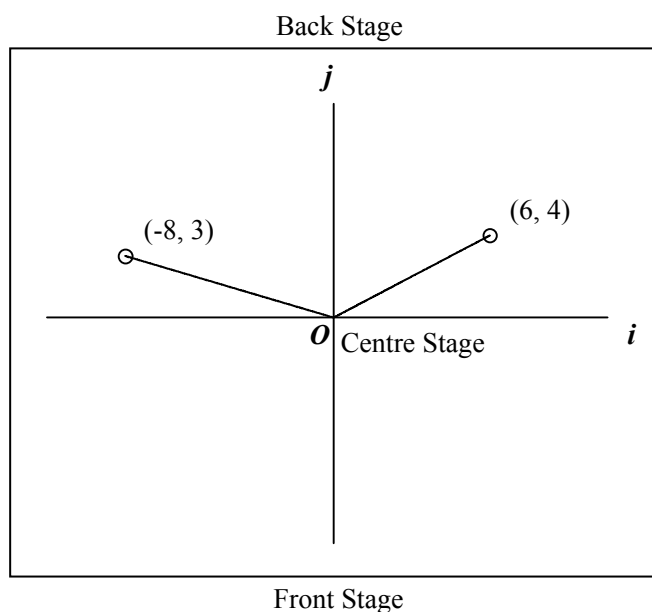
- e.** Use vectors to prove that points M , N and P are the vertices of a right-angled triangle.

3 marks

Total 11 marks

Question 2

Two dancers, Ari, A , and Ben, B , are standing on stage at the start of a performance. Their position coordinates, in metres, in relation to point O at the centre of the stage are shown in the diagram below.



- a. Write vectors $\vec{OA}$ and $\vec{OB}$ in terms of $\underline{i}$ and $\underline{j}$ to describe the positions of Ari and Ben at the start of the performance.

1 mark

- b. Find the obtuse angle AOB in degrees correct to one decimal place.

2 marks

As the performance starts spotlight, r , is beamed onto the stage. The path the spotlight follows around the stage is given by the equation $\underline{r} = 10 \cos(t) \underline{i} + 5 \sin(t) \underline{j}$, $t \geq 0$.

- c. Write a vector that describes the position of spotlight r initially.

1 mark

- d. Show that both Ari and Ben are standing in the path traced out by spotlight r .

3 marks

- e. How long after the spotlight passes Ari does it reach Ben? Write your answer in seconds correct to two decimal places.

2 marks

A second spotlight, s , starts moving at the same time as spotlight r . It follows a path given by the equation $\underline{s} = 5 \sin(t) \underline{i} + 10 \cos(t) \underline{j}$, $t \geq 0$.

- f. Find the times and position coordinates of the points on stage where the spotlights meet. Write your answers correct to two decimal places.

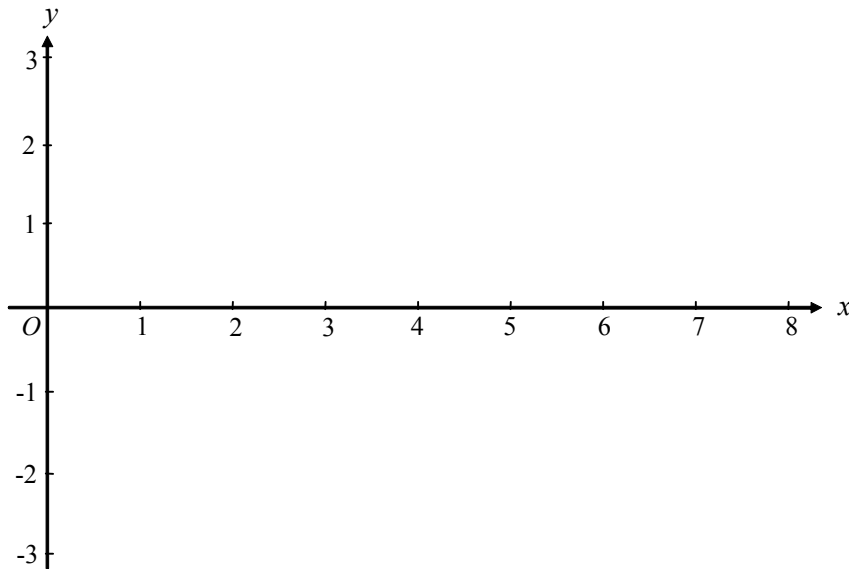
3 marks

Total 12 marks

Question 3

Consider the function $f : D \rightarrow R$ where $f(x) = 0.5 \operatorname{cosec}\left(\frac{\pi}{4}(2-x)\right)$

- a. i.** On the axes below, sketch a graph of f over the interval $[0, 8]$, labelling all features clearly.



2 marks

- ii.** Determine the domain and range of f over this interval.

2 marks

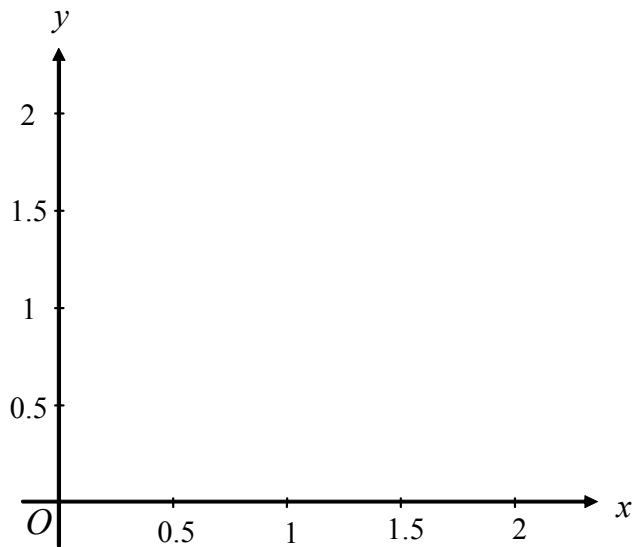
- b.** An equivalent rule for f is $f_1(x) = \frac{1}{a \cos(bx+c)}$ where $a, b, c \in R$

Give values for a , b , and c .

2 marks

- c. Let $D = [0, 2)$.

Sketch f and f^{-1} on the axes below, clearly showing the key features.



1 mark

- d. Write a definite integral that will give the area enclosed by f and f^{-1} . Using your graphics calculator, evaluate this integral correct to three decimal places.

3 marks

Total 10 marks

Question 4

A box of mass m kg is dropped from a hot air balloon. Its motion is retarded by a variable force of $\frac{mv}{5}$ newton, where v m/s is the velocity of the box t seconds after it is dropped.

- a. Taking vertically downwards as positive, show that the differential equation $\frac{dv}{dt} = \frac{5g - v}{5}$, where $g = 9.8 \text{ m/sec}^2$ is the acceleration due to gravity, applies to this situation.

2 marks

- b. Hence, show that $t = 5 \log_e \left(\frac{5g}{5g - v} \right)$

2 marks

- c. Show that at time t the velocity of the box is $5g(1 - e^{-0.2t})$ m/s.

2 marks

- d.** Write an expression for the limiting velocity of the box. Show how you deduced your result.

2 marks

- e.** Determine the time taken for the box to reach half its limiting velocity. Write your answer in seconds correct to two decimal places.

2 marks

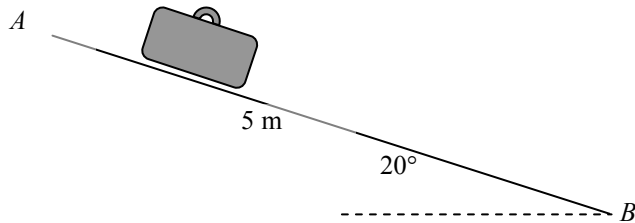
- f.** Find the distance travelled by the box in the first 10 seconds of motion. Write your answer correct to the nearest metre.

3 marks

Total 13 marks

Question 5

Baggage handlers use ramps to transport luggage. Ramp AB is 5 metres in length and inclined at an angle of 20° to the horizontal. A 20 kg suitcase, initially at rest at A , slides down ramp AB under the force of gravity. The coefficient of friction between the suitcase and the ramp is 0.2. Take $g = 9.8 \text{ m/sec}^2$.



- a. On the diagram above, draw all forces acting on the suitcase as it slides down the ramp.

1 mark

- b. Show that the suitcase slides down the ramp with an acceleration of 1.51 m/s^2 .

2 marks

- c. Find the time taken for the suitcase to reach point B . Write your answer in seconds correct to two decimal places.

2 marks

- d. Some time later, an identical 20 kg suitcase, initially at rest at A , is pushed down the ramp with a force of $100 - 200t$ newtons for the first 0.5 seconds of motion. Show that at time t , $0 < t < 0.5$, the acceleration of this suitcase is $6.51 - 10t \text{ m/s}^2$.

2 marks

- e. Find the speed of the suitcase when $t = 0.5$. Write your answer in m/s, correct to two decimal places.

2 marks

- f. Determine the speed of this suitcase when it reaches point B . Write your answer in m/s, correct to two decimal places.

3 marks

12 marks

END OF QUESTION AND ANSWER BOOK